Contents

Lesson 1

Put-Call Parity

Reading: *Derivatives Markets* 9.1–9.2

I expect one or more exam questions based on this lesson.

1.1 Review of derivative instruments

This review section will not be directly tested on. It is background material from Exam FM/2 that you are expected to already know. If you do not wish to review, proceed immediately to Section 1.2, page 10.

1.1.1 Forwards

A forward is an agreement to buy something at a future date for a certain price. We will use the notation $F_{t,T}$ to indicate the price to be paid at time T in a forward agreement made at time t to buy an item at time T. Notice that no payment is made at time t; the only payment made is at time T. If you purchase a forward on a stock at time t, you will pay $F_{t,T}$ at time T and receive the stock.

In practice, there may be no physical transfer of the stock. Instead, settlement may be achieved by your receiving the difference of the price of the stock at time T and $F_{t,T}$. If we let S_k be the price of the stock at time k, then you would receive $S_T - F_{t,T}$, which may be positive or negative. Thus the payoff is a linear function of the value of the stock at time T. Suppose we assume $F_{t,T} = 40$. If the stock price at time T is 30, you then pay 10. If the stock price at time T is 50, you then receive 10. The payoff at time T as a function of the stock price at time T, S_T, is shown in Figure 1.1

A forward agreement is a customized contract. A futures contract, in contrast, is a standardized exchange-traded contract which is similar to a forward in that it is an agreement to pay a certain amount at time T for a certain asset. However, a futures contract is marked to market daily; a payoff of its value is made between the parties each day. There are other differences between forwards and futures. However, in this course, where we are only interested in pricing them, we will not differentiate between them.

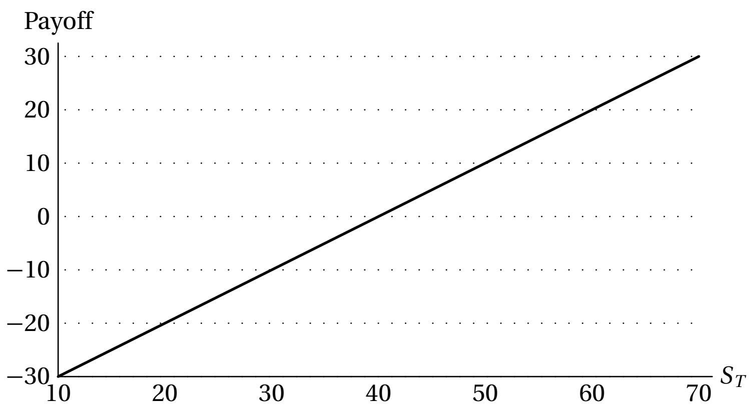


Figure 1.1: Payoff on a forward having price 40. The payoff is $S_T - F_{t,T}$, where $F_{t,T} = 40$.

Let us determine $F_{t,T}$. As we do so, we shall introduce assumptions and terminology used throughout the course. One assumption we make throughout the course is that there is a risk-free interest paying investment, an investment which will never default. If you invest K in this investment, you are sure to receive $K(1+i)$ at the end of a year, where i is some interest rate. We will always express interest as a continuously compounded rate instead of as an annual effective rate, unless we say otherwise. We will use the letter r for this rate. In Financial Mathematics, the letter δ is used for this concept, but we will be using δ for a different (although related) concept. As you learned in Financial Mathematics, $1+i=e^r$. So if the risk-free effective annual return was 5%, we would say that $r=\ln 1.05=0.04879$ rather than 0.05. In real life, Treasury securities play the role of a risk-free interest paying investment.

Here are the assumptions we make:

1. It is possible to borrow or lend any amount of money at the risk-free rate.

2. There are no transaction charges or taxes.

3. *Arbitrage* is impossible.

An *Arbitrage* is a set of transactions which when combined have no cost, no possibility of loss and at least some possibility (if not certainty) of profit. In order to make arbitrage impossible, if there are two sets of transactions leading to the same end result, they must both have the same price, since otherwise you could enter into one set of transactions as a seller and the other set as a buyer and be assured a profit with no possibility of loss.

Forwards on stock

We will consider 3 possibilities for the stock:

1. The stock pays no dividends.

2. The stock pays discrete dividends.

3. The stock pays continuous dividends.

Forwards on nondividend paying stock We begin with a forward on a nondividend paying stock. The forward agreement is entered at time 0 and provides for transferring the stock at time T. Let S_t be the price of the stock at time t. There are two ways you can own the stock at time T:

Two Methods for Owning Non-Dividend Paying Stock at Time T

	Method #1: *Buy stock at time 0 and hold it to time T*	Method #2: *Buy forward on stock at time 0 and hold it to time T*
Payment at time 0	S_0	0
Payment at time T	0	$F_{0,T}$

By the principle of no arbitrage, the two ways must have the same cost. The cost at time 0 of the first way is S_0, the price of the stock at time 0. The cost of the second way is a payment of $F_{0,T}$ at time T. Since you can invest at the risk-free rate, every dollar invested at time 0 becomes e^{rT} dollars at time T. You can pay $F_{0,T}$ at time T by investing $F_{0,T}e^{-rT}$ at time 0. We thus have

$$F_{0,T}e^{-rT}=S_0$$
$$F_{0,T}=S_0e^{rT}$$

We have priced this forward agreement.

Forwards on a stock with discrete dividends Now let's price a forward on a dividend paying stock. This means it pays known cash amounts at known times. There are the same two ways to own a stock at time T:

Two Methods for Owning Dividend Paying Stock at Time T

	Method #1: *Buy stock at time 0 and hold it to time T*	Method #2: *Buy forward on stock at time 0 and hold it to time T*
Payment at time 0	S_0	0
Payment at time T	0	$F_{0,T}$

However, if you use the first way, you will also have the stock's dividends accumulated with interest, which you won't have with the second way. Therefore, the price of the second way as of time T must be cheaper than the price of the first way as of time T by the accumulated value of the dividends. Equating the 2 costs, and letting CumValue(Div) be the accumulated value at time T of dividends from time 0 to time T, we have

$$F_{0,T} = S_0 e^{rT} - \text{CumValue(Div)} \tag{1.1}$$

Forwards on a stock index with continuous dividends We will now consider an asset that pays dividends at a continuously compounded rate which get reinvested in the asset. In other words, rather than being paid as cash dividends at certain times, the dividends get reinvested in the asset continuously, so that the investor ends up with additional shares of the asset rather than cash dividends. The continuously compounded dividend rate will be denoted δ.

