Practice Exam 1

1. You are given:

(i) The following life table.

x	l_x	d_x
50	1000	20
51		
52		35
53		37

(ii) ${}_2q_{52} = 0.07508$.

Determine d_{51}.

(A) 20 (B) 21 (C) 22 (D) 24 (E) 26

2. For a Continuing Care Retirement Community, you are given the following annual transition probability matrix:

	To			
From	Independent Living Unit	Temporary Nursing Facility	Permanent Nursing Facility	Gone
Independent Living Unit	0.7	0.1	0.2	0
Temporary Nursing Facility	0.7	0.1	0.1	0.1
Permanent Nursing Facility	0	0	0.8	0.2
Gone	0	0	0	1

You are given the following costs per year in each unit, payable at the beginning of the year.

Independent Living Unit	50
Temporary Nursing Facility	100
Permanent Nursing Facility	200

Determine the actuarial present value at 6% of 3 years of costs for someone currently in the Temporary Nursing Facility.

(A) 184 (B) 210 (C) 221 (D) 234 (E) 292

3. For a fully discrete 2-year term insurance on (60) with benefit b payable at the end of the year of death, you are given

(i)

t	p_{60+t-1}
1	0.98
2	0.96

(ii) The annual benefit premium is 25.41.

(iii) $i = 0.05$.

Determine the revised annual benefit premium if an interest rate of $i = 0.04$ is used.

(A) 25.65 (B) 25.67 (C) 25.70 (D) 25.72 (E) 25.74

4. You are given:

(i) ${}_tp_x'^{(1)} = 10/(10+t)$

(ii) ${}_tp_x'^{(2)} = \big(10/(10+t)\big)^3$

Determine $q_x^{(1)}$.

(A) 0.068 (B) 0.074 (C) 0.079 (D) 0.083 (E) 0.091

5. For a special whole life insurance paying at the moment of death, you are given:

(i) If death occurs in the first 10 years, the benefit is the refund of the single benefit premium with interest at a rate of $\delta' = 0.03$.

(ii) If death occurs after the first 10 years, the benefit is 1000 only.

(iii) $\mu_x(t) = \begin{cases} 0.01 & t \le 10 \\ 0.02 & t > 10 \end{cases}$

(iv) $\delta = 0.06$

Determine the single benefit premium.

(A) 131 (B) 132 (C) 133 (D) 134 (E) 135

6. A drill has two gears, and fails only if both gears fail. The gears are independent, and time to failure of each one is uniformly distributed over $(0, 20]$.

A 3-year warranty covers the drill.

Determine the average amount of time to the earlier of failure of the drill or expiration of the warranty.

(A) 2.5725 (B) 2.5738 (C) 2.7750 (D) 2.8763 (E) 2.9775

7. A continuous deferred life annuity pays 1 per year starting at time 10. If death occurs before time 10, the single benefit premium is refunded without interest at the moment of death. You are given:

(i) $\mu = 0.02$

(ii) $\delta = 0.05$

Calculate the single benefit premium for this annuity.

(A) 8.19 (B) 8.29 (C) 8.38 (D) 8.48 (E) 8.58

Exercises continue on the next page . . .

8. You are given that $A_x = 0.4 + 0.01x$ for $x < 60$.

Calculate ${}_{20}V_{30}$.

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

9. Taxis arrive in a Poisson process at a hotel at a rate of 2 per minute. Fares paid by riders follow a gamma distribution with probability density function

$$f(x) = \frac{x^4 e^{-x/2}}{768}$$

Fares are independent of each other and of the number of taxis.

Calculate the variance of aggregate fares collected by all taxis in an hour.

(A) 3,600 (B) 7,200 (C) 10,800 (D) 14,400 (E) 18,000

10. A special 9-year term insurance pays the following benefit at the end of the year of death:

Year of death t	1	2	3	4	5	6	7	8	9
Benefit b_t	1	2	3	4	5	4	3	2	1

You are given the following values for increasing and decreasing term insurances:

n	$(IA)^1_{x:\overline{n}\rceil}$	$(DA)^1_{x:\overline{n}\rceil}$
4	0.5	0.7
5	0.8	1.0
9	2.3	2.8
10	2.9	3.7

Determine the actuarial present value of the term insurance.

(A) 0.7 (B) 0.8 (C) 1.3 (D) 1.4 (E) 1.7

11. Cars arrive at a toll booth in a Poisson process at the rate of 6 per minute.

Determine the probability that the third car will arrive between 30 and 40 seconds from now.

(A) 0.16 (B) 0.19 (C) 0.24 (D) 0.28 (E) 0.58

12. Each night, your supper has one of four main courses: beef, chicken, fish, or pasta. Your main course is always different from the one of the previous night. Each of the 3 main courses not eaten the previous night are equally likely.

Determine the probability that you will have pasta three nights from now, given that you had beef tonight.

