

# ACE Study Manual - SOA Exam MFE / CAS Exam 3F - SAMPLE CHAPTER

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## 1 Chapter 9: Parity and Other Option Relationships (and Appendix B.1: Interest Rates)

Expected Number of MFE Questions: 2-3 (10%)

Difficulty Level of these Questions: Average (ranking=5 on 1-9 scale)

### 1.1 Section 9.1: Put-Call Parity

Main Idea: Put-Call Parity (in all its forms)

Significance: Most Important (rating=5 on 1-5 scale)

Concept of Emphasis: Formula in 6th point for general put-call parity

- Recall that synthetic forwards must be priced consistently with actual forwards.
- A synthetic forward consists of buying a call and selling a put, each with the same  $K$  (exercise price) and  $T$  (time to expiration).
- An actual forward has zero premium at  $t=0$ , but the synthetic forward has a net option premium of  $C$  (call price) -  $P$  (put price) at  $t=0$ .
- With an actual forward, we pay  $F$  (the forward price) at  $t=T$ , but with a synthetic forward, we pay  $K$  at  $t = T$ , regardless of the value of  $S(T)$ , the stock value at that time. This is because of the following: if  $S(T) > K$ , we'd exercise the call and buy the stock for  $K$ ; however, if  $S(T) < K$ , the counterparty would exercise the put, and sell the stock to us for  $K$ .
- Thus, because synthetic forwards and actual forwards must be priced consistently,  $C(K, T) - P(K, T) + PV_{0,T}(K) = 0 + PV_{0,T}(F_{0,T})$ . This equation has  $t=0$  as the valuation time, where the left-hand side corresponds to the synthetic forward and the right-hand side corresponds to the actual forward.
- Restating the above equation, we have the standard put-call parity equation:  $C(K, T) - P(K, T) = PV_{0,T}(F_{0,T} - K)$ . Assuming a continuously compounded interest rate  $r$ , this can also be written as:  $C(K, T) - P(K, T) = e^{-rT}(F_{0,T} - K)$ .
- Note that put-call parity applies to European options only, and not necessarily to American options (which can be exercised prior to maturity).

- (Sample Exam: Problem #1)

You were given  $\delta = 0, S = \$60, C - P = \$0.15, T = 4, K = \$70$ , and need to find  $r$ , the continuously compounded risk-free interest rate.

By put-call parity (with no dividends),  $C - P = S - Ke^{-rT}$ .

Thus,  $\$0.15 = \$60 - \$70e^{-r(4)}$ . Solving for  $r$  gives the solution:  $r \approx .039$ .

Answer: A

### 1.1.1 Options on Stocks

Concepts of Emphasis: Formula in 1st point for put-call parity for stocks with discrete dividends, Formula in 2nd point for put-call parity for stocks with continuous dividends

- If the underlying asset is a stock  $S$  with discrete dividends, denoted by  $Divs$ , then  $e^{-rT}F_{0,T} = S_0 - PV_{0,T}(Divs)$ , and the above put-call parity relationship may be written as:  $C(K, T) - P(K, T) = [S_0 - PV_{0,T}(Divs)] - Ke^{-rT}$ .
- If the underlying asset is a stock index  $S$ , which has dividends paid continuously at a rate  $\delta$  that is proportional to the level of the index ( $\delta$  is also known as the dividend yield), then  $e^{-rT}F_{0,T} = S_0e^{-\delta t}$ , and the above put-call parity relationship may be written as:  $C(K, T) - P(K, T) = S_0e^{-\delta t} - Ke^{-rT}$ .
- Rearranging the standard put-call-parity equation, we can create the value for a synthetic call option:  $C(K, T) = [S_0 - PV_{0,T}(Divs) - Ke^{-rT}] + P(K, T)$ . This says that a call is equivalent to a long position on the underlying asset  $S_0 - PV_{0,T}(Divs) - Ke^{-rT}$ , insured by buying a put.
- Likewise, we can create the value for a synthetic put option:  $P(K, T) = -[S_0 - PV_{0,T}(Divs) - Ke^{-rT}] + C(K, T)$ . This says that a put is equivalent to a short position on the underlying asset  $S_0 - PV_{0,T}(Divs) - Ke^{-rT}$ , insured by buying a call.
- If a stock has no dividends, we can write  $PV_{0,T}(Divs) = 0$ ; If a stock index pays no dividends, then  $\delta = 0$ .
- (May 07 Exam: Problem #1)

You were given  $S = \$52.00, D = D_{2/12} = D_{5/12}, C = \$4.50, P = \$2.45, K = \$50.00, T = 6/12, r = .06$ , and need to find  $D$ .