This could be a model for a stock index. A stock index consists of many stocks paying dividends at various times. As a simplification, we assume these dividends are uniform.

We will often model a single stock, not just a stock index, as if it had continuous reinvested dividends, using δ instead of explicit dividends.

Let's price a forward on a stock index paying dividends at a rate of δ. If you buy the stock index at time 0 and it pays continuous dividends at the rate δ, you will have $e^{\delta T}$ shares of the stock index at time T. To replicate this with forwards, since *the forward pays no dividends*, you would need to enter a forward agreement to purchase $e^{\delta T}$ shares of the index. So here are two ways to own $e^{\delta T}$ shares of the stock index at time T:

Two Methods for Owning $e^{\delta T}$ Shares of Continuous Dividend Paying Stock Index at Time T

	Method #1: *Buy stock index at time 0 and hold it to time T*	Method #2: *Buy $e^{\delta T}$ forwards on stock index at time 0 and hold it to time T*
Payment at time 0	S_0	0
Payment at time T	0	$e^{\delta T} F_{0,T}$

Thus to equate the two ways, you should buy $e^{\delta T}$ units of the forward at time 0. At time T, the accumulated cost of buying the stock index at time 0 is $S_0 e^{rT}$, while the cost of $e^{\delta T}$ units of the forward agreement is $F_{0,T} e^{\delta T}$, so we have

$$\begin{aligned} F_{0,T} e^{\delta T} &= S_0 e^{rT} \\ F_{0,T} &= S_0 e^{(r-\delta)T} \end{aligned} \tag{1.2}$$

We see here that δ and r work in opposite directions. We will find in general that δ tends to work like a negative interest rate.

Quiz 1-1 [1] For a stock, you are given:

(i) It pays quarterly dividends of 0.20.
(ii) It has just paid a dividend.
(iii) Its price is 50.
(iv) The continuously compounded risk-free interest rate is 5%.

Calculate the forward price for an agreement to deliver 100 shares of the stock six months from now.

A forward on a commodity works much like a forward on a nondividend paying stock. If the commodity produces income (for example if you can lease it out), the income plays the role of a dividend; if the commodity requires expenses (for example, storage expenses), these expenses work like a negative dividend.

Forwards on currency

For forwards on currency, it is assumed that each currency has its own risk-free interest rate. The risk-free interest rate for the foreign currency plays the role of a continuously compounded dividend on a stock.

For example, suppose a forward agreement at time 0 provides for the delivery of euros for dollars at time T. Let $r_\$$ be the risk-free rate in dollars and $r_€$ the risk-free rate in euros. Let x_0 be the \$/€ rate at time 0. The two ways to have a euro at time T are:

1. Buy $e^{-r_€ T}$ euros at time 0 and let them accumulate to time T.
2. Buy a forward for 1 euro at time T, with a payment of $F_{0,T}$ at time T.

The first way costs $x_0 e^{-r_€ T}$ dollars at time 0. The second way can be funded in dollars at time 0 by putting aside $e^{-r_\$ T} F_{0,T}$ dollars at time 0. Thus

$$\begin{aligned} e^{-r_\$ T} F_{0,T} &= x_0 e^{-r_€ T} \\ F_{0,T} &= e^{(r_\$ - r_€)T} x_0 \end{aligned}$$

Quiz 1-2 A yen-denominated forward agreement provides for the delivery of \$100 at the end of 3 months. The continuously compounded risk-free rate for dollars is 5%, and the continuously compounded risk-free rate for yen is 2%. The current exchange rate is 110¥/\$.

Calculate the forward price in yen for this agreement.

A summary of the forward formulas is in Table 1.1. This table makes the easy generalization from agreements entered at time 0 to agreements entered at time t.

It is also possible to have forwards on bonds, and bond interest would play the same role as stock dividends in such a forward. More typical are forward rate agreements, which guarantee an interest rate.

1.1.2 Call and put options

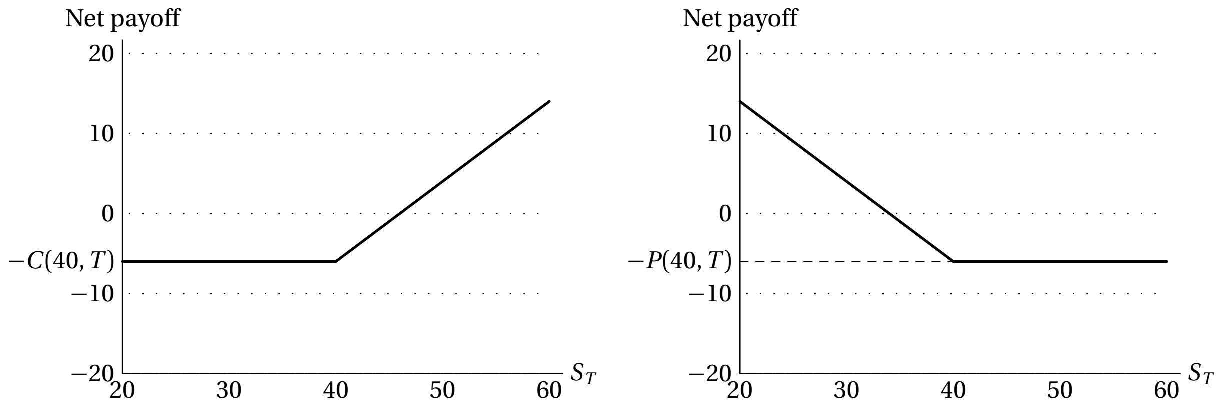
A call option permits, but does not require, the purchaser to pay a specific amount K at time T in return for an asset. In a call option, there are two cash flows: a payment of the value of the call option is made at time 0, and a possible payment of K in return for an asset is made at time T. The payment at time 0, which is the premium of the call, is denoted by $C(K,T)$.[2] At time T, rather than actually paying K and receiving the asset, a monetary

[1] Solutions to quizzes are at the end of each lesson, after the solutions to the exercises.

