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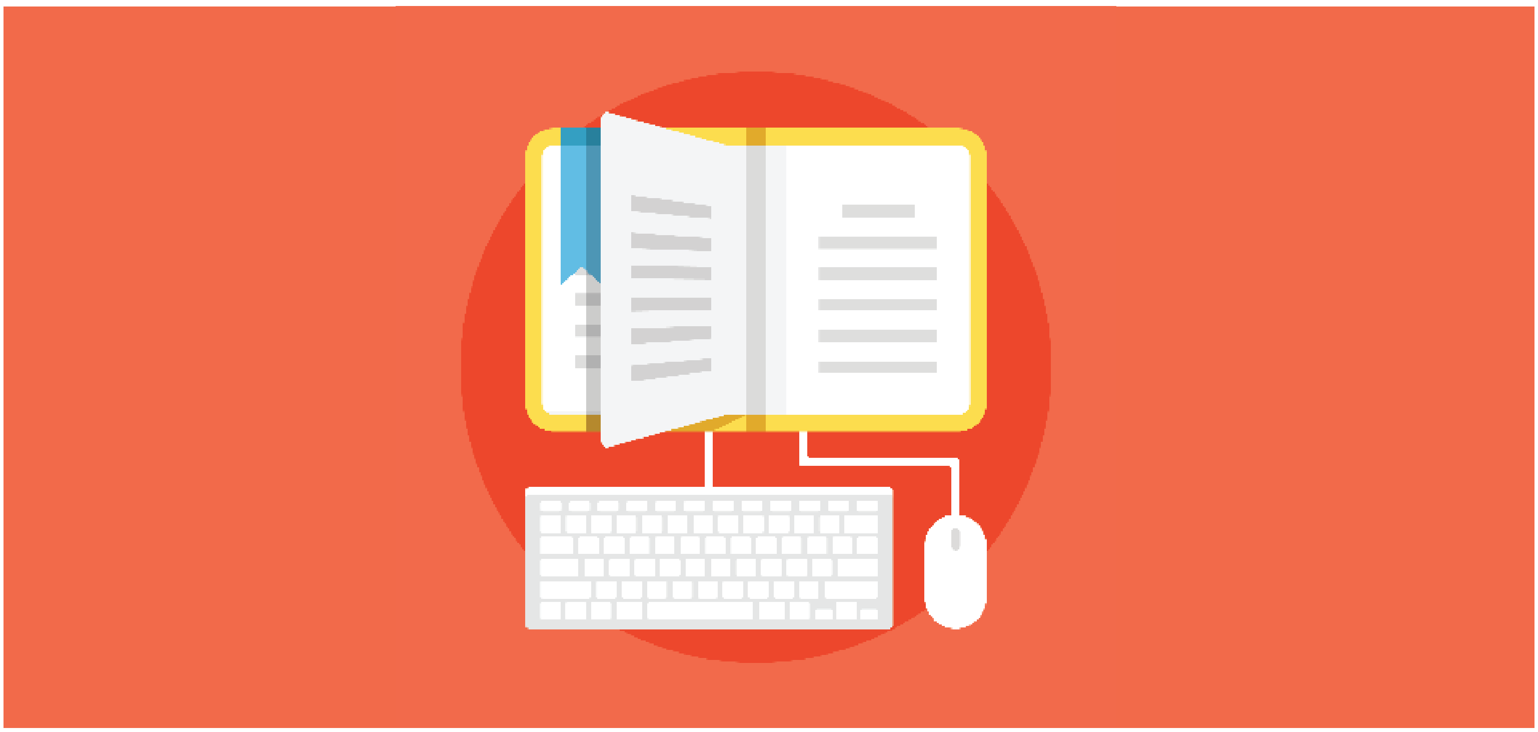

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Introduction

Welcome to Exam P! This manual will prepare you for the exam. You are given 30 bite-size chunks, each with explanations, techniques, examples, and lots of exercises. If you learn the techniques in these 30 lessons, you will be well-prepared for the exam.

A prerequisite to this exam is calculus. Heavy calculus techniques are not tested, but you must know the basics. A short review of integration techniques follows this introduction. If these look totally unfamiliar to you, you must review your calculus before proceeding further!

Exercises in this manual

There are several types of exercises in this manual:

1. Questions from the SOA 330.[1] These are probably the most representative of what will appear on your exam. The lower numbered ones come from the exams released between 2000 and 2003. The higher numbered ones are probably retired questions from the CBT (computer based testing) data bank, the question bank from which the questions on your CBT exam are taken from.

2. Questions from the five released exams 2000–2003 not contained in the SOA 330. Those exams tested both probability and calculus, so only about half the questions are relevant. Almost all of the relevant questions are in the SOA 330, but a small number of them are not in that list.

3. Questions from the 1999 Sample Exam. This sample exam was published before the first exam under the 2000 syllabus. The 2000 syllabus was a drastic syllabus change and was accompanied by a new exam question style. This sample exam reflects the new style of question, but most of the questions never appeared on a real exam, so the questions may not be totally realistic exam questions.

4. Questions from pre-2000 released exams. The probability syllabus was hardly different before 2000—it is extremely stable—but the style of questions was much different. Questions before 2000 tended to be stated purely mathematically, whereas starting in 2000 almost all questions have a practical sounding context. Thus a pre-2000 exam question might be:

    A and B are two events. You are given that $P[A] = 0.8$, $P[B] = 0.6$, and $P[A \cup B] = 0.9$.

    What is $P[A \cap B]$?

   The same question appearing on a 2000 or later exam would read:

    A survey of cable TV subscribers finds:
    - 80% of subscribers watch the Nature channel.
    - 60% of subscribers watch the History channel.
    - 90% of subscribers watch at least one of the Nature or History channels.

    Calculate the percentage of subscribers that watch both the Nature and the History channels.

   Notice, among other things, that the final line of the question is no longer a question; it is always a directive. Some of the post-2000 released exam questions still asked questions. When these questions were incorporated into the SOA 330, they changed the question to a directive. Questions are no longer used.