By put-call parity (with discrete dividends),  $C - P = (S - PV(Divs)) - Ke^{-rT}$ .

Thus,  $\$4.50 - \$2.45 = (\$52.00 - De^{-.06(2/12)} - De^{-.06(5/12)}) - \$50.00e^{-.06(6/12)}$ .

Solving for  $D$  gives the solution:  $D \approx \$0.73$ .

Answer: B

- (May 07 Exam: Problem #4)

You were given  $S = \$50.00$ ,  $\delta = .08$ ,  $r = .04$ ,  $T = 1$ , and a table of four different call prices  $C$ , each with varying strike price  $K$ .

For each given call, we wish to determine whether or not it is optimal to exercise a corresponding put option  $P$  (also with  $T = 1$ ) immediately, which would be the case if and only if  $\$K - \$50 > P$ .

By put-call parity (with continuous dividends),  $C - P = Se^{-\delta T} - Ke^{-rT}$ , or  $P = C - Se^{-\delta T} + Ke^{-rT}$ .

For the first call, where  $C = \$9.12$  and  $K = \$40$ , we get  $P = \$1.39$ ; since the immediate payoff of  $-\$10 < \$1.39$ , we do not exercise immediately.

For the second call, where  $C = \$4.91$  and  $K = \$50$ , we get  $P = \$6.79$ ; since the immediate payoff of  $\$0 < \$6.79$ , we do not exercise immediately.

For the third call, where  $C = \$0.71$  and  $K = \$60$ , we get  $P = \$12.20$ ; since the immediate payoff of  $\$10 < \$12.20$ , we do not exercise immediately.

For the fourth call, where  $C = \$0.00$  and  $K = \$70$ , we get  $P = \$21.10$ ; since the immediate payoff of  $\$20 < \$21.10$ , we do not exercise immediately.

Thus, it is not optimal to exercise any of these put options.

Answer: E

### 1.1.2 Options on Currencies

Concept of Emphasis: Formula in 4th point for put-call parity on exchange rates

- Suppose we can pay dollars to obtain some foreign currency.
- Let  $r$  be the dollar-denominated interest rate and  $r_f$  be the foreign-denominated interest rate, and let  $x_0$  be the current exchange rate in terms of dollars per 1 unit of foreign currency.
- The dollar-denominated forward price  $F$  for 1 unit of foreign currency is:  $F_{0,T} = x_0 e^{(r-r_f)T}$ .
- Then, put-call parity states:  $C(K,T) - P(K,T) = x_0 e^{(r-r_f)T} e^{-rT} - Ke^{-rT} = x_0 e^{-r_f T} - Ke^{-rT}$ .

### 1.1.3 Options on Bonds

Concept of Emphasis: Formula in 1st point for put-call parity on coupon-paying bonds

- If the underlying asset is a bond  $B$  with coupons, denoted by  $Coups$ , then  $e^{-rT} F_{0,T} = B_0 - PV_{0,T}(Coups)$ , and the above put-call parity relationship may be written as:  $C(K,T) - P(K,T) = [B_0 - PV_{0,T}(Coups)] - Ke^{-rT}$ .
- If the bond has no coupons, the above equation still works, but with  $PV_{0,T}(Coups) = 0$ .

## 1.2 Section 9.2: Generalized Parity and Exchange Options

Main Idea: Extending traditional options (where  $K$  is fixed/cash) to exchange options (where  $K$  is variable/another asset)

Significance: Less Important than Average, as better covered in McDonald Section 14.6 when discussing types of exotic options (rating=2 on 1-5 scale)

Concepts of Emphasis: Formula in 6th point for exchange call option payoff, Formula in 7th point for exchange put option payoff, Formula in 8th point for Generalized Parity