[2] I am following McDonald textbook notation, which is not consistent. At this point, C and P have two arguments, but later on, they will have three arguments: S, K, and T, where S is the stock price. When we get to Black-Scholes (Lesson 9), it will have six arguments. Sometimes the arguments will be omitted, and sometimes C will be used generically for both call and put options.

Table 1.1: Forward prices at time t for settlement at time T

Underlying asset	Forward price
Non-dividend paying stock	$S_t e^{r(T-t)}$
Dividend paying stock	$S_t e^{r(T-t)} - \text{CumValue(Divs)}$
Stock index	$S_t e^{(r-\delta)(T-t)}$
Currency, denominated in currency d for delivery of currency f	$x_t e^{(r_d-r_f)(T-t)}$



(a) Payoff net of initial investment on call option

(b) Payoff net of initial investment on put option

Figure 1.2: Payoff net of initial investment (ignoring interest) on option with a strike price of 40

settlement of the difference is usually made. If S_T is the value of the asset at time T, the payoff is $\max(0, S_T - K)$. The one who sells the call option is called the writer of the call option.

A put option is the counterpart to a call option. It permits, but does not require, the purchaser to sell an asset at time T for a price of K. Thus there is a payment at time 0 of the premium of the put option, and a possible receipt of K in return for the asset at time T. The payment at time 0, which is the premium of the put, is denoted $P(K,T)$. The monetary settlement at time T if the asset is then worth S_T is $\max(0, K - S_T)$.

K is called the *strike* price.

Figure 1.2 shows the net payoff of a call or a put, assuming the strike price is 40. The cost of the option is added to the payoff at the time T. Interest is ignored, although we won't ignore it when we price the option.

The options we have described are *European options*. European options can only be exercised at time T, not before. In contrast, *American options* allow the option to be exercised at any time up to time T.

1.1.3 Combinations of options

Various strategies involving buying or selling more than one option are possible. These are discussed in chapter 3 of the McDonald textbook, and you learn about them in Course FM/2. It is probably not necessary to know about them for this exam, since exam questions about them will define them for you. Nevertheless, here is a short summary of them.

When we buy X, we are said to be *long* X, and when we sell X, we are said to be *short* X. Long and short may be used as adjectives or verbs.

Option strategies involving two options may involve buying an option and selling another option of the same kind (both calls or both puts), buying an option of one kind and selling one of the other kind, or buying two options of different kinds (which would be selling two options of different kinds from the 's perspective). We will be assuming European options for simplicity.

Spreads: buying an option and selling another option of the same kind

An example of a spread is buying an option with one strike price and selling an option with a different strike price. Such a spread is designed to pay off if the stock moves in one direction, but subject to a limit.

Bull spreads A *bull spread* pays off if the stock moves up in price, but subject to a limit. To create a bull spread with calls, buy a K_1-strike call and sell a K_2-strike call, $K_2 > K_1$. Then at expiry time T,

1. If $S_T \le K_1$, neither option pays.
2. If $K_1 < S_T \le K_2$, the lower-strike option pays $S_T - K_1$, which is the net payoff.
3. If $S_T > K_2$, the lower-strike option pays $S_T - K_1$ and the higher-strike option pays $S_T - K_2$, so the net payoff is the difference, or $K_2 - K_1$.

To create a bull spread with puts, buy a K_1-strike put and sell a K_2-strike put, $K_2 > K_1$. Then at expiry time T,

1. If $S_T \le K_1$, the lower-strike option pays $K_1 - S_T$ and the higher-strike option pays $K_2 - S_T$, for a net payoff of $K_1 - K_2 < 0$.
2. If $K_1 < S_T \le K_2$, the lower-strike option is worthless and the higher-strike option pays $K_2 - S_T$, so the net payoff is $S_T - K_2 < 0$.
3. If $S_T > K_2$ both options are worthless.

Although all the payoffs to the purchaser of a bull spread with puts are non-positive, they increase with increasing stock price. Since the position has negative cost (the option you bought is cheaper than the option you sold), you will gain as long as the absolute value of the net payoff is lower than the net amount received at inception, and the absolute value of the net payoff decreases as the stock price increases.

A diagram of the net profit on a bull spread is shown in Figure 1.3a.

Bear spreads A *bear spread* pays off if the stock price moves down in price, but subject to a limit. To create a bear spread with puts, buy a K_2-strike put and sell a K_1-strike put, $K_2 > K_1$. Then

1. If $S_T \ge K_2$, neither option pays.
2. If $K_2 > S_T \ge K_1$, the higher-strike option pays $K_2 - S_T$.
3. If $S_T < K_1$, the higher-strike option pays $K_2 - S_T$ and the lower-strike option pays $K_1 - S_T$ for a net payoff of $K_2 - K_1$.

To create a bear spread with calls, buy a K_2-strike call and sell a K_1-strike call, $K_2 > K_1$. Then

1. If $S_T \ge K_2$, the higher-strike option pays $S_T - K_2$ and the lower-strike option pays $S_T - K_1$ for a net payoff of $K_1 - K_2 < 0$.
2. If $K_2 > S_T \ge K_1$, the higher-strike option is worthless and the lower-strike option pays $S_T - K_1$ for a net payoff of $K_1 - S_T < 0$.
3. If $S_T < K_1$, both options are worthless.