I was thinking of rewording all the pre-2000 questions in the current style. But I am not good at creating contexts. I'd probably just copy contexts from existing questions, and it would get boring. Even though the style of these old questions is different, they are based on the same material, so working them out will help you learn the material you need to know for this exam.

5. Original questions. I try to write my questions in the current style, but I often copy contexts from SOA 330 questions.

The solutions to exercises are not necessarily the best methods for solving them. Sometimes, a technique in a later lesson may provide a shortcut. This is particularly true for exercises in the earlier lessons using standard distributions as illustrations. For example, an exercise in an early lesson may ask for the mean of a conditional distribution involving an exponential; after learning in the exponential lesson that an exponential is memoryless, this sort of question may be trivial. *However, if you find a better way to do an exercise that does not involve techniques from later lessons, please contact the author at the same address as the one for sending errata.*

This manual has six practice exams. All the questions on these practice exams are original.

Useful features of this manual

As noted above, there is a very brief calculus note right after this introduction, for reviewing integration techniques.

There is an index at the end of the manual. If you ever remember a term and don't remember where you saw it, refer to the index. If it isn't in the index and you think it is in the manual, contact the author so that he can add it to the index.

Before the index, there are four cross reference tables. First, a cross reference for the practice exams. Some students prefer to do the practice exam questions as additional practice after each lesson, rather than saving them for final review, and this cross reference table lets you do that.

Second and third, tables showing you where each of the SOA 330 questions is listed as an exercise in the manual. If you are interested in seeing my solution to each of these questions, these tables are the place to go.

Fourth, a table showing you where each of the released exam questions from 2000–2003 is listed as an exercise in the manual.

SOA downloads

On the SOA website, you will find the syllabus for the exam. This syllabus has links to other useful material. You'll find links to the 5 released exams and the SOA 330 questions and answers.

There is an important link to *Risk and Insurance* study note, authored by Judy Feldman Anderson and Robert L. Brown. At this writing, the URL of the study note is `http://www.soa.org/files/pdf/P-21-05.pdf`. This note gives you background information on how insurance works. You will not be tested directly on this study note, but insurance is used very often as the context of a question, so you should understand basic insurance terminology.

This study note has some probability examples, so you won't understand it in its entirety if you have never studied probability. Still, you should read the nonmathematical parts (in bed, or at some other leisure time) as soon as you can, ignoring the mathematical examples, since the exercises in this manual often have insurance contexts. At the point indicated in the manual in Lesson 8, you will have all the probability background you need to understand the study note's mathematics, and should read the study note in its entirety.

Another important link is to the table of the cumulative distribution function of the standard normal distribution. At this writing, it is at `http://www.soa.org/files/pdf/P-05-05tables.pdf`. You do not need this table until Lesson 23, but you should download it by then. For your convenience, a standard normal distribution table is provided in Appendix B.

Errata

Please report any errors you find. Reports may be sent to the publisher (`mail@studymanuals.com`) or directly to me (`errata@aceyourexams.net`).

An errata list will be posted at `http://errata.aceyourexams.net`

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Calculus Notes

The exam will not test deep calculus. You will be expected to know how to differentiate and integrate polynomials, simple rational functions, logarithms, and exponentials, but trigonometric functions and integrals will rarely appear. Fancy integration techniques are not needed.

We will review logarithmic differentiation and basic integration techniques.

Logarithmic differentiation When a function $f(x)$ is a product or quotient of several functions, and you need to evaluate its derivative at a point, it is often easier to use logarithmic differentiation rather than to differentiate the function directly. Logarithmic differentiation is based on the formula for the derivative of a logarithm:

$$\frac{\mathrm{d}\ln f(x)}{\mathrm{d}x} = \frac{\mathrm{d}f(x)/\mathrm{d}x}{f(x)}$$

It follows that

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = f(x)\left(\frac{\mathrm{d}\ln f(x)}{\mathrm{d}x}\right)$$

In other words, the derivative of a function is the function times the derivative of its logarithm. If the function is a product or quotient of functions, its derivative will be a sum of difference of logarithms of those functions, and that will be easier to differentiate.

Partial fraction decomposition You should know that $x/(1+x) = 1 - 1/(1+x)$, for example, so that

$$\int_0^3 \frac{x\,\mathrm{d}x}{1+x} = \int_0^3 \left(1 - \frac{1}{1+x}\right)\mathrm{d}x = 3 - \ln(1+x)\Big|_0^3 = 3 - \ln 4$$

Any partial fraction decomposition more complicated is unlikely to appear.

This particular integral can also be evaluated using substitution, as we will now discuss.

Substitution There are two types of substitution.

The first type assists you to identify an antiderivative. Suppose you have

$$\int_0^3 xe^{-x^2/2}\,\mathrm{d}x$$

You may realize that the integrand is negative the derivative of $e^{-x^2/2}$. When you differentiate $e^{-x^2/2}$, by the chain rule, you multiply by the derivative of $-x^2/2$, which is $-x$, and obtain $-xe^{-x^2/2}$. So the integral is $-e^{-x^2/2}$ evaluated at 3 minus the same evaluated at 0. But if you didn't recognize this integrand, you could substitute $y = -x^2/2$ and get:

$$\begin{aligned}
\mathrm{d}y &= -x\,\mathrm{d}x \\
x = 0 &\Rightarrow y = 0 \\
x = 3 &\Rightarrow y = -4.5 \\
\int_0^3 xe^{-x^2/2}\,\mathrm{d}x &= \int_0^{-4.5} -e^y\,\mathrm{d}y \\
&= -e^y\Big|_0^{-4.5} \\
&= 1 - e^{-4.5}
\end{aligned}$$