- What if the strike  $K$  is not necessarily cash, but instead could be some other asset?
- Now, we'll have the option of exchanging one asset for another asset, rather than exchanging an asset for cash (or vice-versa).
- Let the underlying asset A have price  $S_t$  and the strike asset B have price  $Q_t$ .
- In the context of call options, assume that we currently hold asset B, and that we have the option of giving it up to get asset A instead.
- In the context of put options, assume that we currently hold asset A, and that we have the option of giving it up to get asset B instead.
- Thus, the call option price at time  $t$  is:  $C(S_t, Q_t, T - t)$ , where  $T - t$  is the remaining time to expiration; At time  $t = T$ , the payoff of this call option is  $C(S_T, Q_T, 0) = \max(0, S_T - Q_T)$ ; that is, we will make the exchange if asset A is worth more than asset B at  $t = T$ .
- Similarly, the put option price at time  $t$  is:  $P(S_t, Q_t, T - t)$ , where  $T - t$  is the remaining time to expiration; At time  $t = T$ , the payoff of this put option is  $P(S_T, Q_T, 0) = \max(0, Q_T - S_T)$ ; that is, we will make the exchange if asset B is worth more than asset A at  $t = T$ .
- Finally, the generalized parity equation at time  $t$  for European options is:  
$$C(S_t, Q_t, T - t) - P(S_t, Q_t, T - t) = PV_{t,T}(F_{t,T}(S) - F_{t,T}(Q)).$$

### 1.2.1 Options to Exchange Stock

Concept of Emphasis: Idea in 1st point that stock options will have variable strike prices

- Stock options may be constructed so that the strike price is the price of a stock index, instead of being a fixed cash amount.
- These options have payoffs when a company's own stock outperforms that of the index. Note that the index may be a single stock, perhaps that of a competitor.
- Note that a call option from a company perspective is like a put option from a competitor's perspective, and vice-versa.

### 1.2.2 What are Calls and Puts?

Concept of Emphasis: Idea in 1st point that calls can be interpreted as puts and vice-versa

- Labeling an option as a call or put is arbitrary, because calls can also be interpreted as puts and vice-versa.
- We traditionally think of a call option on stock as the right to buy one stock share  $S$  by paying  $K$ , which we will do when  $S > K$ .

- However, this same exchange can be construed as a put option on dollars, whereby we have the right to sell  $K$  for one stock share  $S$ , which we will do when  $S > K$ .
- Similarly, we traditionally think of a put option on stock as the right to sell one stock share  $S$  for  $K$ , which we will do when  $S < K$ .
- However, this same exchange can be construed as a call option on dollars, whereby we have the right to buy  $K$  by paying one stock share  $S$ , which we will do when  $S < K$ .
- (Sample Exam: Problem #3)

You were given that the time- $T$  payoff from the insurance company to the policyholder is  $\pi(1 - y\%)max[S(T)/S(0), (1 + g\%)^T]$ , where  $T = 1, g = 3, \delta = 0, S(0) = \$100, K = \$103$ , and  $P = \$15.21$ , the price of a 1-year European option on the stock index. You need to find  $y$ .

Thus, the policyholder's payoff (at  $T = 1$ ) is  $\pi(1 - y\%)max[S(1)/\$100, 1.03] = \frac{\pi}{\$100}(1 - y\%)max[S(1), \$103] = \frac{\pi}{\$100}(1 - y\%)[max(\$103 - S(1), 0) + S(1)]$ .

Thus, the policyholder's premium (at  $t = 0$ ) is  $\pi = \frac{\pi}{\$100}(1 - y\%)[P + S(0)] = \frac{\pi}{\$100}(1 - y\%)[\$15.21 + \$100]$ .

Note that  $S(0)$  is the time-0 price for a time-1 payoff of  $S(1)$  (since  $\delta = 0$ ), and  $P$  is the time-0 price for a time-1 payoff of  $max(\$103 - S(1), 0)$ .

Thus, dividing through by  $\pi$ , we have  $1 = 1.1521(1 - y\%)$ . Thus,  $y \approx 13.2\%$ .

Answer: 13.2% (Note: No letter choices given for this question)

### 1.2.3 Currency Options

Concepts of Emphasis: Formulae in 5th point for payoffs of currency options, Formula in 6th point for reconciling differences in currency denomination and scale

- The discussion about calls and puts in the previous section can be applied to currency exchange options, whereby one country's currency is exchanged for another country's currency.
- In the discussion below, assume the exchange is between U.S. dollars and European euros.
- An option is considered 'dollar-denominated' if both the strike price and the option's premium are quoted in dollars; likewise, an option is considered 'euro-denominated' if both the strike price and the option's premium are quoted in euros.
- Let  $x_0$  be the current exchange rate specifying the number of dollars per 1 euro, and let  $x_1$  be this same exchange rate one-year ahead. Also, let  $K$  be the dollar-denominated strike price, so that  $\frac{1}{K}$  is the euro-denominated strike price. Similarly,  $\frac{1}{x_0}$  and  $\frac{1}{x_1}$  would be the time-0 and time-1 exchange rates that specify the number of euros per 1 dollar.
- Now, consider a 1-year dollar-denominated call option on euros, which has time-1 payoff:  $max(0, x_1 - K)$ . Also, consider a 1-year euro-denominated put option on dollars, which has time-1 payoff:  $max(0, \frac{1}{K} - \frac{1}{x_1})$ .

