Questions Bank for SOA Exam MFE/ CAS Exam 3F 3rd Edition

Electronic Product/ No Returns

Preface

Despite the fact that Exam MFE is the exam with the least syllabus coverage and exam questions, it is never the easiest. It requires not only the strong mathematical background and solid conceptual understandings on derivatives market, but also extraordinary intuitive approaches and thorough familiarity on the covered readings. In some challenging sittings like in spring 2008, the exam even required the candidates to think beyond the readings, testing them on concepts and calculation which is not obviously learned in the readings.

There are not many equations or mathematical formulae to be memorized other than Black-Scholes formula with its modifications and stochastic processes in this exam. This makes the exam preparation even harder, since memorization and direct substitutions of figures into the formulas are inadequate for a 6 or above. As such, this questions bank serves as mock questions covering the complete syllabus of the exam to adapt and prepare the candidates for a better understanding of the material and more effective tackling of the exams.

This questions bank consisting of 340 questions with full solutions covers all the syllabus coverage based on the textbooks Derivatives Markets by Robert McDonald. All the questions are self-originated and are intended to exhaust all the potential questions that could be raised in the real exams.

Should you have any questions, please feel free to raise them to me at <u>anjstudymanual@gmail.com</u>. I will reply you within 1-3 business days. If you find it necessary, you may also add me in your contact list of messenger for further clarification of your problems.

All the best for the exam, and enjoy the exercises.

Alvin Soh

Part 1. Put-Call Parity and Other Option Relationships

- 1. You are given that S_0 = \$47, and a \$45-strike call option 6 months is priced at \$4.2675. The stock pays no dividend. If the annual continuously compounded risk-free rate is 4%, find the price of a \$45-strike put option expiring in 6 months.
 - (A) \$1.221
 - (B) \$1.275
 - (C) \$1.328
 - (D) \$1.376
 - (E) \$1.429
- 2. You are given the following information:
 - The forward price of ABC stock is \$83.8342.
 - The K-strike put option on 1 unit of the stock is \$3.9909
 - The K-strike call option on 1 unit the stock is C
 - The stock pays annual continuously compounded dividend of 1%
 - The options expire in 6 months

Find the annual continuously compounded risk-free rate, r.

(A)
$$-2\ln\left[\frac{C-3.9909}{83.4161-K}\right]$$

(B) $-2\ln\left[\frac{C-3.9909}{83.8342-K}\right]$
(C) $-2\ln\left[\frac{C-3.9909}{84.2544-K}\right]$
(D) $-0.5\ln\left[\frac{C-3.9909}{83.8342-K}\right]$
(E) $-0.5\ln\left[\frac{C-3.9909}{83.8342-K}\right]$

84.2544

- 3. Which of the followings is false about put-call parity?
 - (A) Buying a call and selling a put with the strike equal to the forward price creates a synthetic forward contract
 - (B) The synthetic forward contract created by put-call parity using a call and a put with K equals to the forward price must have a zero price
 - (C) Put-call parity holds for both European and American options
 - (D) In the absence of risk-free rate and dividend yield, the difference of premiums of call option and the put option with $K = F_{0,T}$ must equal to zero
 - (E) No answers above are correct.

- 4. Which of the followings is incorrect?
 - (A) Early exercise of call option on nondividend-paying stock is irrational even if the call option is deep-in-the- money
 - (B) In the absence of dividend yield and risk-free rate, the put-call parity holds for American options
 - (C) Early exercise of put option on nondividend-paying stock is rational, since it accelerate the receipt of the strike price when the put option is in-the-money
 - (D) With no arbitrage argument, the upper bound for the value of the American put option is C + PV(K) – S, where C is the premium of American call option with the same strike price and date to expiration, and PV(K) is the present value of the strike price, S is the current stock price
 - (E) No answer is given in (A), (B), (C), and (D)
- 5. You are given that:
 - C is the 6-month European option with the strike price, K
 - P is the 6-month European option with the strike price, K
 - PV(D) is the present value of the dividends
 - PV(K) is the present value of the strike price

Which of the followings is true?