The second type simplifies an integral or makes it doable. We evaluated $\int_0^3 \frac{x\,\mathrm{d}x}{1+x}$ above using partial fraction decomposition. An alternative would be to set $y = 1 + x$, $\mathrm{d}y = \mathrm{d}x$. The bounds of the integral become 1 and 4, since $0 + 1 = 1$ and $3 + 1 = 4$, and we get

$$\begin{aligned}
\int_0^3 \frac{x\,\mathrm{d}x}{1+x} &= \int_1^4 \frac{(y-1)\,\mathrm{d}y}{y} \\
&= \int_1^4 \left(1 - \frac{1}{y}\right) \mathrm{d}y \\
&= \left(y - \ln y\right)\Big|_1^4 \\
&= (4-1) + (\ln 4 - \ln 1) = 3 - \ln 4
\end{aligned}$$

As another example, suppose you have

$$\int_0^1 x(1-x)^8 \mathrm{d}x$$

You could expand $(1-x)^8$, multiply it by x, and integrate 9 terms. But it is easier to substitute $y = 1 - x$ and get

$$\begin{aligned}
\mathrm{d}y &= -\mathrm{d}x \\
x = 0 &\Rightarrow y = 1 \\
x = 1 &\Rightarrow y = 0
\end{aligned}$$

$$\begin{aligned}
\int_0^1 x(1-x)^8 \mathrm{d}x &= \int_1^0 -(1-y)y^8\,\mathrm{d}y \\
&= \int_0^1 (y^8 - y^9)\mathrm{d}y \\
&= \frac{1}{9} - \frac{1}{10} = \frac{1}{90}
\end{aligned}$$

Another technique to evaluate this integral is integration by parts, differentiating x and integrating $(1-x)^8$.

Integration by parts When integrating by parts, we use $\int u\,\mathrm{d}v = uv - \int v\,\mathrm{d}u$. We integrate one expression and evaluate its product with the other, then differentiate the other and evaluate the integral with the derivative and the integrated expression.

If you are given

$$\int_0^1 x(1-x)^8 \mathrm{d}x$$

set $u = x$ and $\mathrm{d}v = (1-x)^8\,\mathrm{d}x$ and you get

$$\begin{aligned}
\int_0^1 x(1-x)^8 \mathrm{d}x &= -\frac{x(1-x)^9}{9}\Big|_0^1 + \int_0^1 \frac{(1-x)^9}{9}\mathrm{d}x \\
&= -\left.\frac{(1-x)^{10}}{90}\right|_0^1 = \frac{1}{90}
\end{aligned}$$

A technique that is helpful, especially if integration by parts has to be repeated, is tabular integration. Set up a table. Each row has two columns. The first row has the two expressions, the one you want to differentiate (left column) and the one you want to integrate (right column). On each successive row, differentiate the left column entry from the previous row and integrate the right column entry from the previous row. Keep doing this until the left column entry is 0. Then the antiderivative is the alternating sum of the products of the k^{th} entry of the left column and the $k+1^{\text{st}}$ entry of the second column, $k = 1, 2, \ldots$ up to but not including the final 0 in the left column. The sign of each summand is $(-1)^{k-1}$.

For example, suppose you want to evaluate

$$\int_0^3 x^2 e^{x/5} \mathrm{d}x$$

The table looks like this:

$$\begin{array}{ccc}
+ & x^2 & e^{x/5} \\
- & 2x & 5e^{x/5} \\
+ & 2 & 25e^{x/5} \\
 & 0 & 125e^{x/5}
\end{array}$$

The entries connected with a line are multiplied, and added or subtracted as indicated by the sign. The result is

$$\int_0^3 x^2 e^{x/5} \mathrm{d}x = \left(5x^2 e^{x/5} - 50x e^{x/5} + 250 e^{x/5}\right)\Big|_0^3 = 14.20723$$

We often integrate xe^{-ax}. If you wish, you may memorize the following results, especially the first one, so that you don't have to integrate by parts each time you need them:

$$\int_0^\infty x e^{-ax}\,\mathrm{d}x = \frac{1}{a^2} \quad \text{for } a > 0 \tag{1}$$

$$\int_0^\infty x^2 e^{-ax}\,\mathrm{d}x = \frac{2}{a^3} \quad \text{for } a > 0 \tag{2}$$

Lesson 2

Combinatorics

In this lesson, we will consider a probability model in which the sample space consists of a set of n mutually exclusive events each having equal probability. When describing this space, we will say that one of the events happens "at random". Whenever we say one of several events happens at random and do not specify probabilities, we mean that each of the events has an equal probability of occurring.

Examples of this probability model are

1. An urn contains n balls of several colors. If we say that a ball is drawn at random, it means that the probability of drawing that ball is $1/n$.
2. You are given a standard deck of 52 cards. A card is drawn at random. That means that the probability of drawing a specific card is $1/52$.
3. A used car lot has 50 cars. Four of them are lemons. A car is bought at random. That means that the probability of buying a specific car is $1/50$.
4. Twenty people are invited to a party. People arrive at random. That means, among other things, that each person has a $1/20$ probability of arriving first.

Older exam questions from the pre-2000 era frequently use urn models. Newer exam questions are framed in more practical terms, like the third and fourth models.

A combinatorics question involves asking for the probability of an event in these models. Examples from each of the four models above are

1. An urn contains 4 blue balls and 5 white balls. Three balls are drawn at random without replacement. Calculate the probability of selecting at least one ball of each color.
2. Five cards are drawn at random from a standard deck of cards. Calculate the probability of drawing two pairs of cards, each pair having two cards the same denomination, with the two denominations not equal.
3. Three cars are sold. Calculate the probability that one of the cars is a lemon.
4. Eleven of the people are men and nine are women. Calculate the probability that of the first three people to arrive, exactly two are women.[1]

To work out these questions, we take advantage of the fact that all events are equally likely. Here is how we calculate the probability.

1. We count up the total number of different events that can occur. Call that A.
2. We count up the number of events of interest. Call that B.
3. The probability is B/A.

To carry out this program, we have to count events. Two concepts are useful for this purpose: *permutations* and *combinations*.