- Although these two options are exercised under the same condition (i.e. - when  $x_1 > K$ ), both the currency of denomination and the scale are different. These differences can be reconciled by using the following formula:  $C_{\$}(x_0, K) = x_0 K P_E(\frac{1}{x_0}, \frac{1}{K})$ , where  $\$$  and  $E$  refer to dollar-dominated and euro-denominated options, respectively.
- (May 09 Exam: Problem #9)

You were given  $x_0 = .011\$/Y$  (where  $Y$ =Japanese Yen),  $T = 4$ ,  $K = \$.008$ ,  $P = \$.0005$  (where  $P$  is the price of a dollar-denominated European put option on yen),  $r = .03$ ,  $r_f = .015$  (where  $f$  stands for 'foreign' from the U.S. perspective), and need to calculate  $P_f$ , the price of a Yen-denominated European put option on dollars (with  $T = 4$  and  $1/K = 125Y$ ).

Adjusting for currency and scale differences, the relationship between  $P_f$  and  $C$ , where  $C$  is the price of a dollar-denominated European call option on yen, is:

$$P_f(\frac{1}{x_0}, \frac{1}{K}, T) = \frac{C(x_0, K, T)}{x_0 K} = \frac{C}{(.011)(.008)}.$$

Now, we can find  $C$  in terms of  $P$  by put-call parity:

$$C = P + x_0 e^{-r_f T} - K e^{-r T} = .0005 + .011 e^{-(.015)(4)} - .008 e^{-(.03)(4)} = .003764.$$

$$\text{Thus, } P_f = \frac{.003764}{(.011)(.008)} \approx 43Y.$$

Answer: E

### 1.3 Section 9.3: Comparing Options on Style, Maturity, and Strike

Main Idea: Bounds and ordering relationships among similar but varying types of calls (and puts)

Significance: More Important than Average (rating=4 on 1-5 scale)

#### 1.3.1 European v. American Options

Concepts of Emphasis: Inequality in 3rd point comparing American and European call options, Inequality in 4th point comparing American and European put options

- Recall that a European option may only be exercised at expiration, whereas an American option can be exercised at any time. Note that some options can be exercised any time after an initial period during which exercise is not allowed - these are called Bermuda options.
- Thus, American options must be at least as valuable as otherwise equivalent European options.
- Thus,  $C_{Amer}(S, K, T) \geq C_{Eur}(S, K, T)$ .
- Also,  $P_{Amer}(S, K, T) \geq P_{Eur}(S, K, T)$ .

#### 1.3.2 Maximum and Minimum Option Prices

Concepts of Emphasis: Bounds in 5th point for both American and European call options, Bounds in 6th point for both American and European put options

- Here, we focus initially on European option prices.

- Note that option prices (for calls and puts) can never be negative, since they reflect options rather than obligations to exercise.
- A call price, at any point in time, cannot exceed the stock price, but must be at least as great as the price implied by Put-Call-Parity (where  $P=0$ ).
- A put price, at any point in time, cannot exceed the strike price, but must be at least as great as the price implied by Put-Call-Parity (where  $C=0$ ).
- Thus, for call options,  $S \geq C_{Amer}(S, K, T) \geq C_{Eur}(S, K, T) \geq \max[0, PV_{0,T}(F_{0,T} - K)]$ .
- Also, for put options,  $K \geq P_{Amer}(S, K, T) \geq P_{Eur}(S, K, T) \geq \max[0, PV_{0,T}(K - F_{0,T})]$ .
- (Sample Exam: Problem #26)

You were given  $\delta = 0, K = 100, T = .5, r = .10$ , and are given four graphs of  $S$  (x axis) versus  $\pi$  (y axis), where  $S$  is the current stock price and  $\pi$  is the current price of some option.