(A) 1/4 (B) 7/27 (C) 5/19 (D) 4/15 (E) 1/3

13. You are given

(i) $e_{85:\overline{20}\rceil} = 18$
(ii) $e_{84:\overline{21}\rceil} = 18.4$
(iii) Uniform distribution of deaths is assumed between integral ages.

Determine m_{84}.

(A) 0.0313 (B) 0.0316 (C) 0.0318 (D) 0.0321 (E) 0.0323

Exercises continue on the next page . . .

14. You are given:

(i) The probability that a milk carton on the shelf is purchased on any day is 20%.
(ii) Milk cartons are discarded after being on the shelf for 7 days.

Determine the average number of full days a purchased milk carton is on the shelf.

(A) 1.69 (B) 1.73 (C) 2.14 (D) 2.18 (E) 2.63

15. For a fully discrete whole life insurance of 1000 with annual premiums payable for life, you are given:

(i) ${}_8V = 210.10$
(ii) ${}_9V = 232.22$
(iii) ${}_{10}V = 255.40$
(iv) $q_{x+8} = q_{x+9}$
(v) $q_{x+10} = 1.1 q_{x+9}$
(vi) $i = 0.04$

Determine ${}_{11}V$.

(A) 276.82 (B) 277.58 (C) 278.35 (D) 279.14 (E) 279.69

16. You are given:

(i) $q_{80} = 0.1$
(ii) $q_{81} = 0.2$
(iii) The force of mortality is constant between integral ages.

Calculate $\mathring{e}_{80.5:\overline{1}|}$.

(A) 0.93 (B) 0.94 (C) 0.95 (D) 0.96 (E) 0.97

17. For a fully continuous whole life insurance of 1000:

(i) The contract premium is 25.
(ii) The variance of loss at issue is 2,000,000.
(iii) $\delta = 0.06$

Employees are able to obtain this insurance for a 20% discount.

Determine the variance of loss at issue for insurance sold to employees.

(A) 1,281,533 (B) 1,295,044 (C) 1,771,626 (D) 1,777,778 (E) 1,825,013

18. You are given:

(i) $c_{40} = 1$
(ii) $c_x = vq_x + vp_x(1 + c_{x+1})$

Determine c_{30}

(A) $\ddot{a}_{30:\overline{10}|} + A^{1}_{30:\overline{10}|}$ (B) $\ddot{a}_{30:\overline{10}|} + A_{30:\overline{10}|}$ (C) $a_{30:\overline{10}|} + A^{1}_{30:\overline{10}|}$ (D) $a_{30:\overline{10}|} + A_{30:\overline{10}|}$ (E) $a_{30:\overline{10}|} + A_{30:\overset{1}{\overline{10}|}}$

Exercises continue on the next page ...

19. Brilliant ideas come to you in a Poisson process at a rate of $1/(2\sqrt{t})$, where t is the amount of time in hours.

Determine the amount of time t for which there is a 95% probability that you will come up with at least one brilliant idea before time t.

(A) 3 (B) 5 (C) 7 (D) 9 (E) 11

20. For a 10-year deferred life annuity-due of 1000 per year, you are given:

(i) Premiums are payable at the beginning of the first 10 years.
(ii) The annual benefit premium payable for 10 years is 800.
(iii) Percent of premium expenses are 5%.
(iv) Expenses incurred with each annuity payment are 10.

Determine the expense-loaded premium.

(A) 848 (B) 850 (C) 851 (D) 853 (E) 854

21. For a fully continuous 5-year deferred whole life insurance of 1000, you are given:

(i) If death occurs during the deferral period, premiums are refunded without interest.
(ii) $\mu_x(t) = 0.02$ for $t \geq 0$.
(iii) $\delta = 0.04$
(iv) Premiums are payable for life.

Calculate the annual benefit premium.

(A) 14.82 (B) 14.90 (C) 14.96 (D) 15.00 (E) 15.05

22. You are given:

(i) $l_{50.5} = 1200$
(ii) $l_{51} = 1100$
(iii) Deaths between integral ages are assumed to follow the hyperbolic assumption.

Determine l_{50}.

(A) 1280 (B) 1290 (C) 1300 (D) 1310 (E) 1320

23. For a fully discrete special life insurance to (45),

(i) The benefit is 1000 if death occurs before age 65, 500 otherwise.
(ii) Annual premiums are payable for the first 20 years only.
(iii) $i = 0.05$
(iv) $A_{45} = 0.2$
(v) ${}_{20}E_{45} = 0.3$
(vi) $\ddot{a}_{45:\overline{20}|} = 12.6$

Determine the annual benefit premium.

(A) 11.24 (B) 11.90 (C) 13.05 (D) 13.16 (E) 15.87

Exercises continue on the next page . . .

24. For a fully continuous whole life insurance of 1000 on 2 independent lives (x) and (y), you are given

(i) Benefits are payable at the moment of the second death.
(ii) Premiums are payable while both are alive.
(iii) $\mu_x(t) = 0.02$ for all t.
(iv) $\mu_y(t) = 0.03$ for all t.
(v) $\delta = 0.05$

Determine the annual benefit premium.