- (A) $C+K+PV(D)-S \ge P$
- (B) $P-S-PV(D)+K \leq C$
- (C) $P + S PV(K) \leq C$
- (D) $C + S PV(D) \le P$
- (E) $P + S K \ge C$

1. This is a simple Put-Call Parity question.

 $S_0 = 47, K = 45, t = 0.5, C(47, 45, 0.5) = 4.2675, r = 0.04$

Plug in all this into the Put-Call Parity equation, we have

 $C(47, 45, 0.5) - P(47, 45, 0.5) = S_0 - Ke^{-rt}$

 $4.2675 - P(47, 45, 0.5) = 47 - 45e^{-(0.04)(0.5)}$

Solving for this, we get P(47, 45, 0.5) = 1.3764. (D)

2. This is testing on the Put-Call Parity but leaving the answer in symbol form.

$$F = 83.8342, K = K, P = 3.9909, r = 0.01, t = 0.5$$

$$F^{p} = S_{0}e^{-\delta t} = S_{0}e^{(r-\delta t)}e^{-rt} = Fe^{-rt}$$

Plug in all this into the Put-Call Parity equation and solve for r, the risk free rate

$$C - P = Fe^{-rt} - Ke^{-rt}$$

$$C - 3.9909 = 83.8342e^{-r(0.5)} - Ke^{-r}$$

$$\Rightarrow e^{-0.5r} = \frac{C - 3.9909}{83.8342 - K}$$

$$\Rightarrow r = -2\ln\left[\frac{C - 3.9909}{83.8342 - K}\right]$$
The answer is **(B)**.

- 3. **(C)** is incorrect as put-call parity only holds for European options. With American options, we can exercise them earlier; while for forward, it is only effective at the expiration date.
- 4. **(D)** is incorrect as the upper bound of an American put is K.
- 5. The formula for Put-call parity is C P = S PV(D) PV(K).

 $C + K + PV(D) - S \ge P$ is correct if r equals to 0 or some other values. When r=0, the left side of the equation is equal to the right side of the equation. When r is some other value rather than 0 but positive, K will be larger than PV(K), and thus the left side is larger than the right side.

The answer is thus (A).

- 6. Before the issuance of the liquidating dividend, the stock is \$100, and right after the liquidating dividend, the stock is \$0.
 - (A) is incorrect as the exercise of the call will render a payoff of 0 since S = K. You will get the stock and thus the dividend, but this is not the payoff from the option.
 - (B) is incorrect as right after the issuance of the dividend, S = 0, so the payoff is 0 100 = 100.
 - (C) is correct as right before the issuance of dividend, the stock price is 100, so the payoff is 0.
 - (D) is incorrect as the payoff should be 0.

Part 3. Black-Scholes Formula and Option Greeks

- 1. Which of the followings are conditions for Black-Scholes framework?
 - I. The continuously compounded returns on the stock are normally distributed
 - II. The continuously compounded returns on the stock are not necessarily independent
 - III. The stock price follows lognormal distribution
 - IV. The movement magnitude of the stock price is correlated with the stock price
 - (A) I, II, III
 - (B) I, II, IV
 - (C) I, III, IV
 - (D) II, III, IV
 - (E) No answer is given in (A), (B), (C), and (D)
- 2. Which of the followings are conditions for Black-Scholes framework?
 - I. The volatility must follow a constant drift over time
 - II. The risk-free rate must be according to Vasicek Model
 - III. It is possible to short-sell costlessly and to borrow at the risk-free rate
 - IV. The future dividends must be expressed as fixed dividend yield
 - (A) I
 - (B) I, IV
 - (C) II, III
 - (D) III, IV
 - (E) No answer is given in (A), (B), (C), and (D)
- 3. Which of the followings are true?
 - I. An at-the-money option is more sensitive to the stock price than an out-of-the money option
 - II. The share equivalent of an out-of-the-money option is greater than that of an in-themoney option
 - III. The deep-in-the-money call option has delta approaching to 1
 - IV. The deep-in-the-money put option has delta approaching to -1.
 - (A) I, II, III
 - (B) I, II, IV
 - (C) I, III, IV
 - (D) II, III, IV
 - (E) No answer is given in (A), (B), (C) and (D)