Each graph shows a shaded region of option prices that are within permissible bounds. Out of the four option types given (European Call, American Call, European Put, and American Put), you are to identify which graph corresponds with each option type. Note that is possible for more than 1 option type to correspond with the same graph (and therefore, there may be some graphs that do not correspond to any option type). Also, if more than one graph is possible, you are to choose the one with the smallest shaded region.

For call options, the appropriate bounds are

$$S(0) \geq C_{Amer} \geq C_{Eurp} \geq \max[0, PV_{0,T}(F_{0,T} - K)].$$

Here,  $PV_{0,T}(K) = Ke^{-rT} = 100e^{-(.10)(.5)} \approx 95.12$ , and  $PV_{0,T}(F_{0,T}) = S$  because the stock pays no dividends. In addition, also because  $\delta = 0$ , it is never optimal to exercise the call option early, and therefore,  $C_{Amer} = C_{Eurp}$ .

Putting this all together, both  $C_{Eurp}$  and  $C_{Amer}$  must be between  $S - 95.12$  and  $S$ . This is consistent with graph II so that only answer choices D and E remain as possibilities. Note that  $C_{Eurp}$  and  $C_{Amer}$  are also always between  $S - 100$  and  $S$ , but this provides a larger shaded region (thus, graph II is chosen over graph I).

For put options, the appropriate bounds are

$$K \geq P_{Amer} \geq P_{Eurp} \geq \max[0, PV_{0,T}(K - F_{0,T})].$$

Unlike for calls, even with  $\delta = 0$ , it is possible for early exercise to be optimal, so that  $P_{Amer}$  may be greater than  $P_{Eurp}$ .

We know, though, that  $K = 100$  is an upper bound for  $P_{Amer}$ , so that graph III is the appropriate choice for  $P_{Amer}$ . This rules out answer Choice E.

The only viable answer choice remaining is D, where graph II corresponds to both types of calls, graph III corresponds to American puts, and graph IV corresponds to European puts.

Answer: D

### 1.3.3 Early Exercise for American Options

Concept of Emphasis: Idea in 1st point that an American call option on a stock without dividends should never be exercised early

- An American call option on a nondividend-paying stock should never be exercised early.
- However, if the stock pays dividends, and dividends are sufficiently great, it can be optimal to exercise an American call option early. If early exercise is optimal, one should do so at the latest possible moment before the ex-dividend date.
- It may also be optimal to exercise early for an American put option on a nondividend-paying stock (or on a stock paying dividends).

### 1.3.4 Time to Expiration

Concepts of Emphasis: Idea in 1st point that American options do not increase in value when time-to-expiration decreases, Idea in 2nd point that European options on stocks without dividends do not increase in value when time-to-expiration decreases

- For American options (both calls and puts), options with more time-to-expiration should be worth at least as much as otherwise equivalent options with less time-to-expiration.
- For European call options on nondividend-paying stocks, options with more time-to-expiration should be worth at least as much as otherwise equivalent options with less time-to-expiration. Recall that European call options on nondividend-paying stocks are valued the same as otherwise equivalent American call options.
- For European call options on stocks paying dividends, it is possible for shorter-term options to be worth more than otherwise equivalent longer-term options.
- Also, for European put options (on stocks with or without dividends), it is also possible for shorter-term options to be worth more than otherwise equivalent longer-term options.

### 1.3.5 Different Strike Prices

Concepts of Emphasis: Inequalities in 2nd/3rd points comparing call/put options of varying strike prices, Inequalities in 4th/5th points comparing differences in call/put options at varying strike prices, Inequalities in 6th/7th points comparing ratios of differences in call/put options to differences in strike prices at varying strike prices, Inequalities in 10th point for satisfying convexity definition for call/put options

- Assume there are three strike prices  $K_1 < K_2 < K_3$ , corresponding to three otherwise equivalent call options and also three otherwise equivalent put options. The statements to follow apply to both European and American options.
- It must be true that:  $C(K_1) \geq C(K_2) \geq C(K_3)$ .
- It must be true that:  $P(K_1) \leq P(K_2) \leq P(K_3)$ .
- It must be true that:  $C(K_1) - C(K_2) \leq K_2 - K_1$  and  $C(K_2) - C(K_3) \leq K_3 - K_2$ .
- It must be true that:  $P(K_2) - P(K_1) \leq K_2 - K_1$  and  $P(K_3) - P(K_2) \leq K_3 - K_2$ .
- For European options (both calls and puts), the above inequalities can be strengthened further. Specifically,  $C(K_1) - C(K_2) \leq PV(K_2 - K_1)$ ,  $C(K_2) - C(K_3) \leq PV(K_3 - K_2)$ ,  $P(K_2) - P(K_1) \leq PV(K_2 - K_1)$ , and  $P(K_3) - P(K_2) \leq PV(K_3 - K_2)$ .