Permutations are different orderings of a set. For example, for the set {1,2,3}, there are six permutations: {1,2,3}, {1,3,2}, {2,1,3}, {2,3,1}, {3,1,2}, {3,2,1}. For a set of n elements, there are n ways to select the first element. Then there are $n-1$ elements left. There are $n-1$ ways to select the second element. There are $n-2$ ways to select the third element. And so on. We conclude that there are $n(n-1)(n-2)\cdots 1 = n!$ permutations of n elements.

[1]In combinatorics questions, the word "exactly" is understood even if it is not stated. If we want the probability of 2 or 3 women, we ask for the probability of at least 2 women.

Example 2A You are given the letters A, C, E, and F. The letters are written in random order. How many different 4-letter sequences can be produced?

Solution: There are $4! = \boxed{\textbf{24}}$ permutations of four letters. □

Example 2B You are given the letters A, C, E, and F. You write them in random order.

Calculate the probability that you wrote a proper English word.

Solution: Only two English words may be formed, FACE and CAFE. As we discussed above, the probability is the number of events of interest (2) divided by the total number of events that may occur (24), or $\boxed{\textbf{1/12}}$. □

Sometimes the set has duplicate elements. We may be interested in distinct permutations. For example, if the set is {A, B, C, C}, we may not want to count the permutation {A,B,C,C} with the two Cs interchanged as a distinct permutation. For each distinct series of letters, there are two possible orders of the two Cs. If we wish to not count these as different permutations, we must divide the total number of permutations, 4!, by 2 to obtain the number of distinct permutations, which is 12. More generally, if the set has n elements with k_1 elements of type 1, k_2 elements of type 2, ..., k_j elements of type j, then the number of distinct permutations is

$$\frac{n!}{k_1!k_2!\cdots k_j!}$$

Example 2C Consider the set {1,1,2,2,2,3,4}. How many distinct permutations does this set have?

Solution: This set has 7 elements with two 1s and three 2s. The number of permutations is

$$\frac{7!}{2!3!1!1!} = \boxed{\textbf{420}}$$ □

Combinatorics often deals with random samples. A *random sample* is a selection of n elements from a set. This selection may be with replacement or without replacement. For example, if cards are drawn from a standard deck of cards, a card may be drawn, put back in the deck, then another card drawn, put back in the deck, and so on. In that case, selection is with replacement. If the cards are not put back in the deck after being drawn, that is selection without replacement. The vast majority of combinatorics questions are selection without replacement. Counting permutations is useful for either type of question. Let's illustrate its use for selection with replacement.

Example 2D An actuarial financial reporting department consists of 2 fellows, 3 associates, and 4 students. Each month, the chief actuary of the company takes one member of this department, selected at random, out for lunch.

Calculate the probability that over a period of 3 months, the chief actuary has lunch with a fellow, an associate, and a student.

Solution: This is selection with replacement; a member of the department may go out for lunch more than once.

The total number of possible outcomes is $9^3 = 729$.

The number of outcomes with three different classes are $(2)(3)(4) = 24$. However, there are 3! orders in which these outcomes may occur: for example, fellow may be first, or associate, or student. So there are $24(6) = 144$ outcomes of interest, and the probability is $144/729 = \boxed{\textbf{0.197531}}$. □

Combinations are the number of ways to select k elements from a set of n elements. When counting combinations, the order of the elements doesn't matter; we only want to know the number of distinct subsets of size k from a set of size n. For example, suppose the set is {1,2,3,4,5}, and we want to calculate the number of distinct subsets of size 2. They are {1,2}, {1,3}, {1,4}, {1,5}, {2,3}, {2,4}, {2,5}, {3,4}, {3,5}, {4,5}. There are a total of 10 combinations. More generally, the number of combinations of size k from a set of size n can be calculated as follows. There are n ways to select the first element, $n-1$ ways to select the second element, and so on, down to $n-k+1$ ways to select the k^{th} element. However, we are ignoring order. Each combination occurs in $k!$ different orders (the number of permutations of k elements). So the number of combinations of size k from a set of size n is

$$\frac{n(n-1)\cdots(n-k+1)}{k!} = \frac{n!}{k!(n-k)!}$$

where the second expression is derived from the first by multiplying numerator and denominator by $(n-k)!$. The second expression is called a *binomial coefficient*, and is denoted by $\binom{n}{k}$.

Example 2E An actuarial pricing department has six employees. Three of them are selected to work on a new product.

Determine the number of possible teams.

Solution:

$$\binom{6}{3} = \frac{6!}{3!3!} = \frac{720}{6^2} = \boxed{\mathbf{20}}$$

□

Counting combinations is very useful for solving combinatorics questions involving selection without replacement. A typical question involves n elements of j different types, n_i of type i. The question asks for the probability that a draw of k elements contains k_i of type i, $i = 1, 2, \ldots, j$, where $\sum k_i = k$. To solve this question, the total number of ways of selecting k elements from n elements is $\binom{n}{k}$. The number of ways to select k_i elements of type i from n_i elements, $i = 1, 2, \ldots, j$ is $\prod_{i=1}^{j}\binom{n_i}{k_i}$. The probability is the latter divided by the former.

Example 2F A used car lot has 50 cars. Four of them are lemons.

Eight customers each buy one car.

Calculate the probability that exactly two customers bought lemons.

Solution: The total number of outcomes is $\binom{50}{8}$. The number of outcomes of interest are the ones for which 6 customers bought one of the 46 good cars and 2 bought one of the lemons. That number is $\binom{46}{6}\binom{4}{2}$.

$$\frac{\binom{46}{6}\binom{4}{2}}{\binom{50}{8}}$$

To save calculations and avoid overflow, you should cancel factors to the extent possible. For example, $50!/46! = (50)(49)(48)(47)$.

$$\frac{\frac{46!}{40!6!}(6)}{\frac{50!}{42!8!}} = \frac{46!42!8!6}{40!6!50!} = \frac{(42)(41)(8)(7)(6)}{(50)(49)(48)(47)} = \boxed{\mathbf{0.104681}}$$

□

In these examples, you must be careful to account for all of the items you are selecting. In the previous example, where there were 8 customers, the numerators of the binomial coefficients in the numerator of the fraction must add up to 50 and the denominators must add up to 8. Avoid the mistake of only putting $\binom{4}{2}$ in the numerator and forgetting the $\binom{46}{6}$ factor.