(A) 9.57 (B) 12.50 (C) 16.07 (D) 18.37 (E) 21.43

25. For a fully discrete whole life insurance of 1000 on (x), you are given:

(i) $\ddot{s}_{x:\overline{20}|} = 36$
(ii) ${}_{20}k_x = 0.1$
(iii) $i = 0.04$
(iv) Percent of premium expenses are 5% in all years.
(v) Per 1000 expenses are 1 in all years.
(vi) There are no withdrawals.

Determine the contract premium for which ${}_{20}AS$ is 3% of the accumulated contract premiums.

(A) 3.98 (B) 4.07 (C) 4.11 (D) 4.18 (E) 4.25

26. For two lives (50) and (60) with independent future lifetimes:

(i) $\mu_{50}(t) = 0.002t$
(ii) $\mu_{60}(t) = 0.003t$

Calculate ${}_{20}q^{\;1}_{50:60} - {}_{20}q^{\;\;\;2}_{50:60}$.

(A) 0.17 (B) 0.18 (C) 0.30 (D) 0.31 (E) 0.37

27. You are given that $\mu_x = 0.002x + 0.005$.

Calculate ${}_{5|}q_{20}$.

(A) 0.042 (B) 0.044 (C) 0.046 (D) 0.048 (E) 0.050

28. In a three-decrement model, decrements (1) and (2) are uniformly distributed between integral ages in the associated single decrement tables. Decrement (3) occurs only at the end of a year. You are given:

(i) $l_x^{(\tau)} = 1000$
(ii) $d_x^{(1)} = 90$
(iii) $q_x'^{(2)} = 2q_x'^{(1)}$
(iv) $q_x'^{(3)} = 3q_x'^{(1)}$

Determine $d_x^{(3)}$.

(A) 214 (B) 216 (C) 240 (D) 270 (E) 288

Exercises continue on the next page . . .

29. For a double decrement model with decrements from death (1) and withdrawal (2), you are given:

(i) The following rates in the double-decrement table for (x):

t	Death $q^{(1)}_{x+t-1}$	Withdrawal $q^{(2)}_{x+t-1}$
1	0.003	0.20
2	a	0.15
3	$2a$	0.10

(ii) ${}_3q^{(1)}_x = 0.017985$.

Determine a.

(A) 0.004 (B) 0.005 (C) 0.006 (D) 0.007 (E) 0.008

30. A special 10-year term insurance pays 1 at the moment of death. It also refunds the single benefit premium without interest if the insured survives 10 years. You are given:

(i) $\mu = 0.01$
(ii) $\delta = 0.06$

Calculate the single benefit premium.

(A) 0.12 (B) 0.13 (C) 0.14 (D) 0.15 (E) 0.16

Solutions to the above questions begin on page 1171.

Appendix A. Solutions to the Practice Exams

Answer Key for Practice Exam 1

1	B	11	B	21	D
2	D	12	B	22	E
3	C	13	D	23	B
4	C	14	C	24	C
5	E	15	D	25	C
6	E	16	B	26	B
7	B	17	C	27	A
8	D	18	D	28	B
9	D	19	D	29	D
10	D	20	C	30	C

Practice Exam 1

1. [**Lesson 3**] $0.07508 = {}_2q_{52} = (d_{52}+d_{53})/l_{52} = 72/l_{52}$, so $l_{52} = 72/0.07508 = 959$. But $l_{52} = l_{50} - d_{50} - d_{51} = 1000 - 20 - d_{51}$, so $d_{51} = \boxed{\mathbf{21}}$. **(B)**

2. [**Lesson 50**] The state vector after 1 year is $\begin{pmatrix}0.7 & 0.1 & 0.1 & 0.1\end{pmatrix}$. The state vector after 2 years is $\begin{pmatrix}0.56 & 0.08 & 0.23 & 0.13\end{pmatrix}$. Thus the expected cost of the second year (before taking present values) is

$$0.7(50)+0.1(100)+0.1(200)=65$$

and the expected cost of the third year is

$$0.56(50)+0.08(100)+0.23(200)=82$$

The actuarial present value of 3 years of costs is

$$100+\frac{65}{1.06}+\frac{82}{1.06^2}=\boxed{\mathbf{234.30}} \qquad \textbf{(D)}$$

3. [**Lesson 22**] We determine b.

$$\begin{aligned}
\ddot{a}_{60:\overline{2}|} &= 1+\frac{0.98}{1.05}=1.93333\\
A^{1}_{60:\overline{2}|} &= \frac{0.02}{1.05}+\frac{(0.98)(0.04)}{1.05^2}=0.054603\\
25.41 &= \frac{bA^{1}_{60:\overline{2}|}}{\ddot{a}_{60:\overline{2}|}}
\end{aligned}$$

$$b = \frac{25.41(1.93333)}{0.054603} = 899.69$$

Now we recalculate at 4%.