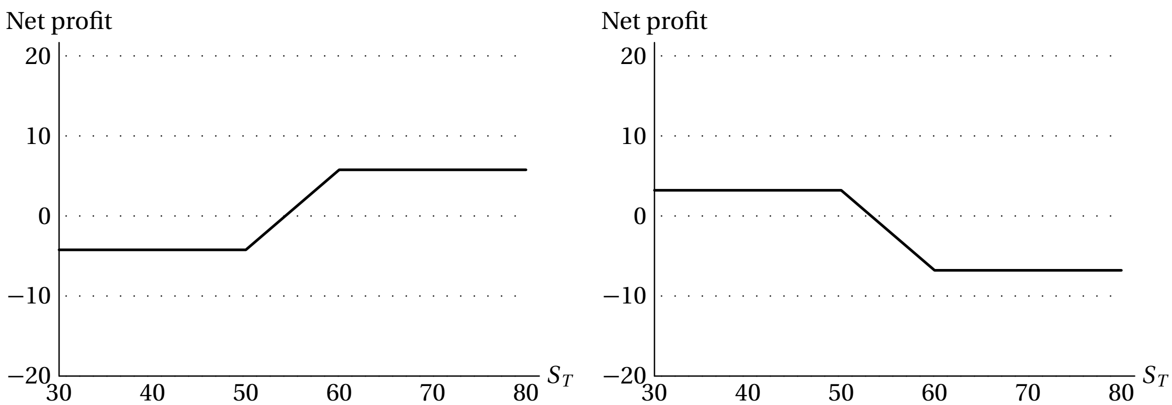


Figure 1.3: Net profits (price accumulated at interest plus final settlement) of spreads using European options, assuming strike prices of 50 and 60

Although all the payoffs for a bear spread with calls are non-positive, they increase with decreasing stock price. Since the option you bought is cheaper than the option you sold, you will gain if the stock price is lower.

A diagram of the net profit on a bear spread is shown in Figure 1.3b.

A bull spread from the perspective of the purchaser is a bear spread from the perspective of the writer.

Ratio spreads A *ratio spread* involves buying n of one option and selling m of another option of the same type where $m \neq n$. It is possible to make the net initial cost of this strategy zero.

Box spreads A *box spread* is a four-option strategy consisting of buying a bull spread of calls with strikes K_2 and K_1 ($K_2 > K_1$) and buying a bear spread of puts with strikes K_2 and K_1. This means buying and selling the following:

Strike	Bull Spread	Bear Spread
K_1	Buy call	Sell put
K_2	Sell call	Buy put

Assuming European options, whenever you buy a call and sell a put at the same strike price, exercise by one of the parties is certain (unless the stock price is the strike price at maturity, in which case both options are worthless), so it is equivalent to a forward. Thus a box spread consists of an agreement to buy the stock for K_1 and sell it for K_2, with definite profit $K_2 - K_1$. If priced correctly, there will be no gain or loss regardless of the stock price at maturity.

Butterfly spreads A *butterfly spread* is a three-option strategy, all options of the same type, consisting of buying n bull spreads with strike prices K_1 and $K_2 > K_1$ and selling m bull spreads with strike prices K_2 and $K_3 > K_2$, with m and n selected so that (for calls) if the stock price S_T at expiry is greater than K_1, the payoffs net to zero. (If puts are used, arrange it so that the payoffs are zero if $S_T < K_3$.)

Let's work out the relationship between m and n for a butterfly spread with calls.

- For the n bull spreads which you buy with strikes K_2 and K_1, the payoff is $K_2 - K_1$ when the expiry stock price is $K_3 > K_2 > K_1$.

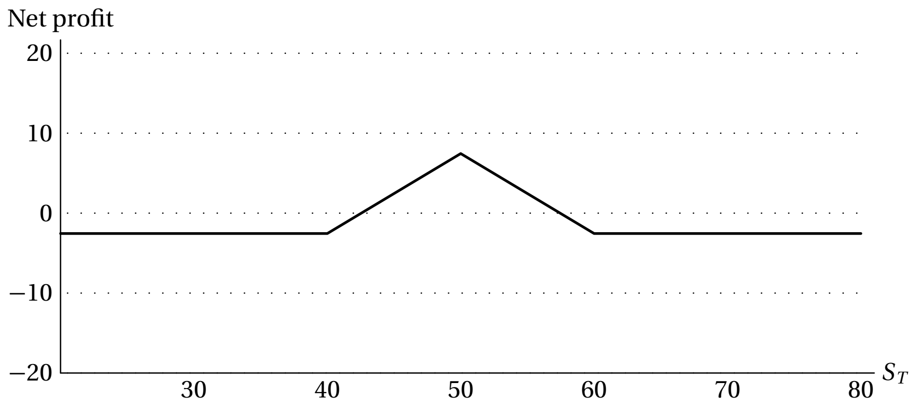


Figure 1.4: Net profit on butterfly spread. European call options with strikes 40, 50, and 60 are used.

- For the m bull spreads which you sell with strikes K_3 and K_2, the payoff is $K_3 - K_2$ when the expiry stock price is $K_3 > K_2$.

Adding up the payoffs:

$$n(K_2 - K_1) - m(K_3 - K_2) = 0$$
$$n(K_2 - K_1) = m(K_3 - K_2)$$
$$\frac{n}{m} = \frac{K_3 - K_2}{K_2 - K_1}$$

In the simplest case, $m = n$ and K_2 is half way between K_1 and K_3; this is a *symmetric* butterfly spread. Otherwise the butterfly spread is asymmetric.

The payoff on a butterfly spread is 0 when $S_T < K_1$. It then increases with slope n until it reaches its maximum at $S_T = K_2$, at which point it is $n(K_2 - K_1)$. It then decreases with slope m until it reaches 0 at $S_T = K_3$, and is 0 for all $S_T > K_3$.

A diagram of the net profit is in Figure 1.4. There is no free lunch, so the 3 options involved must be priced in such a way that a butterfly spread loses money if the final stock price is below K_1 or above K_3. This implies that option prices as a function of strike prices must be *convex*. This will be discussed on page 40.