What if you don't care what the other 6 customers got? In other words, what if you wanted the probability that at least 2 lemons were bought? In that case, you'd have to sum up the probabilities of 2, 3, or 4 lemons. Alternatively, you can sum up the probabilities of 0 or 1 lemons and subtract that probability from 1.

Binomial coefficients have many nice properties. One property that you will find helpful is that they are symmetric:

$$\binom{n}{k} = \binom{n}{n-k}$$

Thus instead of calculating $\binom{100}{98}$, you can calculate $\binom{100}{2}$. It follows that for odd n,

$$\sum_{i=0}^{(n-1)/2}\binom{n}{i} = \sum_{i=(n+1)/2}^{n}\binom{n}{i}$$

For your amusement, the classical mismatched hat problem is presented in a sidebar. This is a problem solved by combinatorics and applying the inclusion-exclusion principle in all its generality. Nothing as hard as this problem is going to appear on an exam, so you may skip it; it is only included for your amusement.

Probability of n Mismatched Hats

Example 2G n people come to a party, each with a hat. On the way out, they each take a hat at random. Calculate the probability that none of them get their own hat.

Solution: We will calculate the probability that at least one of them gets their own hat, and then we'll take the complement. Let A_i be the event that person i gets their own hat. Then

$$P[\cup A_i] = \sum P[A_i] - \sum P[A_i \cap A_j] + \sum P[A_i \cap A_j \cap A_k] - \cdots$$

Consider the m^{th} term on the right, the sum with the intersection of m events. It has $\binom{n}{m}$ summands, since there are $\binom{n}{m}$ ways to select m A_is from all n. For each of these $\binom{n}{m}$ summands, there is only one way for all m to get their own hats, and $(n-m)!$ ways for the other $n-m$ people to get their hats, so the summand, the probability of m specific people getting their own hats, equals $(n-m)!/n!$. Therefore, the m^{th} term on the right, the sum of the probabilities of all intersections with m events, equals

$$\binom{n}{m}\frac{(n-m)!}{n!} = \frac{n!}{(n-m)!m!}\left(\frac{(n-m)!}{n!}\right) = \frac{1}{m!}$$

Summing up all the terms, the probability that at least one gets their own hat is

$$P[\cup A_i] = \sum_{i=1}^{n} \frac{(-1)^{n-1}}{m!}$$

and the probability that none gets their own hat is

$$1 - \sum_{i=1}^{n} \frac{(-1)^{n-1}}{m!}$$

This approaches $1 - 1/e$ as $n \to \infty$. □

Table 2.1: Summary of combinatorics concepts

- The number of permutations of n elements is $n!$.
- If there are n_i elements of type i, $i = 1, \ldots, j$ among $n = \sum_{i=1}^{j} n_i$ elements, the number of distinct permutations is
$$\frac{n!}{\prod_{i=1}^{j} n_i!}$$
- The number of combinations of k out of n elements is
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$
- When k elements are drawn from a set of n elements of j distinct types, with n_i elements of type i and $\sum_{i=1}^{j} n_i = n$, the probability of selecting k_i elements of type i, $i = 1, \ldots, j$, with $k = \sum_{i=1}^{j} k_i$ is
$$\frac{\prod_{i=1}^{j}\binom{n_i}{k_i}}{\binom{n}{k}}$$

Exercises

2.1. [**110-S83:1**] Suppose a box contains 4 blue, 5 white, 6 red, and 7 green balls. In how many of all possible samples of size 5, chosen without replacement. will every color be represented?

(A) 1,260 (B) 1,680 (C) 2,520 (D) 7,560 (E) 15,120

2.2. [**110-S85:1**] In a particular softball league each team consists of 5 women and 5 men. In determining a batting order for the 10 players, a woman must bat first, and successive batters must be of opposite sex. How many different batting orders are possible for a team?

(A) 5! (B) 2(5!) (C) $(5!)^2$ (D) $\frac{10!}{2}$ (E) 10!

2.3. A program has five speakers. You are determining the order in which they speak. Speaker #2 should speak before, but not necessarily immediately before, Speaker #5.

Determine the number of different orders for the speakers.

2.4. [**Sample:141**] Thirty items are arranged in a 6-by-5 array as shown.

A_1	A_2	A_3	A_4	A_5
A_6	A_7	A_8	A_9	A_{10}
A_{11}	A_{12}	A_{13}	A_{14}	A_{15}
A_{16}	A_{17}	A_{18}	A_{19}	A_{20}
A_{21}	A_{22}	A_{23}	A_{24}	A_{25}
A_{26}	A_{27}	A_{28}	A_{29}	A_{30}

Calculate the number of ways to form a set of three distinct items such that no two of the selected items are in the same row or same column.

(A) 200 (B) 760 (C) 1200 (D) 4560 (E) 7200

2.5. Calculate the probability that a selection of five people from 7 men and 3 women will have 3 men and 2 women.

2.6. [**Sample:247**] Each week, a subcommittee of four individuals is formed from among the members of a committee comprising seven individuals. Two subcommittee members are then assigned to lead the subcommittee, one as chair and the other as secretary.

Calculate the maximum number of consecutive weeks that can elapse without having the subcommittee contain four individuals who have previously served together with the same subcommittee chair.

(A) 70 (B) 140 (C) 210 (D) 420 (E) 840

2.7. [**110-S83:39**] A box contains 10 white marbles and 15 black marbles. If 10 marbles are selected at random and without replacement, what is the probability that x of the 10 marbles are white for $x = 0, 1, \ldots, 10$?

(A) $\frac{x}{10}$ (B) $\binom{10}{x}\left(\frac{2}{5}\right)^x\left(\frac{3}{5}\right)^{10-x}$ (C) $\frac{\binom{10}{x}\binom{15}{10-x}}{\binom{25}{10}}$ (D) $\frac{\binom{10}{x}}{\binom{25}{10}}$ (E) $\frac{\binom{10}{x}}{\binom{25}{x}}$

2.8. [**110-S85:35**] A bin of 10 light bulbs contains 4 that are defective. If 3 bulbs are chosen without replacement from the bin, what is the probability that exactly k of the bulbs in the sample are defective?