- Convexity must hold for call options; that is,  $\frac{C(K_1)-C(K_2)}{K_2-K_1} \geq \frac{C(K_2)-C(K_3)}{K_3-K_2}$ .
- Convexity must hold for put options; that is,  $\frac{P(K_2)-P(K_1)}{K_2-K_1} \leq \frac{P(K_3)-P(K_2)}{K_3-K_2}$ .
- Convexity means that premiums decline at decreasing rates when considering call options with progressively higher strike prices, and also when considering put options with progressively lower strike prices.
- Since  $K_2$  is between  $K_1$  and  $K_3$ , we can express  $K_2$  as a weighted average of  $K_1$  and  $K_3$  as follows:  
 $K_2 = \lambda K_1 + (1 - \lambda)K_3$  so that  $\lambda = \frac{K_3 - K_2}{K_3 - K_1}$ .
- Then, the necessary conditions for convexity stated above can be rewritten as:  
 $C(K_2) \leq \lambda C(K_1) + (1 - \lambda)C(K_3)$  and  $P(K_2) \leq \lambda P(K_1) + (1 - \lambda)P(K_3)$ .
- (May 09 Exam: Problem #12)

You were given  $\delta = 0$ ,  $C(K, T)$  and  $P(K, T)$  are call and put options (respectively) on the same stock, and need to determine whether or not each of three inequalities regarding option price bounds are true.

Inequality (I) states that  $0 \leq C(50, T) - C(55, T) \leq 5e^{-rT}$ . We know that  $C(50) \geq C(55)$  so that  $C(50) - C(55) \geq 0$ . We also know that  $C(50) - C(55) \leq 55 - 50 = 5$  and (by the footnote on McDonald page 300)  $C(50) - C(55) \leq PV(55 - 50) = PV(5) = 5e^{-rT}$ . Thus, inequality (I) is true.

Inequality (II) states that  $50e^{-rT} \leq P(45, T) - C(50, T) + S \leq 55e^{-rT}$ . Now, from put-call parity, we can write  $P(K, T) = C(K, T) - PV(S) + PV(K)$  so that  $P(45) = C(45) - S + 45e^{-rT}$ . Then,  $P(45) - C(50) + S = C(45) - S + 45e^{-rT} - C(50) + S = C(45) - C(50) + 45e^{-rT}$ . Thus, inequality (II) can be written:  $50e^{-rT} \leq C(45) - C(50) + 45e^{-rT} \leq 55e^{-rT}$ , or subtracting  $45e^{-rT}$  from all 3 sides,  $5e^{-rT} \leq C(45) - C(50) \leq 10e^{-rT}$ . However, this is NOT true because (following the logic used above for inequality (I)), we know that  $C(45) - C(50) \leq 5e^{-rT}$  rather than the other way around. Thus, inequality (II) is false.

Inequality (III) states that  $45e^{-rT} \leq P(45, T) - C(50, T) + S \leq 50e^{-rT}$ . Following the same logic used above for inequality (II), inequality (III) can be written:  $45e^{-rT} \leq C(45) - C(50) + 45e^{-rT} \leq 50e^{-rT}$ , or subtracting  $45e^{-rT}$  from all 3 sides,  $0 \leq C(45) - C(50) \leq 5e^{-rT}$ . This is now analogous to inequality (I), which was shown to be true. Thus, inequality (III) is true.

Answer: E (I and III only are true)

- (Sample Exam: Problem #2)

You were given  $C(40) = \$11$ ,  $P(40) = \$3$ ,  $C(50) = \$6$ ,  $P(50) = \$8$ ,  $C(55) = \$3$ ,  $P(55) = \$11$ , and suggestions for arbitrage opportunities from Mary and Peter.

Mary suggests a portfolio of 1 long- $C(40)$ , 3 short- $C(50)$ 's, the lending of \$1, and  $m$

long- $C(55)$ 's.