- 4. Which of the followings are true?
 - I. The gamma for a deep-in-the-money call option is near to 1
 - II. The gamma for an at-the-money put option is always the greatest compared to an deep-in-the-money put option and an deep-out-of-the-money put option
 - III. The gammas for a pair of identical call and put option are always the same
 - IV. The gamma of a longer time to expiration has a greater tail than those with shorter time to expiration
 - (A) I, II
 - (B) II, III
 - (C) III, IV
 - (D) I, IV
 - (E) No answer is given in (A), (B), (C), and (D)
- 5. Which of the followings are true?
 - I. Vega is always a positive value
 - II. Vega is higher when the time to expiration is higher
 - III. Vega is always increasing with increasing stock price
 - IV. Vega for a pair of identical call and put option are always the same
 - (A) I, II, III
 - (B) I, II, IV
 - (C) I, III, IV
 - (D) II, III, IV
 - (E) No answer is given in (A), (B), (C), and (D)

- 1. Il is wrong. The returns have to be independent. (C)
- is wrong. The volatility has to be a constant.
 II is wrong. There is no specific model required to model the risk free rate. The framework holds as long as the rate is fixed.

IV is wrong. The dividend can be expressed in dollar amount as well. Refer to Question 5 and 6!

Only III is correct. (E)

3. I is true.

II is false. The share equivalent/delta, Δ is greater for in-the-money options.

III is true. This is because when the call option is deep-in-the-money, it is very likely to be exercised, whereby pay off equals the value of the stock.

IV is true. The Δ is approaching -1 because the put is very likely to be exercised, and hence the option is likely to be SOLD instead of purchased.

The answer is **(C)**.

4. I is false. The gamma is approaching 0 instead of 1. II is true. Generally, the gamma of an option with the strike price nearer to the current price is higher, because the change of delta is more sensitive when the stock price is near to the strike.

III is true. This can be proven by put-call parity.

IV is true. Generally, the curve for gamma for a longer time to expiration is "flatter", because with more time to expiration, there is more "opportunity" for the option value to change; so is to the delta.

The answer is (E)

5. I is true. Mathematically, $\frac{\partial C}{\partial \sigma} = Se^{-r(T-t)}N'(d_1)\sqrt{T-t} \ge 0$.

Il is true. This is because with longer time to expiration, the change of volatility can "cooperate" with the time to affect the option value.

III is false. In fact, after a certain value of the stock price, the vega is decreasing.

IV is true. It can be shown by manually differentiating of the options with respect to σ .

The answer is **(B)**.

Part 5. Exotic Options

- 1. You are given the following underlying stock prices:
 - Jan 2009- \$85 (date of issue)
 - Apr 2009- \$87
 - July 2009- \$93
 - Oct 2009- \$90
 - Jan 2010- \$95 (today)

Which of the following options has the highest payoff?

- (A) 1-year arithmetic average strike call
- (B) 1-year arithmetic average strike put
- (C) 1-year geometric average strike call
- (D) 1-year geometric average strike put
- (E) 1-year \$90-strike call option
- 2. Assume that the Black-Scholes framework holds. You are given that a nondividend-paying stock follows the ito's process:

$$\frac{dS_t}{S_t} = 0.1dt + 0.15dZ(t)$$

where $\{Z(t)\}$ is a standard Brownian motion.

You are given that:

- An arithmetic average strike call option was issued at t = 0
- Two random normal numbers are used to model the price of the underlying stock at t= 0.25 and 0.5:

 $Z_1 = 0.76, Z_2 = 0.43$

- The option expires in 6 months
- The current stock price is \$100
- The strike price is the average of the stock price at the end of every 3 months

Find the payoff of the option at t = 0.5.