(A) $\frac{\binom{4}{k}\binom{6}{3-k}}{\binom{10}{3}}$ (B) $\binom{3}{k}\left(\frac{4}{10}\right)^k\left(\frac{6}{10}\right)^k$ (C) $\left(\frac{4}{10}\right)^k\left(\frac{6}{10}\right)^{3-k}$ (D) $\frac{\binom{4}{k}}{\binom{10}{4}}$ (E) $\frac{\binom{6}{3-k}}{\binom{10}{3}}$

Exercises continue on the next page . . .

2.9. **[110-S88:16]** What is the probability that a hand of 5 cards chosen randomly and without replacement from a standard deck of 52 cards contains the king of spades, exactly 1 other king, and exactly 2 queens?

(A) $\dfrac{\binom{4}{2}\binom{4}{2}\binom{44}{1}}{\binom{52}{5}}$ (B) $\dfrac{\binom{3}{1}\binom{4}{2}\binom{44}{1}}{\binom{52}{5}}$ (C) $\dfrac{\binom{7}{3}\binom{44}{1}}{\binom{52}{5}}$ (D) $\dfrac{\binom{3}{1}\binom{4}{2}\binom{11}{1}}{\binom{52}{5}}$ (E) $\dfrac{\binom{3}{1}\binom{4}{2}}{\binom{52}{5}}$

2.10. **[110-S85:3]** An urn contains 3 red balls, 2 green balls, and 1 yellow ball. Three balls are selected at random and without replacement from the urn. What is the probability that at least 1 color is NOT drawn?

(A) $\frac{2}{5}$ (B) $\frac{7}{10}$ (C) $\frac{5}{6}$ (D) $\frac{11}{12}$ (E) $\frac{19}{20}$

2.11. **[110-S85:24]** An urn contains 4 balls numbered 0 through 3. One ball is selected at random and removed from the urn and not replaced. All balls with nonzero numbers less than that of the selected ball are also removed from the urn. Then a second ball is selected at random from those remaining in the urn. What is the probability that the second ball selected is numbered 3?

(A) $\frac{1}{4}$ (B) $\frac{7}{24}$ (C) $\frac{1}{3}$ (D) $\frac{11}{24}$ (E) $\frac{13}{24}$

2.12. **[110-S88:7]** An urn contains 10 balls: 5 are white, 3 are red, and 2 are black. Three balls are drawn at random, with replacement, from the urn. What is the probability that all 3 balls are different colors?

(A) 0.03 (B) 0.09 (C) 0.18 (D) 0.40 (E) 0.84

2.13. **[110-S92:8]** There are 97 men and 3 women in an organization. A committee of 5 people is chosen at random, and one of these 5 is randomly designated as chairperson. What is the probability that the committee includes all 3 women and has one of the women as chairperson?

(A) $\dfrac{3(4!97!)}{2(100!)}$ (B) $\dfrac{5!97!}{2(100!)}$ (C) $\dfrac{3(5!97!)}{2(100!)}$ (D) $\dfrac{3!5!97!}{100!}$ (E) $\dfrac{3^3 97^2}{100^5}$

2.14. **[110-S92:34]** If four dice are rolled, what is the probability of obtaining two identical odd numbers and two identical even numbers?

(A) $\frac{1}{144}$ (B) $\frac{1}{72}$ (C) $\frac{1}{36}$ (D) $\frac{1}{24}$ (E) $\frac{1}{6}$

2.15. **[110-W96:7]** A class contains 8 boys and 7 girls. The teacher selects 3 of the children at random and without replacement.

Calculate the probability that the number of boys selected exceeds the number of girls selected.

(A) $\frac{512}{3375}$ (B) $\frac{28}{65}$ (C) $\frac{8}{15}$ (D) $\frac{1856}{3375}$ (E) $\frac{36}{65}$

2.16. In order to test its administrative system, an insurance company selects 5 policies from its portfolio of 100 policies. Of the 100 policies, 70 are in the preferred underwriting class, 25 are in the standard underwriting class, and 5 are in the rated underwriting class.

Calculate the probability that the 5 selected policies include at least one policy from each rating class.

2.17. **[Sample:132]** A store has 80 modems in its inventory, 30 coming from Source A and the remainder from Source B. Of the modems from Source A, 20% are defective. Of the modems from Source B, 8% are defective.

Calculate the probability that exactly two out of a random sample of five modems from the store's inventory are defective.

(A) 0.010 (B) 0.078 (C) 0.102 (D) 0.105 (E) 0.125

Exercises continue on the next page . . .

2.18. **[Sample:151]** From 27 pieces of luggage, an airline luggage handler damages a random sample of four.

The probability that exactly one of the damaged pieces of luggage is insured is twice the probability that none of the damaged pieces are insured.

Calculate the probability that exactly two of the four damaged pieces are insured.

(A) 0.06 (B) 0.13 (C) 0.27 (D) 0.30 (E) 0.31

2.19. **[Sample:177]** In a group of 25 factory workers, 20 are low-risk and five are high-risk.

Two of the 25 factory workers are randomly selected without replacement.

Calculate the probability that exactly one of the two selected factory workers is low-risk.

(A) 0.160 (B) 0.167 (C) 0.320 (D) 0.333 (E) 0.633

2.20. **[Sample:210]** On a block of ten houses, k are not insured. A tornado randomly damages three houses on the block.

The probability that none of the damaged houses are insured is $1/120$.

Calculate the probability that at most one of the damaged houses is insured.

(A) $1/5$ (B) $7/40$ (C) $11/60$ (D) $49/60$ (E) $119/120$

2.21. On a block of n houses, 4 are not insured. A tornado randomly damages three houses on the block.

The probability that none of the damaged houses is insured is $1/55$.