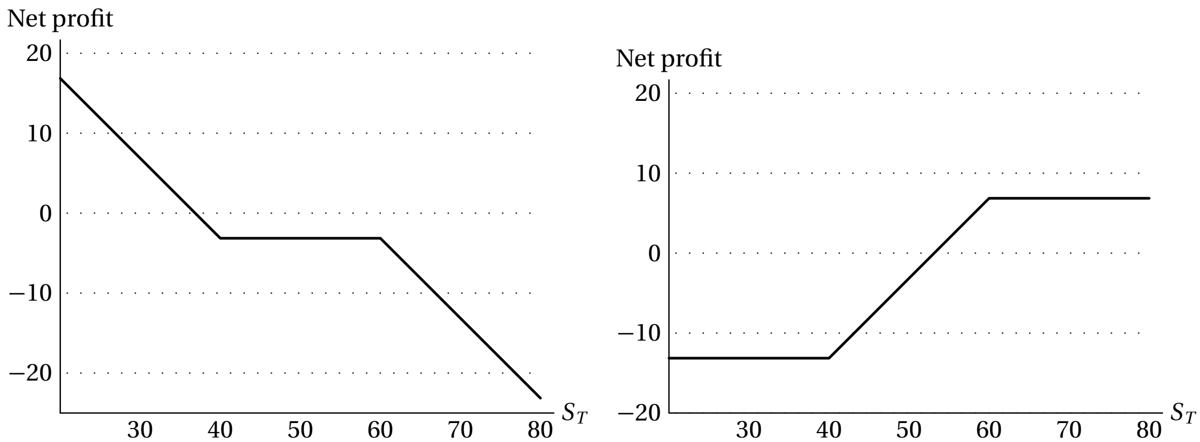
Calendar spreads *Calendar spreads* involve buying and selling options of the same kind with different expiry dates. They will be discussed in Subsection 11.1.2.

Collars: buying one option and selling an option of the other kind

In a collar, you sell a call with strike K_2 and buy a put with strike $K_1 < K_2$. Note that if we allow $K_1 = K_2$, exercise of the option is guaranteed at expiry time T, unless $S_T = K_1$ in which case both options are worthless. There is no risk, and if the options are priced fairly no profit or loss is possible. This will be the basis of put-call parity, which we discuss in the next section.

A collar's payoff increases as the price of the underlying stock decreases below K_1 and decreases as the price of the underlying stock increases above K_2. Between K_1 and K_2, the payoff is flat. A diagram of the net profit is in Figure 1.5a.

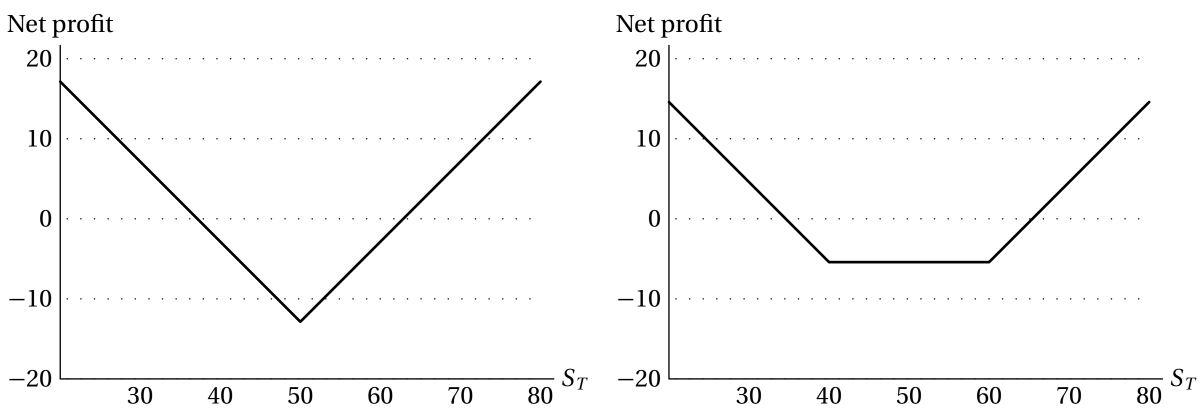
If you own the stock together with a collar, the result is something like a bull spread, as shown in Figure 1.5b.



(a) Net profit on collar

(b) Net profit on collared stock

Figure 1.5: Net profits (price accumulated at interest plus final settlement) of a collar and of a collared stock using European options, assuming strike prices of 40 and 60 and current stock price of 50



(a) Net profit on straddle

(b) Net profit on strangle

Figure 1.6: Net profits (price accumulated at interest plus final settlement) of a straddle and of a strangle using European options, assuming strike prices of 50 for straddle and 40 and 60 for strangle

Straddles: buying two options of different kinds

In a straddle, you buy a call and a put, both of them at-the-money ($K = S_0$). If they both have the same strike price, the payoff is $|S_T - S_0|$, growing with the absolute change in stock price, so this is a bet on volatility. You have to pay for both options. To lower the initial cost, you can buy a put with strike price K_1 and buy a call with strike price $K_2 > K_1$; then this strategy is called a *strangle*.

Diagrams of the net profits of a straddle and a strangle are shown in Figure 1.6.

We're done with review.

1.2 Put-call parity

In this lesson and the next, rather than presenting a model for stocks or other assets so that we can price options, we discuss general properties that are true regardless of model. In this lesson, we discuss the relationship between the premium of a call and the premium of a put.

Suppose you bought a European call option and sold a European put option, both having the same underlying asset, the same strike, and the same time to expiry. *In this entire section, we will deal only with European options, not American ones,* so henceforth "European" should be understood. As above, let the value of the underlying asset be S_t at time t. You would then pay $C(K,T)-P(K,T)$ at time 0. Interestingly, an equivalent result can be achieved without using options at all! Do you see how?

The point is that at time T, one of the two options is sure to be exercised, unless the price of the asset at time T happens to exactly equal the strike price($S_T=K$), in which case both options are worthless. Whichever option is exercised, you pay K and receive the underlying asset:

- If $S_T>K$, you exercise the call option you bought. You pay K and receive the asset.
- If $K>S_T$, the counterparty exercises the put option you sold. You receive the asset and pay K.
- If $S_T=K$, it doesn't matter whether you have K or the underlying asset.