- (A) \$35
- (B) \$40
- (C) \$45
- (D) \$50
- (E) \$55
- 3. Which of the following is false?
 - (A) As the frequency of averaging increases, the value of a average price option declines
 - (B) As the frequency of averaging increases, the value of a average strike option increases
 - (C) If the frequency of averaging is 1, the value of the average strike put is always zero
 - (D) A geometric average price call is always more valuable than another identical arithmetic average price call
 - (E) No answer is given in (A), (B), (C), and (D)

- 4. Suppose that S = \$100, K = \$110, r = 0.07, σ = 0.25, δ = 0, T = 1. Construct a standard 2period binomial tree and find the price of an arithmetic average price put option.
 - (A) \$7
 - (B) \$8
 - (C) \$9
 - (D) \$10
 - (E) \$11
- 5. Suppose that S = \$50, r = 0.06, σ = 0.3, δ = 0, T = 0.5. Find the price of an arithmetic average strike call option using 2-period binomial pricing model.
 - (A) Less than \$0.50
 - (B) At least \$0.50, but less than \$1.00
 - (C) At least \$1.00, but less than \$1.50
 - (D) At least \$1.50, but less than \$2.00
 - (E) At least \$2.00

1. For (A),

 $\frac{87+93+90+95}{4} = 91.25$

The payoff is max(95-91.25,0) = 3.75

For (B), the payoff is max(91.25-95,0) = 0

For (C), $[(87)(93)(90)(95)]^{0.25} = 91.19942 \approx 91.20$ The payoff is $\max(95-91.20,0) = 3.8$ For (D), the payoff is $\max(91.20-95,0) = 0$ For (E), the payoff is $\max(95-90,0) = 5$

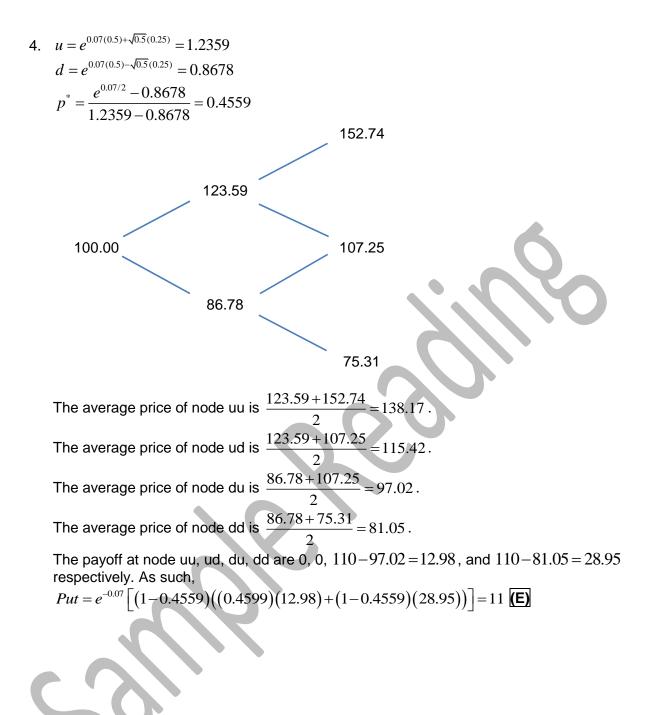
The answer is (E).