$$\ddot{a}_{60:\overline{2}|} = 1 + \frac{0.98}{1.04} = 1.94231$$
$$A^{1}_{60:\overline{2}|} = \frac{0.02}{1.04} + \frac{(0.98)(0.04)}{1.04^2} = 0.055473$$
$$899.69P^{1}_{60:\overline{2}|} = 899.69\left(\frac{0.055473}{1.94231}\right) = \boxed{\mathbf{25.695}} \qquad \textbf{(C)}$$

4. [**Lesson 43**]

$$
\begin{aligned}
{}_tp_x^{(\tau)} &= \left(\frac{10}{10+t}\right)\left(\frac{10}{10+t}\right)^3 = \left(\frac{10}{10+t}\right)^4 \\
\mu_x^{(1)}(t) &= -\frac{\mathrm{d}\ln {}_tp_x^{\prime(1)}}{\mathrm{d}t} = \frac{1}{10+t} \\
q_x^{(1)} &= \int_0^1 {}_tp_x^{(\tau)}\mu_x^{(1)}(t)\mathrm{d}t \\
&= \int_0^1 \left(\frac{10}{10+t}\right)^4\left(\frac{1}{10+t}\right)\mathrm{d}t \\
&= \int_0^1 \frac{10^4\mathrm{d}t}{(10+t)^5} \\
&= -\left(\frac{10^4}{4}\right)\left(\frac{1}{(10+t)^4}\right)\Bigg|_0^1 \\
&= \left(\frac{10^4}{4}\right)\left(\frac{1}{10^4} - \frac{1}{11^4}\right) \\
&= \boxed{\mathbf{0.079247}} \qquad \textbf{(C)}
\end{aligned}
$$

5. [**Lesson 11**] The actuarial present value of one unit of a 10-year deferred whole life insurance is ${}_{10}E_x\,\bar{A}_{x+10}$. The force of mortality is constant after 10 years, so

$$\bar{A}_{x+10} = \frac{\mu}{\mu+\delta} = \frac{0.02}{0.02+0.06} = 0.25$$

The pure endowment factor ${}_{10}E_x$ is computed using the mortality rate in effect for the first ten years, so it is $e^{-(0.01+0.06)(10)} = e^{-0.7}$. Therefore, the APV of the 10-year deferred whole life insurance is

$$1000_{10|}\bar{A}_x = 250e^{-0.7}$$

Let P be the single benefit premium. The $\delta' = 0.03$ interest for the benefit in the first ten years partially offsets the $\delta = 0.06$ discount factor, so the APV of the first ten years of insurance is

$$\frac{P\mu}{\mu+\delta-\delta'}\left(1-e^{-10(\mu+\delta-\delta')}\right) = \frac{0.01P}{0.04}\left(1-e^{-0.4}\right)$$

We now solve for P.

$$\begin{aligned}P &= 250e^{-0.7} + P\Big(0.25(1-e^{-0.4})\Big)\\ &= 124.1463 + 0.08242P\\ P &= \frac{124.1463}{1-0.0842} = \boxed{\mathbf{135.30}} \qquad \textbf{(E)}\end{aligned}$$

6. [**Lesson 36**] We want the 3-year temporary future lifetime of the last survivor of the two gears.

$$\begin{aligned}\mathring{e}_{\overline{0:0}:\overline{3}|} &= \int_0^3 {}_tp_{\overline{0:0}}\,\mathrm{d}t\\ &= \int_0^3 \left(1 - {}_tq_0^2\right)\mathrm{d}t\\ &= \int_0^3 \left(1 - \left(\frac{t}{20}\right)^2\right)\mathrm{d}t\\ &= 3 - \left.\frac{t^3}{3(20^2)}\right|_0^3\\ &= 3 - \frac{27}{1200} = \boxed{\mathbf{2.9775}} \qquad \textbf{(E)}\end{aligned}$$

7. [**Lesson 17**] We equate the single benefit premium P and the value of the sum of the annuity and the insurance for the first 10 years.

$$\begin{aligned}P &= {}_{10|}\bar{a}_x + P\bar{A}^{\,1}_{x:\overline{10}|}\\ &= \int_{10}^{\infty} e^{-0.07t}\,\mathrm{d}t + P\int_0^{10} e^{-0.07t}\,0.02\,\mathrm{d}t\\ &= \frac{e^{-0.7}}{0.07} + P\left(\frac{2}{7}\right)\left(1-e^{-0.7}\right)\\ &= \frac{0.4965853}{0.07} + P\left(\frac{2}{7}\right)(0.5034147) = 7.09408 + 0.14383P\\ P &= \frac{7.09408}{1-0.14383} = \boxed{\mathbf{8.286}} \qquad \textbf{(B)}\end{aligned}$$

8. [**Lesson 29**] By the insurance ratio formula (29.2),

$$\begin{aligned}A_{30} &= 0.4 + 0.01(30) = 0.7\\ A_{50} &= 0.4 + 0.01(50) = 0.9\\ {}_{20}V_{30} &= \frac{A_{50}-A_{30}}{1-A_{30}} = \frac{0.9-0.7}{1-0.7} = \boxed{\mathbf{\frac{2}{3}}} \qquad \textbf{(D)}\end{aligned}$$