Therefore, there are two ways to receive S_T at time T:

1. Buy a call option and sell a put option at time 0, and pay K at time T.
2. Enter a forward agreement to buy S_T, and at time T pay $F_{0,T}$, the price of the forward agreement.

By the no-arbitrage principle, these two ways must cost the same. Discounting to time 0, this means

$$C(K,T)-P(K,T)+Ke^{-rT}=F_{0,T}e^{-rT}$$

or

Put-Call Parity

$$C(K,T)-P(K,T)=e^{-rT}(F_{0,T}-K) \tag{1.3}$$

Here's another way to derive the equation. Suppose you would like to have the maximum of S_T and K at time T. What can you buy now that will result in having this maximum? There are two choices:

- You can buy a time-T forward on the asset, and buy a put option with expiry T and strike price K. The forward price is $F_{0,T}$, but since you pay at time 0, you pay $e^{-rT}F_{0,T}$ for the forward.
- You can buy a risk-free investment maturing for K at time T, and buy a call option on the asset with expiry T and strike price K. The cost of the risk-free investment is Ke^{-rT}.

Both methods must have the same cost, so

$$e^{-rT}F_{0,T}+P(S,K,T)=Ke^{-rT}+C(S,K,T)$$

which is the same as equation (1.3).

The Put-Call Parity equation lets you price a put once you know the price of a call.

Let's go through specific examples.

1.2.1 Stock put-call parity

For a nondividend paying stock, the forward price is $F_{0,T} = S_0 e^{rT}$. Equation (1.3) becomes

$$C(K,T) - P(K,T) = S_0 - Ke^{-rT}$$

The right hand side is the present value of the asset minus the present value of the strike.

EXAMPLE 1A A nondividend paying stock has a price of 40. A European call option allows buying the stock for 45 at the end of 9 months. The continuously compounded risk-free rate is 5%. The premium of the call option is 2.84.

Determine the premium of a European put option allowing selling the stock for 45 at the end of 9 months.

ANSWER: Don't forget that 5% is a continuously compounded rate.

$$\begin{aligned} C(K,T) - P(K,T) &= S_0 - Ke^{-rT} \\ 2.84 - P(K,T) &= 40 - 45e^{-(0.05)(0.75)} \\ 2.84 - P(K,T) &= 40 - 45(0.963194) = -3.34375 \\ P(K,T) &= 2.84 + 3.34375 = \boxed{\mathbf{6.18375}} \end{aligned}$$

□

A convenient concept is the *prepaid forward*. This is the same as a regular forward, except that the payment is made at the time the agreement is entered, time $t = 0$, rather than at time T. We use the notation $F^P_{t,T}$ for the value at time t of a prepaid forward settling at time T. Then $F^P_{0,T} = e^{-rT} F_{0,T}$. More generally, if the forward maturing at time T is prepaid at time t, $F^P_{t,T} = e^{-r(T-t)} F_{t,T}$, so we can translate the formulas in Table 1.1 into the ones in Table 1.2. In this table $\mathrm{PV}_{t,T}$ is the present value at time t of a payment at time T. Using prepaid forwards, the put-call parity formula becomes

Table 1.2: Prepaid Forward Prices at time t for settlement at time T

Underlying asset	Forward price	Prepaid forward price
Non-dividend paying stock	$S_t e^{r(T-t)}$	S_t
Dividend paying stock	$S_t e^{r(T-t)} - \mathrm{CumValue(Divs)}$	$S_t - \mathrm{PV}_{t,T}(\mathrm{Divs})$
Stock index	$S_t e^{(r-\delta)(T-t)}$	$S_t e^{-\delta(T-t)}$
Currency, denominated in currency d for delivery of currency f	$x_t e^{(r_d - r_f)(T-t)}$	$x_t e^{-r_f(T-t)}$

$$\boxed{C(K,T) - P(K,T) = F^P_{0,T} - Ke^{-rT}} \tag{1.4}$$

Using this, let's discuss a dividend paying stock. If a stock pays discrete dividends, the formula becomes

$$C(K,T) - P(K,T) = S_0 - \mathrm{PV}_{0,T}(\mathrm{Divs}) - Ke^{-rT} \tag{1.5}$$

EXAMPLE 1B A stock's price is 45. The stock will pay a dividend of 1 after 2 months. A European put option with a strike of 42 and an expiry date of 3 months has a premium of 2.71. The continuously compounded risk-free rate is 5%.

Determine the premium of a European call option on the stock with the same strike and expiry.

Answer: Using equation (1.5),

$$C(K,T) - P(K,T) = S_0 - \mathrm{PV}_{0,T}(\mathrm{Divs}) - Ke^{-rT}$$

$PV_{0,T}(\mathrm{Divs})$, the present value of dividends, is the present value of the dividend of 1 discounted 2 months at 5%, or $(1)e^{-0.05/6}$.

$$\begin{aligned} C(42, 0.25) - 2.71 &= 45 - (1)e^{-0.05/6} - 42e^{-0.05(0.25)} \\ &= 45 - 0.991701 - 42(0.987578) = 2.5300 \\ C(42, 0.25) &= 2.71 + 2.5300 = \boxed{5.2400} \end{aligned}$$

□

Quiz 1-3 A stock's price is 50. The stock will pay a dividend of 2 after 4 months. A European call option with a strike of 50 and an expiry date of 6 months has a premium of 1.62. The continuously compounded risk-free rate is 4%.

Determine the premium of a European put option on the stock with the same strike and expiry.

Now let's consider a stock with continuous dividends at rate δ. Using prepaid forwards, put-call parity becomes

$$C(K,T) - P(K,T) = S_0 e^{-\delta T} - Ke^{-rT} \tag{1.6}$$

Example 1C You are given:

(i) A stock's price is 40.
(ii) The continuously compounded risk-free rate is 8%.
(iii) The stock's continuous dividend rate is 2%.

A European 1-year call option with a strike of 50 costs 2.34.

Determine the premium for a European 1-year put option with a strike of 50.