Calculate the probability that all of the damaged houses are insured.

2.22. **[Sample:211]** In a casino game, a gambler selects four different numbers from the first twelve positive integers. The casino then randomly draws nine numbers without replacement from the first twelve positive integers. The gambler wins the jackpot if the casino draws all four of the gambler's selected numbers.

Calculate the probability that the gambler wins the jackpot.

(A) 0.002 (B) 0.255 (C) 0.296 (D) 0.573 (E) 0.625

2.23. **[Sample:170]** An insurance agent meets twelve potential customers independently, each of whom is equally likely to purchase an insurance product. Six are interested only in auto insurance, four are interested only in homeowners insurance, and two are interested only in life insurance.

The agent makes six sales.

Calculate the probability that two are for auto insurance, two are for homeowners insurance, and two are for life insurance.

(A) 0.001 (B) 0.024 (C) 0.069 (D) 0.097 (E) 0.500

2.24. **[Sample:258]** Six claims are to be randomly selected from a group of thirteen different claims, which includes two workers compensation claims, four homeowners claims and seven auto claims.

Calculate the probability that the six claims selected will include one workers compensation claim, two homeowners claims and three auto claims.

(A) 0.025 (B) 0.107 (C) 0.153 (D) 0.245 (E) 0.643

2.25. **[Sample:259]** A drawer contains four pairs of socks, with each pair a different color. One sock at a time is randomly drawn from the drawer until a matching pair is obtained.

Calculate the probability that the maximum number of draws is required.

(A) 0.0006 (B) 0.0095 (C) 0.0417 (D) 0.1429 (E) 0.2286

Solutions

2.1. We need to pick 1 apiece from 3 colors and 2 from the fourth color. The number of ways to do this is

$$\begin{aligned}&\binom{4}{2}\binom{5}{1}\binom{6}{1}\binom{7}{1}+\binom{4}{1}\binom{5}{2}\binom{6}{1}\binom{7}{1}+\binom{4}{1}\binom{5}{1}\binom{6}{2}\binom{7}{1}+\binom{4}{1}\binom{5}{1}\binom{6}{1}\binom{7}{2}\\ &=(6)(5)(6)(7)+(4)(10)(6)(7)+(4)(5)(15)(7)+(4)(5)(6)(21)=\boxed{7{,}560}\qquad\textbf{(D)}\end{aligned}$$

2.2. There are 5! orders for the men and 5! orders for the women, and the orders for men are independent of the orders for women, so there are $\boxed{(5!)^2}$ orders. **(C)**

2.3. Consider the positions of Speakers #2 and #5 in the 5! = 120 orders. Half of them have #2 before #5. So there are $\boxed{60}$ different orders satisfying the constraint of Speaker #2 before Speaker #5.

2.4. First select 3 out of 5 columns. There are $\binom{5}{3}=10$ ways to do this. Then select one element from the first column (6 ways), one from the second column not on the same row (5 ways), and one from the third column not on the same row as either of the first two (4 ways). Total number of ways is $10(6)(5)(4)=\boxed{1200}$. **(C)**

2.5. There are $\binom{10}{5}$ ways to select five from ten, and of these, $\binom{7}{3}\binom{3}{2}$ have 3 men and 2 women.

$$\frac{\binom{7}{3}\binom{3}{2}}{\binom{10}{5}}=\frac{7!3!5!5!}{4!3!2!1!10!}=\frac{5\cdot 120}{(10)(9)(8)(2)}=\frac{5}{12}=\boxed{0.416667}$$

2.6. There are $\binom{7}{4}=35$ ways to pick 4 out of 7. And there are 4 different individuals who can be chair. So the number of possible distinct picks of 4 and unique chairs is $(35)(4)=\boxed{140}$. **(B)**

2.7. We will select x out of 10 and $10-x$ out of 15, so the number of ways to select x white marbles is

$$\binom{10}{x}\binom{15}{10-x}$$

and the total number of ways of selecting 10 marbles is

$$\binom{25}{10}$$

so the probability is the former expression over the latter. **(C)**

2.8. We choose k from 4 and $3-k$ from 6, and there are a total of $\binom{10}{3}$ ways of choosing 3 bulbs from 10. So the probability of k defective is

$$\frac{\binom{4}{k}\binom{6}{3-k}}{\binom{10}{3}}\qquad\textbf{(A)}$$

2.9. There is only one way to select the king of spades. There are $\binom{3}{1}$ ways to select a king from the other three kings. There are $\binom{4}{2}$ ways to select two queens from four. There are 44 cards that are not kings and queens, and we need to select one card from them; there are $\binom{44}{1}$ ways to do that. So the total number of ways to get these 5 cards is $\binom{3}{1}\binom{4}{2}\binom{44}{1}$, out of $\binom{52}{5}$ ways to select 5 cards. The answer is **(B)**

2.10. Let's calculate the probability that all 3 colors are drawn. That means 1 red, 1 green, and 1 yellow. There are 3 ways to draw a red, 2 ways to draw a green, and 1 way to draw a yellow: a total of $3\times 2\times 1=6$ ways to draw one of each color. Total number of ways to draw 3 balls is $\binom{6}{3}=20$. So there are 14 ways to miss a color. The probability of missing a color is $\frac{14}{20}=\boxed{\frac{7}{10}}$. **(B)**

2.11. If 3 is removed on the first draw, then the second ball won't be 3.
If 2 is removed, the probability of 3 is $1/2$, since only 0 and 3 will be left in the urn.
If 1 is removed, the probability of 3 is $1/3$, since 0, 2, and 3 will be left in the urn.
If 0 is removed, the probability of 3 is $1/3$, since 1, 2, and 3 will be left in the urn.
The probability of 3 is $\frac{1}{4}(0 + 1/2 + 1/3 + 1/3) = \frac{7}{24}$. **(B)**

2.12. The probability of picking a white is 0.5; the probability of red is 0.3; and the probability of black is 0.2. So the probability of picking one apiece is $(0.5)(0.3)(0.2) = 0.03$. There are 6 different orders to pick them in, so the probability of picking 3 different colors is $6(0.03) = \boxed{0.18}$. **(C)**