Peter suggests a portfolio of 2 long- $C(55)$ 's and 2 short- $P(55)$ 's, 1 long- $C(40)$  and 1 short- $P(40)$ , the lending of \$2, and the combination of  $p$  short- $C(50)$ 's and  $p$  long- $P(50)$ 's.

Note that a third person John believes there are no arbitrage opportunities possible.

To determine whether or not John, Mary, and Peter are correct, we must determine (in turn) whether or not the described portfolios produce arbitrage opportunities.

Let's consider Mary's portfolio first, noting that  $m=2$  (so that the number of calls bought matches the number sold). Mary's profit scenarios are shown in the following table:

Table 1: Mary's Portfolio: Profit Table

Action	Initial Cash Flow	$S_T < 40$	$40 \leq S_T < 50$	$50 \leq S_T < 55$	$S_T \geq 55$
Buy 1 $C(40)$	-\$11	\$0	$S_T - 40$	$S_T - 40$	$S_T - 40$
Sell 3 $C(50)$ 's	\$18	\$0	\$0	$-3(S_T - 50)$	$-3(S_T - 50)$
Buy 2 $C(55)$ 's	-\$6	\$0	\$0	\$0	$2(S_T - 55)$
Lend \$1	-\$1	$\$e^{rT}$	$\$e^{rT}$	$\$e^{rT}$	$\$e^{rT}$
TOTAL	\$0	$=\$e^{rT}$	$\geq \$e^{rT}$	$\geq \$e^{rT}$	$=\$e^{rT}$

Next, let's consider Peter's portfolio, noting that  $p=3$  (so that the number of calls bought matches the number sold). Peter's profit scenarios are shown in the following table:

Table 2: Peter's Portfolio: Profit Table

Action	Initial Cash Flow	$S_T < 40$	$40 \leq S_T < 50$	$50 \leq S_T < 55$	$S_T \geq 55$
Buy 1 $C(40)$ , Sell 1 $P(40)$	-\$8	$-(40 - S_T)$	$S_T - 40$	$S_T - 40$	$S_T - 40$
Sell 3 $C(50)$ 's, Buy 3 $P(50)$ 's	\$6	$3(50 - S_T)$	$3(50 - S_T)$	$-3(S_T - 50)$	$-3(S_T - 50)$
Buy 2 $C(55)$ 's, Sell 2 $P(55)$ 's	\$16	$-2(55 - S_T)$	$-2(55 - S_T)$	$-2(55 - S_T)$	$2(S_T - 55)$
Lend \$2	-\$2	$\$2e^{rT}$	$\$2e^{rT}$	$\$2e^{rT}$	$\$2e^{rT}$
TOTAL	\$0	$=\$2e^{rT}$	$=\$2e^{rT}$	$=\$2e^{rT}$	$=\$2e^{rT}$

Since both Mary and Peter have positive profit (in every cell in the TOTAL rows of their respective tables), regardless of what the value of  $S_T$  is, both Mary and Peter are correct, and therefore John is incorrect.

Answer: D

### 1.3.6 Exercise and Moneyness

Concept of Emphasis: Definition in 1st point of 'in the money' options

- In-the-money options have positive payoffs if exercised immediately.
- When comparing two in-the-money options, if it is optimal to exercise the less in-the-money option, it will also be optimal to exercise an otherwise equivalent more in-the-money option. This is true for both calls and puts.

### 1.4 Recommended McD. Chapter 9 Examples/Tables/Figures to Review

Examples: 1, 2, 5, 7, 8, and 9; Tables: 3, 6, 7, and 8; Figures: 2

### 1.5 Recommended McD. Chapter 9 End-of-Chapter Exercises to Do

1, 2, 4, 5, 6, 8, 9, 10, 11, and 12.

### 1.6 Appendix B.1: The Language of Interest Rates

Main Idea: Distinguishing between effective annual rates and continuously compounded rates

Significance: Least Important, since this is just a review of what you learned in Exam FM (rating=1 on 1-5 scale)

Concepts of Emphasis: Formula in 1st point for an accumulation factor using effective annual interest, Formula in 2nd point for an accumulation factor using continuously compounded interest

- If  $r$  is an effective annual rate, and \$1 is invested now, it will be worth  $(1 + r)^n$  in  $n$  years.
- If  $r$  is an annualized continuously compounded rate, and \$1 is invested now, it will be worth  $e^{rn}$  in  $n$  years.
- Note: if a rate is quoted as 'continuously compounded,' you should assume it is an annualized rate unless otherwise instructed.

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