Answer: Using equation (1.6),

$$\begin{aligned} C(K,T) - P(K,T) &= S_0 e^{-\delta T} - Ke^{-rT} \\ 2.34 - P(K,T) &= 40e^{-0.02} - 50e^{-0.08} \\ &= 40(0.9801987) - 50(0.9231163) = -6.94787 \\ P(K,T) &= 2.34 + 6.94787 = \boxed{9.28787} \end{aligned}$$

□

Quiz 1-4 You are given:

(i) A stock's price is 57.
(ii) The continuously compounded risk-free rate is 5%.
(iii) The stock's continuous dividend rate is 3%.

A European 3-month put option with a strike of 55 costs 4.46.

Determine the premium of a European 3-month call option with a strike of 55.

1.2.2 Synthetic stocks and Treasuries

Since the put-call parity equation includes terms for stock (S_0) and cash (K), we can create a synthetic stock with an appropriate combination of options and lending. With continuous dividends, the formula is

$$\begin{aligned} C(K,T)-P(K,T) &= S_0e^{-\delta T}-Ke^{-rT} \\ S_0 &= \left(C(K,T)-P(K,T)+Ke^{-rT}\right)e^{\delta T} \end{aligned} \tag{1.7}$$

For example, suppose the risk-free rate is 5%. We want to create an investment equivalent to a stock with continuous dividend rate of 2%. We can use any strike price and any expiry; let's say 40 and 1 year. We have

$$S_0=\left(C(40,1)-P(40,1)+40e^{-0.05}\right)e^{0.02}$$

So we buy $e^{0.02}=1.02020$ call options and sell 1.02020 put options, and buy a Treasury for 38.8178. At the end of a year, the Treasury will be worth $38.8178e^{0.05}=40.808$. An option will be exercised, so we will pay $40(1.02020)=40.808$ and get 1.02020 shares of the stock, which is equivalent to buying 1 share of the stock originally and reinvesting the dividends.

If dividends are discrete, then they are assumed to be fixed in advance, and the formula becomes

$$\begin{aligned} C(K,T)-P(K,T) &= S_0-\text{PV(dividends)}-Ke^{-rT} \\ S_0 &= C(K,T)-P(K,T)+\underbrace{\text{PV(dividends)}+Ke^{-rT}}_{\text{amount to lend}} \end{aligned} \tag{1.8}$$

For example, suppose the risk-free rate is 5%, the stock is 40, and the period is 1 year. The dividends are 0.5 apiece at the end of 3 months and at the end of 9 months. Then their present value is

$$0.5e^{-0.05(0.25)}+0.5e^{-0.05(0.75)}=0.97539$$

To create a synthetic stock, we buy a call, sell a put, and lend $0.97539+40e^{-0.05}=39.0246$. At the end of the year, we'll have 40 plus the accumulated value of the dividends. One of the options will be exercised, so the 40 will be exchanged for one share of the stock.

To create a synthetic Treasury[3], we rearrange the equation as follows:

$$\begin{aligned} C(K,T)-P(K,T) &= S_0e^{-\delta T}-Ke^{-rT} \\ Ke^{-rT} &= S_0e^{-\delta T}-C(K,T)+P(K,T) \end{aligned} \tag{1.9}$$

We buy $e^{-\delta T}$ shares of the stock and a put option and sell a call option. Using $K=40$, $r=0.05$, $\delta=0.02$, and 1 year to maturity again, the total cost of this is $Ke^{-rT}=40e^{-0.05}=38.04918$. At the end of the year, we sell the stock for 40 (since one option will be exercised). This is equivalent to investing in a one-year Treasury bill with maturity value 40.

If dividends are discrete, then they are assumed to be fixed in advance and can be combined with the strike price as follows:

$$Ke^{-rT}+\text{PV(dividends)}=S_0-C(K,T)+P(K,T) \tag{1.10}$$

The maturity value of this Treasury is $K+\text{CumValue(dividends)}$. For example, suppose the risk-free rate is 5%, the stock is 40, and the period is 1 year. The dividends are 0.5 apiece at the end of 3 months and at the end of 9 months. Then their present value, as calculated above, is

$$0.5e^{-0.05(0.25)}+0.5e^{-0.05(0.75)}=0.97539$$

and their accumulated value at the end of the year is $0.97539e^{0.05}=1.02540$. Thus if you buy a stock and a put and sell a call, both options with strike prices 40, the investment will be $Ke^{-0.05}+0.97539=39.0246$ and the maturity value will be $40+1.02540=41.0254$.

[3]Creating a synthetic Treasury is called a *conversion*. Selling a synthetic Treasury by shorting the stock, buying a call, and selling a put, is called a *reverse conversion*.

Quiz 1-5 You wish to create a synthetic investment using options on a stock. The continuously compounded risk-free interest rate is 4%. The stock price is 43. You will use 6-month options with a strike of 45. The stock pays continuous dividends at a rate of 1%. The synthetic investment should duplicate 100 shares of the stock.

Determine the amount you should invest in Treasuries.

1.2.3 Synthetic options

If an option is mispriced based on put-call parity, you may want to create an arbitrage.

Suppose the price of a European call based on put-call parity is C, but the price it is actually selling at is $C' < C$. (From a different perspective, this may indicate the put is mispriced, but let's assume the price of the put is correct based on some model.) You would then buy the underpriced call option and sell a synthesized call option. Since

$$C(S,K,t) = Se^{-\delta t} - Ke^{-rt} + P(S,K,t)$$

you would sell the right hand side of this equation. You'd sell $e^{-\delta t}$ shares of the underlying stock, sell a European put option with strike price K and expiry t, and buy a risk-free zero-coupon bond with a price of Ke^{-rt}, or in other words lend Ke^{-rt} at the risk-free rate. These transactions would give you $C(S,K,t)$. You'd pay C' for the option you bought, and keep the difference.