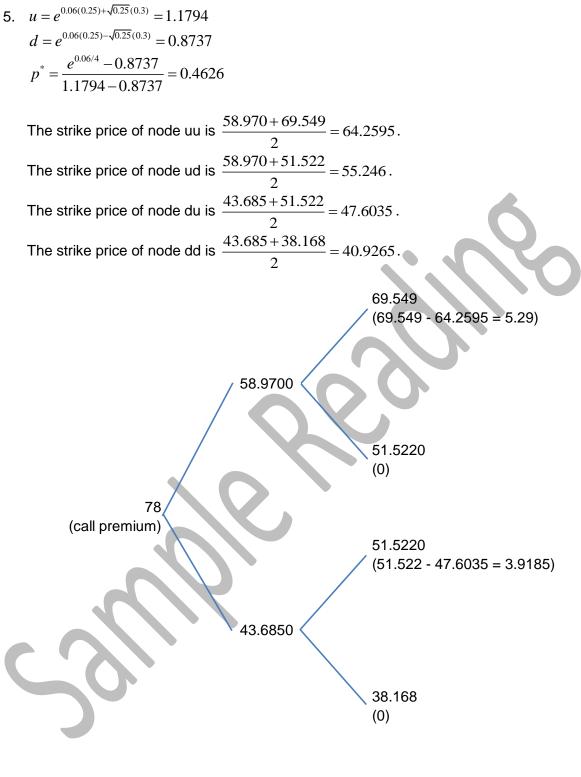
2. $S(0.25) = S(0)e^{0.1(0.25) + \sqrt{0.25}(0.15)(0.76)} = 108.55$ $S(0.5) = S(0.25)e^{0.1(0.25) + \sqrt{0.25}(0.15)(0.43)} = 114.95$

The strike price, K is $\frac{108.55 + 114.95}{2} = 111.75$.

The payoff is max(114.95 - 111.75, 0) = 3.2 (B)

3. The answer is (D). The geometric average is always lower or equal than an arithmetic average. This causes the geometric average price call option less valuable than an identical arithmetic price call option.





 $\therefore Put = e^{-0.06/2} \left[(5.29)(0.4626)^2 + 3.9185(0.4626)(0.5374) \right] = 2.044 \text{ (C)}$

Part 7. Monte Carlo Valuation

1. You are given that a stock price follows a geometric Brownian motion: dS(t) = 0.14S(t)dt + 0.3S(t)dZ(t)

By using the following Uniform (0, 1) random numbers :

0.2918 0.3980 0.8764 0.7100 0.5699

You are given that S(0) = 100. Simulate 5 stock prices at time 10, and find the average of the simulated prices.

(A) Less than \$150

- (B) At least \$150, but less than \$250
- (C) At least \$250, but less than \$350
- (D) At least \$350, but less than \$450
- (E) At least \$450
- 2. You are given that a stock price follows a geometric Brownian motion:

$$dS(t) = 0.2S(t)dt + 0.2S(t)dZ(t)$$

By using the following Uniform (0, 1) random numbers:

0.9251	0.0158	0.6179	0.3859	0.2936

You are given that S(0) = 45. Simulate 5 stock prices at time 3, and find the mean of the simulated prices.

- (A) Less than \$45
- (B) At least \$55, but less than \$65
- (C) At least \$65, but less than \$75
- (D) At least \$75, but less than \$85
- (E) At least \$85
- 3. You are given that a stock price follows lognormal distribution with μ = 0.12 and σ =0.1. By using the following Normal (0, 1) random numbers,

0.4443

Find the mean of the simulated stock price at time 2, if the current stock price is \$20.

- (A) Less than \$20
- (B) At least \$20, less than \$25
- (C) At least \$25, less than \$30
- (D) At least \$30, less than \$35
- (E) At least \$35

1. By Ito's Lemma, we know that: $d \ln S(t) = \left[0.14 - 0.5(0.3)^2 \right] dt + 0.3 dZ(t) = 0.095 dt + 0.3 dZ(t)$

Applying inversion method:

$$Z_{i} = \Phi \left[\frac{\ln \left(\frac{S(t)}{S(0)} \right) - 0.095(10)}{0.3\sqrt{10}} \right]$$
$$\Rightarrow S(t) = S(0)e^{0.095(10) + 0.3\sqrt{10}\Phi^{-1}(Z_{i})}$$

From the given simulated numbers,

$$\Phi^{-1}(0.2918) = -\Phi^{-1}(0.7082) \approx -\Phi^{-1}(0.7088) = -0.55$$

$$\Phi^{-1}(0.398) = -0.26$$

$$\Phi^{-1}(0.8764) = 1.16$$

$$\Phi^{-1}(0.71) = 0.55$$

$$\Phi^{-1}(0.5699) = 0.18$$

Hence, we have:

 $S_1 = 100e^{0.095(10) + 0.3\sqrt{10}(-0.55)} = 153.453$

- $S_2 = 202.049$
- $S_3 = 777.157$
- $S_4 = 435.697$
- $S_5 = 306.719$

The mean is \$375.015. (D)

2. Using Ito's Lemma,

$$d\ln S(t) = \left[0.2 - 0.5(0.2)^2\right] dt + 0.2dZ(t) = 0.18dt + 0.2dZ(t)$$

From the given simulated numbers,

 $\Phi^{-1}(0.9251) = 1.44 \qquad \Phi^{-1}(0.0158) = -2.15 \qquad \Phi^{-1}(0.6179) = 0.3$ $\Phi^{-1}(0.3859) = -0.29 \qquad \Phi^{-1}(0.2936) = -0.54$

Therefore,

 $S_1 = 127.166$ $S_2 = 36.667$ $S_3 = 85.677$ $S_4 = 69.839$ $S_5 = 64.046$

The mean is \$76.68. (D)

3. Notice that the simulated numbers are from normal distribution instead of uniform. As such, we can use the number directly:

 $S_1 = 28.469$ $S_2 = 27.442$ $S_3 = 27.074$

The mean is \$27.66. (C)

Part 8. Brownian Motion and Ito's lemma

1. The price of a stock is governed by the stochastic differential equation:

$$\frac{dS(t)}{S(t)} = 0.09dt + 0.15dZ(t)$$

where $\{Z(t)\}$ is a Brownian motion.

Consider

$$W = \frac{S(1) + S(2)}{2}$$

Find Var(W).

(A)
$$0.11S(0)^2$$

(B) $0.25S(0)^2$
(C) $0.75S(0)^2$
(D) $0.89S(0)^2$

(E)
$$S(0)^2$$

2. The price of a stock is governed by the stochastic differential equation:

$$\frac{dS(t)}{S(t)} = 0.09dt + 0.15dZ(t)$$

where $\{Z(t)\}$ is a Brownian motion.

Consider

$$W = \sqrt{S(1)S(2)}$$

Find $Var(\ln W)$.

(A) 0.045
(B) 0.050
(C) 0.055
(D) 0.060
(E) 0.065

1.
$$Var(W) = \left(\frac{1}{2}\right)^{2} \left\{ E\left[\left(S_{1} + S_{2}\right)^{2}\right] - E\left(S_{1} + S_{2}\right)^{2} \right\}$$

 $E\left[\left(S_{1} + S_{2}\right)^{2}\right] = E\left[S_{1}^{2} + 2S_{1}S_{2} + S_{2}\right]$
 $= E\left[S_{1}^{2}\left(1 + \frac{S_{2}}{S_{1}}\right)^{2}\right] = S_{0}^{2}E\left[\left(\frac{S_{1}}{S_{0}}\right)^{2}\right]E\left[1 + \left(\frac{S_{2}}{S_{1}}\right)^{2}\right]$

Recall that

$$E(S^{a}(t)) = S^{a}(0)e^{\left[a(\alpha-\delta)+\frac{1}{2}a(\alpha-1)\sigma^{2}\right]T}$$

$$\Rightarrow E\left(\frac{S^{a}(t)}{S^{a}(0)}\right) = e^{\left[a(\alpha-\delta)+\frac{1}{2}a(\alpha-1)\sigma^{2}\right]T}$$

$$\Rightarrow E\left(\frac{S(1)^{2}}{S(0)^{2}}\right) = e^{2(0.09)+\frac{1}{2}(2)(1)0.15^{2}} = e^{0.2025}$$

$$\Rightarrow E\left[\left(S_{1}+S_{2}\right)^{2}\right] = S(0)^{2}e^{0.2025}\left[1+e^{0.2025}\right] = 2.7238S(0)^{2}$$