9. [**Lesson 55**] Let X be fare per rider, S aggregate fares. The Poisson parameter per hour is 120. The gamma distribution has $\alpha = 5$, $\theta = 2$, mean $\alpha\theta = 10$, and variance $\alpha\theta^2 = 20$, and therefore second moment $20 + 10^2 = 120$. By the formula for the variance of a compound Poisson distribution, equation (55.2),

$$\operatorname{Var}(S) = \lambda\,\mathbf{E}[X^2] = (120)(120) = \boxed{\mathbf{14{,}400}} \qquad \textbf{(D)}$$

10. [**Lesson 14**] The benefits are a 9-year decreasing insurance minus twice a 4-year decreasing insurance. $2.8-2(0.7)=\boxed{\mathbf{1.4}}$. **(D)**

11. [**Lesson 51**] The probability that the third car will arrive in the interval $(30,40)$ is the probability of at least 3 cars in 40 seconds minus the probability of at least 3 cars in 30 seconds. For 40 seconds, the Poisson parameter is 4 and the probability is

$$1-e^{-4}\left(1+4+\frac{4^2}{2}\right)=1-0.238103$$

For 30 seconds, the Poisson parameter is 3 and the probability is

$$1-e^{-3}\left(1+3+\frac{3^2}{2}\right)=1-0.423190$$

The difference is $0.423190-0.238103=\boxed{\mathbf{0.185087}}$. **(B)**

12. [**Lesson 49**] This is a Markov chain. The transition matrix going in the beef, chicken, fish, pasta order (actually the order doesn't matter) is

$$\begin{pmatrix} 0 & \frac13 & \frac13 & \frac13 \\ \frac13 & 0 & \frac13 & \frac13 \\ \frac13 & \frac13 & 0 & \frac13 \\ \frac13 & \frac13 & \frac13 & 0 \end{pmatrix}$$

The probabilities of fish, chicken, pasta are 1/3 apiece on the first night, making the state vector $(0,\frac13,\frac13,\frac13)$. Multiplying this by the transition matrix, we get for the second night's state vector

$$\begin{pmatrix} 0 & \frac13 & \frac13 & \frac13 \end{pmatrix}\begin{pmatrix} 0 & \frac13 & \frac13 & \frac13 \\ \frac13 & 0 & \frac13 & \frac13 \\ \frac13 & \frac13 & 0 & \frac13 \\ \frac13 & \frac13 & \frac13 & 0 \end{pmatrix}=\begin{pmatrix} \frac13 & \frac29 & \frac29 & \frac29 \end{pmatrix}$$

For the third night's pasta, we multiply the state vector by the last column of the transition matrix (in order to get the last entry of the third state vector):

$$\frac13\left(\frac13\right)+\frac29\left(\frac13\right)+\frac29\left(\frac13\right)+\frac29(0)=\boxed{\mathbf{\frac{7}{27}}}\qquad\textbf{(B)}$$

13. [**Lesson 8**] By the recursive formula (7.5),

$$\begin{aligned}
e_{84:\overline{21}|}&=p_{84}(1+e_{85:\overline{20}|})\\
18.4&=p_{84}(19)\\
p_{84}&=\frac{18.4}{19}\\
q_{84}&=1-\frac{18.4}{19}=\frac{0.6}{19}\\
m_{84}&=\frac{q_{84}}{1-0.5q_{84}}=\frac{0.6/19}{18.7/19}=\boxed{\mathbf{0.03209}}\qquad\textbf{(D)}
\end{aligned}$$

14. [**Lesson 6**] The average number of full days that a milk carton "survives" on the shelf is $e_{0:\overline{7}|}$. The t-day survival probability is ${}_tp_0 = 0.8^t$ for $t \le 7$.

$$e_{0:\overline{7}|} = \sum_{k=1}^{7} 0.8^k = \frac{0.8 - 0.8^8}{1 - 0.8} = 3.16114$$

Now we remove the contribution of cartons on the shelf 7 full days, $7(0.8^7)$, and divide by the probability of being purchased, $1 - 0.8^7$.

$$\frac{3.16114 - 7(0.8^7)}{1 - 0.8^7} = \boxed{\mathbf{2.1424}} \qquad \textbf{(C)}$$

15. [**Lesson 31**] The two recursions for the reserve from time 8 to time 9 and time 9 to time 10 are, with the common mortality rate denoted by q:

$$(210.10 + P)(1.04) - 1000q = 232.22(1 - q)$$
$$(232.22 + P)(1.04) - 1000q = 255.40(1 - q)$$