2.13. There are $\binom{100}{5}$ ways to select 5 from 100. Of these, we want to choose 3 from 3 women and 2 from 97 men. The probability of choosing all 3 women is

$$\frac{\binom{97}{2}}{\binom{100}{5}} = \frac{97!5!}{100!2!}$$

We need to multiply this by $3/5$ for the probability that one of the 3 women is chairperson. The result is

$$\frac{3(97!4!)}{100!2!}$$

which is **(A)**

2.14. The probability of two 1s, or two 2s, etc., is $1/36$. We want two 1s or two 3s or two 5s, and also two 2s or two 4s or two 6s. The probability of two odds is $3(1/36) = 1/12$, and the probability of two evens is $1/12$. There are $\binom{4}{2} = 6$ different pairs of dice on which the two odds can be tossed; once a pair of dice for the odd ones is chosen, the pair for the evens is also determined. So the probability we seek is $6(1/12)^2 = \boxed{1/24}$. **(D)**

Another way to calculate the probability is: there are 6^4 possible rolls. There are 3 ways to pick the odd number, 3 ways to pick the even number, and $\binom{4}{2} = 6$ ways to select the two rolls that are even (the other two are odd), or a total of $(3)(3)(6) = 54$ rolls of interest. Then the probability we seek is $54/6^4 = \boxed{1/24}$.

2.15. The probability of 3 boys is

$$\frac{\binom{8}{3}}{\binom{15}{3}} = \frac{8!/5!3!}{15!/12!3!} = \frac{8!12!}{15!5!} = \frac{8 \cdot 7 \cdot 6}{15 \cdot 14 \cdot 13} = \frac{8}{65}$$

The probability of 2 boys is

$$\frac{\binom{8}{2}\binom{7}{1}}{\binom{15}{3}} = \frac{8 \cdot 7 \cdot 7 \cdot 6/2}{15 \cdot 14 \cdot 13} = \frac{28}{65}$$

The sum of the two probabilities is $\boxed{36/65}$. **(E)**

2.16. It is easier to calculate the probability that not all three classes are included. There are $\binom{100}{5} = 75{,}287{,}520$ combinations. Of these, $\binom{70}{0}\binom{30}{5} = 142{,}506$ do not have preferred, $\binom{25}{0}\binom{75}{5} = 17{,}259{,}390$ do not have standard, and $\binom{5}{0}\binom{95}{5} = 57{,}940{,}519$ do not have rated. From this sum, we must subtract samples missing two underwriting classes, which are double-counted: $\binom{95}{0}\binom{5}{5} + \binom{75}{0}\binom{25}{5} + \binom{30}{0}\binom{70}{5} = 1 + 53{,}130 + 12{,}103{,}014 = 12{,}156{,}145$. The net number of ways to not have all three classes is 63,186,270. The probability of having all three classes is $1 - 63{,}186{,}270/75{,}287{,}520 = \boxed{0.160734}$.

To do it directly, let (a, b, c) be the number of preferred, standard, and rated policies, in that order. The numbers of combinations of each type are:

$$\begin{aligned}
(1,1,3):&\quad \binom{70}{1}\binom{25}{1}\binom{5}{3} = 17{,}500\\
(1,2,2):&\quad \binom{70}{1}\binom{25}{2}\binom{5}{2} = 210{,}000\\
(1,3,1):&\quad \binom{70}{1}\binom{25}{3}\binom{5}{1} = 805{,}000\\
(2,1,2):&\quad \binom{70}{2}\binom{25}{1}\binom{5}{2} = 603{,}750\\
(2,2,1):&\quad \binom{70}{2}\binom{25}{2}\binom{5}{1} = 3{,}622{,}500\\
(3,1,1):&\quad \binom{70}{3}\binom{25}{1}\binom{5}{1} = 6{,}842{,}500
\end{aligned}$$

The sum of the six numbers is 12,101,250, and 12,101,250/75,287,520 = $\boxed{\mathbf{0.160734}}$.

2.17. The number of defective modems is (0.2)(30) + (0.08)(50) = 10. There are $\binom{80}{5}$ ways to pick five modems. Of these, there are $\binom{70}{3}\binom{10}{2}$ ways to pick 3 non-defective ones and 2 defective ones. The probability of picking exactly 2 defective ones is

$$\frac{\binom{70}{3}\binom{10}{2}}{\binom{80}{5}} = \frac{70\cdot 69\cdot 68\cdot 10\cdot 9/3!2!}{80\cdot 79\cdot 78\cdot 77\cdot 76/5!} = \boxed{\mathbf{0.10247}} \qquad \textbf{(C)}$$

2.18. Suppose k pieces of the 27 were insured. Then the probability that none of the 4 is insured is

$$\frac{\binom{27-k}{4}}{\binom{27}{4}}$$

and the probability that exactly 1 is insured is

$$\frac{\binom{k}{1}\binom{27-k}{3}}{\binom{27}{4}}$$

Since the second expression is twice the first,

$$\begin{aligned}
\frac{k(27-k)(26-k)(25-k)}{3!} &= 2\left(\frac{(27-k)(26-k)(25-k)(24-k)}{4!}\right)\\
k &= 0.5(24-k)\\
k &= 8
\end{aligned}$$

The probability that exactly two pieces are insured is

$$\frac{\binom{19}{2}\binom{8}{2}}{\binom{27}{4}} = \frac{(19)(18)(8)(7)/4}{(27)(26)(25)(24)/24} = \boxed{\mathbf{0.2728}} \qquad \textbf{(C)}$$

2.19. There are $\binom{25}{2}$ ways to select two factory workers. Of those, there are $\binom{20}{1}$ ways to select low risk and $\binom{5}{1}$ ways to select high risk.

$$\frac{\binom{20}{1}\binom{5}{1}}{\binom{25}{2}} = \boxed{\mathbf{\frac{1}{3}}} \qquad \textbf{(D)}$$

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