And,

$$E(S_{1}+S_{2})^{2} = [E(S_{1})+E(S_{2})]^{2}$$
$$= S(0)^{2} [e^{0.09}+e^{0.18}]^{2}$$
$$= 2.29139S(0)^{2}$$

$$Var(W) = \frac{1}{4} \left[2.7238S(0)^2 - 2.29139S(0)^2 \right] = 0.1081S(0)^2 \text{(A)}$$

2.
$$Var(\ln W) = Var\left[\ln(S(1) + S(2))^{1/2}\right]$$

 $= \frac{1}{4}Var(\ln S(1) + \ln S(2))$
 $= \frac{\sigma^2}{4}Var(Z(1) + Z(2))$
 $= \frac{\sigma^2}{4}Var(2Z(1) + Z(1)]$
 $= \frac{\sigma^2}{4}Var(3Z(1))$
 $= \frac{3^2\sigma^2}{4}(1)$
 $= \frac{9}{4}\sigma^2 = \frac{9}{4}(0.15)^2 = 0.050625$ [B]

Part 9. Interest Rate Models

- 1. Consider modeling the interest rate using the Cox-Ingersoll-Ross model. You are given that:
 - $dr = 0.2(0.025 r)dt + 0.3\sqrt{r}dZ(t)$
 - The sharpe ratio is 0.15
 - r(0) = 0.03

Find the yield to maturity on an infinitely lived bond.

- (A) Less than 1%
- (B) At least 1%, but less than 2%
- (C) At least 2%, but less than 3%
- (D) At least 3%, but less than 4%
- (E) At least 4%
- 2. Consider modeling the interest rate using Rendelman-Bartter model. You are given that:

$$dr = 0.06rdt + 0.3rdZ(t)$$

Find the probability that the interest rate will be greater than 4% 6 months from now, if the current rate is 5%.

- (A) Less than 0.8
- (B) At least 0.8, but less than 0.85
- (C) At least 0.85, but less than 0.90
- (D) At least 0.90, but less than 0.95
- (E) At least 0.95

1. The sharpe ratio is given by: $\phi = \frac{\overline{\phi}\sqrt{r}}{\sigma}$. Hence, $0.15 = \frac{\bar{\phi}\sqrt{0.03}}{0.3} \Longrightarrow \bar{\phi} = 0.259808$ $\therefore \bar{r} = \frac{2ab}{\bar{a+\phi+\gamma}}$ $=\frac{2(0.2)(0.025)}{0.2+0.259808+\sqrt{(0.2-0.259808)^2}+2(0.3)^2}=0.011258$ The answer is (B) 2. $\mu = 0.06 - 0.5(0.3^2) = 0.015$ $P\{r(0.5) > 0.04\}$ $= P \left\{ Z > \frac{\ln\left(\frac{0.04}{0.05}\right) - 0.015(0.5)}{0.3\sqrt{0.5}} \right\}$ $=1-\Phi(-1.087)$ =1-0.1379= 0.8621The answer is (B).

Author's Biography

Alvin Soh was born in Penang, Malaysia. He is now a final year campus student studying in Petaling Jaya. He is keen in sharing and helping his peers in actuarial exams. His strong passion in mathematics and actuarial career is reflected in his fast progress in actuarial exams. He has passed all the preliminary exams in the mere 1.5 years and he is more than willing to share his study methods and experience to every actuarial student striving to progress in exams. He has passed an advanced level exam (Exam AFE) recently, and is pursuing his CERA and FSA in Finance and Enterprise Risk Management.

The following is the progress of his exams:

Exam Passed	Sitting
P/1- Probability	Spring 2007
FM/2- Financial Mathematics	Fall 2007
MLC- Models for Life Contingencies	Spring 2008
MFE/3F- Models for Financial Economics	Fall 2008
C/4- Construction and Evaluation of Actuarial Models	Fall 2008
AFE- Advanced Finance and Enterprise Risk Management	Spring 2009

He is now preparing for the last fellowship exam by the Society of Actuaries in finance and enterprise risk management track- exam FETE (Financial Economics Theory and Engineering).