We'll solve for q and for P. Subtracting the first equation from the second,

$$\begin{aligned}
22.12(1.04) &= 23.18 - 23.18q \\
q &= -\frac{(22.12)(1.04) - 23.18}{23.18} = 0.00755824 \\
210.10(1.04) + P(1.04) - 7.55824 &= 232.22(1 - 0.00755824) = 230.4648 \\
P(1.04) &= 230.4648 + 7.55824 - 210.10(1.04) = 19.5191 \\
P &= 18.7683 \\
q_{x+10} &= 1.1(0.00755824) = 0.00831406
\end{aligned}$$

Now we calculate ${}_{11}V$.

$${}_{11}V = \frac{(255.40 + 18.7683)(1.04) - 1000(0.00831406)}{1 - 0.00831406} = \boxed{\mathbf{279.1418}} \qquad \textbf{(D)}$$

16. [**Section 8.2**]

$$\begin{aligned}
\mathring{e}_{80.5:\overline{1}|} &= \mathring{e}_{80.5:\overline{0.5}|} + {}_{0.5}p_{80.5}\, \mathring{e}_{81:\overline{0.5}|} \\
&= \frac{\int_{0.5}^{1} 0.9^t \,\mathrm{d}t}{0.9^{0.5}} + 0.9^{0.5} \int_0^{0.5} 0.8^t \,\mathrm{d}t \\
&= \frac{0.9^{0.5} - 1}{\ln 0.9} + \left(0.9^{0.5}\right) \frac{0.8^{0.5} - 1}{\ln 0.8} \\
&= 0.487058 + (0.948683)(0.473116) = \boxed{\mathbf{0.93590}} \qquad \textbf{(B)}
\end{aligned}$$

17. [**Lesson 23**] The variance of loss at issue for a contract premium of 25 is

$$\begin{aligned}
2{,}000{,}000 &= \mathrm{Var}\left(v^T\right)\left(1000 + \frac{25}{0.06}\right)^2 \\
&= \mathrm{Var}\left(v^T\right)(2{,}006{,}944)
\end{aligned}$$

If we replace 25 with 20 (for a 20% discount) in the above formula, it becomes

$$\begin{aligned}\operatorname{Var}({}_0L) &= \operatorname{Var}\left(v^T\right)\left(1000+\frac{20}{0.06}\right)^2\\ &= \operatorname{Var}\left(v^T\right)(1{,}777{,}778)\end{aligned}$$

We see that this is 1,777,778/2,006,944 times the given variance, so the final answer is

$$\operatorname{Var}({}_0L) = \frac{1{,}777{,}778}{2{,}006{,}944}(2{,}000{,}000) = \boxed{\mathbf{1{,}771{,}626}} \qquad \textbf{(C)}$$

18. [**Lesson 19**] Since each c_x pays (1) a one-year insurance (vq_x), (2) a payment of 1 one year from now (vp_x), and (3) next year's c_{x+1}, we need an insurance for the insurance recursion (vq_x) and an annuity for the annuity recursion (vp_x). Since $c_{40} = 1$ rather than 0, it looks like the insurance is an endowment insurance rather than a term insurance. So let's guess that

$$c_x = a_{x:\overline{40-x}|} + A_{x:\overline{40-x}|} \tag{*}$$

and prove it by induction.

First of all, $c_{40} = a_{40:\overline{0}|} + A_{40:\overline{0}|} = 1$ works, since a 0-term annuity is worth nothing and a 0-term endowment insurance pays 1 immediately.

Then we need

$$c_x = vq_x + vp_x(1 + c_{x+1})$$

and expanding c_x with equation (*), we need

$$a_{x:\overline{40-x}|} + A_{x:\overline{40-x}|} = vq_x + vp_x + vp_x\, a_{x+1:\overline{39-x}|} + vp_x\, A_{x+1:\overline{39-x}|} \tag{**}$$

However, by insurance recursion

$$A_{x:\overline{40-x}|} = vq_x + vp_x\, A_{x+1:\overline{39-x}|}$$

and by annuity recursion

$$a_{x:\overline{40-x}|} = vp_x + vp_x\, a_{x+1:\overline{39-x}|}$$

so (**) is true. For $x = 30$, expression (*) is **(D)**

19. [**Lesson 52**] The Poisson parameter at time t for this non-homogeneous Poisson process is

$$\lambda = \int_0^t \frac{\mathrm{d}u}{2\sqrt{u}} = \sqrt{t}$$

We want the probability of 0 to be $1 - 0.95 = 0.05$, so

$$\begin{aligned} e^{-\sqrt{t}} &= 0.05\\ \sqrt{t} &= -\ln 0.05 = \ln 20\\ t &= (\ln 20)^2 = \boxed{\mathbf{8.97}} \qquad \textbf{(D)}\end{aligned}$$

20. [**Lesson 46**] Let G be the expense-loaded premium.

$$G\ddot{a}_{x:\overline{10|}} = 800\ddot{a}_{x:\overline{10|}} + 0.05G\ddot{a}_{x:\overline{10|}} + 10\,{}_{10|}\ddot{a}_x$$
$$0.95G = 800 + 10\left(\frac{{}_{10|}\ddot{a}_x}{\ddot{a}_{x:\overline{10|}}}\right)$$

But since the benefit premium is $1000({}_{10|}\ddot{a}_x/\ddot{a}_{x:\overline{10|}}) = 800$, the last term must be 8.

$$G = \frac{808}{0.95} = \boxed{\mathbf{850.5263}} \qquad \textbf{(C)}$$

21. [**Lesson 21**] Let P be the premium. We need

$$P\bar{a}_x = 1000\,{}_{5|}\bar{A}_x + (P)(\bar{I}\bar{A})^{1}_{x:\overline{5|}}$$

$$\bar{a}_x = \frac{1}{\mu+\delta} = \frac{1}{0.06} = 16.6667$$
$${}_{5|}\bar{A}_x = \frac{\mu e^{-5(\mu+\delta)}}{\mu+\delta} = \frac{e^{-0.3}}{3} = 0.24694$$
$$(\bar{I}\bar{A})^{1}_{x:\overline{5|}} = \int_0^5 t\,e^{-0.06t}\,0.02\mathrm{d}t$$

Using equation (11.4), you can immediately evaluate this integral as $\frac{0.02}{0.06^2}\left(1-\left(1+0.06(5)\right)e^{-(0.06)(5)}\right)$. As an alternative,

$$(\bar{I}\bar{A})^{1}_{x:\overline{5|}} = (\bar{I}\bar{A})_x - {}_5E_x\left((\bar{I}\bar{A})_{x+5} + 5A_{x+5}\right)$$

since the two subtracted items cancel out the increasing insurance after the fifth year. For exponential mortality,

$$(\bar{I}\bar{A})_x = \int_0^\infty \mu t\,e^{-(\mu+\delta)t}\,\mathrm{d}t = \frac{\mu}{(\mu+\delta)^2}$$

by equation (11.2). Here, $\mu = 0.02$ and $\delta = 0.04$, so $(\bar{I}\bar{A})_x = 0.02/0.06^2$. And $\bar{A}_x = \mu/(\mu+\delta) = 0.02/0.06$. Also, ${}_5E_x = e^{-0.06(5)}$. So

$$(\bar{I}\bar{A})^{1}_{x:\overline{5|}} = \frac{0.02}{0.06^2} - e^{-0.06(5)}\left(\frac{0.02}{0.06^2} + \frac{5(0.02)}{0.06}\right)$$
$$= 5.5555 - 5.3504 = 0.2052$$

The premium is

$$P = \frac{1000\,{}_{5|}\bar{A}_x}{\bar{a}_x - (\bar{I}\bar{A})^{1}_{x:\overline{5|}}} = \frac{246.94}{16.6667 - 0.2052} = \boxed{\mathbf{15.001}} \qquad \textbf{(D)}$$

22. [**Section 8.3**]

$$\frac{1}{1200} = \frac{0.5}{l_{50}} + \frac{0.5}{1100}$$
$$\frac{11l_{50}}{12} = 0.5(1100) + 0.5l_{50}$$
$$\frac{5l_{50}}{12} = 550$$
$$l_{50} = \frac{550(12)}{5} = \boxed{\mathbf{1320}} \qquad \textbf{(E)}$$

23. [**Lesson 22**] Let P be the annual benefit premium.

$$\begin{aligned}
A_{45:\overline{20}|} &= 1 - d\,\ddot{a}_{45:\overline{20}|}\\
&= 1 - \left(\tfrac{1}{21}\right)(12.6) = 0.4\\
A^{\,1}_{45:\overline{20}|} &= A_{45:\overline{20}|} - {}_{20}E_{45} = 0.4 - 0.3 = 0.1\\
{}_{20|}A_{45} &= A_{45} - A^{\,1}_{45:\overline{20}|} = 0.2 - 0.1 = 0.1\\
P &= \frac{1000A^{\,1}_{45:\overline{20}|} + 500\,{}_{20|}A_{45}}{\ddot{a}_{45:\overline{20}|}}\\
&= \frac{100+50}{12.6} = \boxed{\mathbf{11.9048}} \qquad \textbf{(B)}
\end{aligned}$$

24. [**Lesson 37**]

$$\begin{aligned}
\bar{A}_{\overline{xy}} &= \bar{A}_x + \bar{A}_y - \bar{A}_{xy}\\
&= \frac{0.02}{0.07} + \frac{0.03}{0.08} - \frac{0.05}{0.10} = 0.16071\\
\bar{a}_{xy} &= \frac{1}{0.02+0.03+0.05} = 10
\end{aligned}$$

The annual benefit premium is $1000(0.16071/10) = \boxed{\mathbf{16.071}}$. **(C)**

25. [**Lesson 48**] Let G be the contract premium. Then the accumulated premiums after percent of premium expenses are $0.95\ddot{s}_{x:\overline{20}|}G = 34.2G$, the accumulated per 1000 expenses are $\ddot{s}_{x:\overline{20}|} = 36$, and the accumulated benefits are $1000\,{}_{20}k_x = 100$, so we equate

$$\begin{aligned}
34.2G - 36 - 100 &= 0.03(36)G\\
33.12G &= 136\\
G &= \frac{136}{33.12} = \boxed{\mathbf{4.10628}} \qquad \textbf{(C)}
\end{aligned}$$

26. [**Section 39.1**] ${}_{20}q^{\,1}_{50:60} - {}_{20}q^{\,2}_{50:60} = {}_{20}q_{50}\,{}_{20}p_{60}$, and

$$\begin{aligned}
{}_{20}q_{50} &= 1 - \exp\left(-\int_0^{20} 0.002t\,\mathrm{d}t\right)\\
&= 1 - e^{-0.001(20)^2} = 1 - 0.670320 = 0.329680\\
{}_{20}p_{60} &= \exp\left(-\int_0^{20} 0.003t\,\mathrm{d}t\right)\\
&= e^{-0.0015(20)^2} = 0.548812\\
{}_{20}q_{50}\,{}_{20}p_{60} &= (0.329680)(0.548812) = \boxed{\mathbf{0.180932}} \qquad \textbf{(B)}
\end{aligned}$$

27. [**Lesson 4**] ${}_{5|}q_{20} = \big(s(25) - s(26)\big)/s(20)$, so we will calculate these three values of $s(x)$. (Equivalently, one could calculate ${}_5p_{20}$ and ${}_6p_{20}$ and take the difference.) The integral of μ_x is

$$\int_0^x \mu_u\,\mathrm{d}u = \left(\frac{0.002u^2}{2} + 0.005u\right)\Bigg|_0^x = 0.001x^2 + 0.005x$$

so

$$\begin{aligned}
s(20) &= \exp\Big(-\big(0.001(20^2)+0.005(20)\big)\Big) = \exp(-0.5) = 0.606531\\
s(25) &= \exp\Big(-\big(0.001(25^2)+0.005(25)\big)\Big) = \exp(-0.75) = 0.472367\\
s(26) &= \exp\Big(-\big(0.001(26^2)+0.005(26)\big)\Big) = \exp(-0.806) = 0.446641
\end{aligned}$$

and the answer is

$$_{5|}q_{20} = \frac{0.472367-0.446641}{0.606531} = \boxed{\mathbf{0.042415}} \qquad \textbf{(A)}$$

28. [**Lesson 44**] From $d_x^{(1)} = 90$, $q_x^{(1)} = \frac{90}{1000} = 0.09$. Then

$$\begin{aligned}
q_x^{(1)} &= q_x'^{(1)}\left(1-\frac{q_x'^{(2)}}{2}\right)\\
0.09 &= q_x'^{(1)}\left(1-q_x'^{(1)}\right)\\
\left(q_x'^{(1)}\right)^2 &- q_x'^{(1)} + 0.09 = 0\\
q_x'^{(1)} &= \frac{1\pm\sqrt{0.64}}{2} = 0.1, 0.9
\end{aligned}$$

The solution 0.9 is rejected since then $q_x'^{(2)} > 1$.

$$q_x'^{(2)} = 0.2 \qquad q_x'^{(3)} = 0.3$$

Since (3) occurs at the end of the year, only $l_x^{(\tau)} p_x'^{(1)} p_x'^{(2)} = (1000)(0.9)(0.8) = 720$ lives are subject to it. So $d_x^{(3)} = 720(0.3) = \boxed{\mathbf{216}}$. **(B)**

29. [**Lesson 41**] We have $_{1|2}q_x^{(1)} = {}_3q_x^{(1)} - q_x^{(1)} = 0.017985 - 0.003 = 0.014985$. Also, $p_x^{(\tau)} = 1 - 0.003 - 0.20 = 0.797$. We set up an equation for $_{1|2}q_x^{(1)}$ and solve.

$$\begin{aligned}
{}_{1|}q_x^{(1)} + {}_{2|}q_x^{(1)} &= {}_{1|2}q_x^{(1)}\\
(0.797)(a) + (0.797)(1-a-0.15)(2a) &= 0.014985\\
0.797a + 1.3549a - 1.594a^2 &= 0.014985\\
1.594a^2 - 2.1519a + 0.014985 &= 0\\
a = \frac{2.1519-\sqrt{4.535129}}{3.188} &= \boxed{\mathbf{0.007}} \qquad \textbf{(D)}
\end{aligned}$$

The other solution to the quadratic is rejected since it is greater than 1.

30. [**Lesson 11**] Let P be the single benefit premium.

$$\begin{aligned}
P &= \int_0^{10} e^{-0.07t}\,0.01\,\mathrm{d}t + Pe^{-0.7}\\
P\left(1-e^{-0.7}\right) &= \frac{1}{7}\left(1-e^{-0.7}\right)\\
P &= \frac{1}{7} = \boxed{\mathbf{0.142857}} \qquad \textbf{(C)}
\end{aligned}$$