1.2.4 Exchange options

The options we've discussed so far involve receiving/giving a stock in return for cash. We can generalize to an option to receive a stock in return for a different stock. Let S_t be the value of the underlying asset, the one for which the option is written, and Q_t be the price of the strike asset, the one which is paid. Forwards will now have a parameter for the asset; $F_{t,T}(Q)$ will mean a forward agreement to purchase asset Q (actually, the asset with price Q_t) at time T. A superscript P will indicate a prepaid forward, as before. Calls and puts will have an extra parameter too:

- $C(S_t, Q_t, T-t)$ means a call option written at time t which lets the purchaser elect to receive S_T in return for Q_T at time T; in other words, to receive $\max(0, S_T - Q_T)$.
- $P(S_t, Q_t, T-t)$ means a put option written at time t which lets the purchaser elect to give S_T in return for Q_T; in other words, to receive $\max(0, Q_T - S_T)$.

The put-call parity equation is then

$$C(S_t, Q_t, T-t) - P(S_t, Q_t, T-t) = F^P_{t,T}(S) - F^P_{t,T}(Q) \tag{1.11}$$

Exchange options are sometimes given to corporate executives. They are given a call option on the company's stock against an index. If the company's stock performs better than the index, they get compensated.

Notice how the definitions of calls and puts are mirror images. A call on one share of Ford with one share of General Motors as the strike asset is the same as a put on one share of General Motors with one share of Ford as the strike asset. In other words

$$P(S_t, Q_t, T-t) = C(Q_t, S_t, T-t)$$

and we could've written the above equation with just calls:

$$C(S_t, Q_t, T-t) - C(Q_t, S_t, T-t) = F^P_{t,T}(S) - F^P_{t,T}(Q)$$

EXAMPLE 1D A European call option allows one to purchase 2 shares of Stock B with 1 share of Stock A at the end of a year. You are given:

(i) The continuously compounded risk-free rate is 5%.
(ii) Stock A pays dividends at a continuous rate of 2%.
(iii) Stock B pays dividends at a continuous rate of 4%.
(iv) The current price for Stock A is 70.
(v) The current price for Stock B is 30.

A European put option which allows one to sell 2 shares of Stock B for 1 share of Stock A costs 11.50.

Determine the premium of the European call option mentioned above, which allows one to purchase 2 shares of Stock B for 1 share of Stock A.

ANSWER: The risk-free rate is irrelevant. Stock B is the underlying asset (price S in the above notation) and Stock A is the strike asset (price Q in the above notation). By equation (1.11),

$$\begin{aligned} C(S,Q,1) &= 11.50 + F^P_{0,T}(S) - F^P_{0,T}(Q) \\ F^P_{0,T}(S) &= S_0 e^{-\delta_S T} = (2)(30)e^{-0.04} = 57.64737 \\ F^P_{0,T}(Q) &= Q_0 e^{-\delta_Q T} = (70)e^{-0.02} = 68.61391 \\ C(S,Q,1) &= 11.50 + 57.64737 - 68.61391 = \boxed{\mathbf{0.5335}} \end{aligned}$$

□

Quiz 1-6 In the situation of Example 1D, determine the premium of a European call option which allows one to buy 1 share of Stock A for 2 shares of Stock B at the end of a year.

1.2.5 Currency options

Let $C(x_0, K, T)$ be a call option on currency with spot exchange rate[4] x_0 to purchase it at exchange rate K at time T, and $P(x_0, K, T)$ the corresponding put option. Putting equation 1.4 and the last formula in Table 1.2 together, we have the following formula:

$$C(x_0, K, T) - P(x_0, K, T) = x_0 e^{-r_f T} - K e^{-r_d T} \tag{1.12}$$

where r_f is the "foreign" risk-free rate for the currency which is playing the role of a stock (the one which can be purchased for a call option or the one that can be sold for a put option) and r_d is the "domestic" risk-free rate which is playing the role of cash in a stock option (the one which the option owner pays in a call option and the one which the option owner receives in a put option).

EXAMPLE 1E You are given:

(i) The spot exchange rate for dollars to pounds is 1.4\$/£.
(ii) The continuously compounded risk-free rate for dollars is 5%.
(iii) The continuously compounded risk-free rate for pounds is 8%.

A 9-month European put option allows selling £1 at the rate of \$1.50/£. A 9-month dollar denominated call option with the same strike costs \$0.0223.

Determine the premium of the 9-month dollar denominated put option.

[4] The word "spot", as in spot exchange rate or spot price or spot interest rates, refers to the current rates, in contrast to *forward* rates.

ANSWER: The prepaid forward price for pounds is

$$x_0 e^{-r_f T} = 1.4e^{-0.08(0.75)} = 1.31847$$

The prepaid forward for the strike asset (dollars) is

$$Ke^{-r_d T} = 1.5e^{-0.05(0.75)} = 1.44479$$

Thus

$$\begin{aligned} C(x_0, K, T) - P(x_0, K, T) &= 1.31847 - 1.44479 = -0.12632 \\ 0.0223 - P(1.4, 1.5, 0.75) &= -0.12632 \\ P(1.4, 1.5, 0.75) &= 0.0223 + 0.12632 = \boxed{\mathbf{0.14862}} \end{aligned}$$

□

Quiz 1-7 You are given:

(i) The spot exchange rate for yen to dollars is 90¥/$.
(ii) The continuously compounded risk-free rate for dollars is 5%.
(iii) The continuously compounded risk-free rate for yen is 1%.

A 6-month yen-denominated European call option on dollars has a strike price of 92¥/$ and costs ¥0.75.

Calculate the premium of a 6-month yen-denominated European put option on dollars having a strike price of 92¥/$.

A call to purchase pounds with dollars is equivalent to a put to sell dollars for pounds. However, the units are different. Let's see how to translate units.

EXAMPLE 1F The spot exchange rate for dollars into euros is $1.05/€. A 6-month dollar denominated call option to buy one euro at strike price $1.1/€1 costs $0.04.

Determine the premium of the corresponding euro-denominated put option to sell one dollar for euros at the corresponding strike price.

ANSWER: To sell 1 dollar, the corresponding exchange rate would be $1/€$\frac{1}{1.1}$, so the euro-denominated strike price is $\frac{1}{1.1} = 0.9091$€/$. Since we're in effect buying 0.9091 of the dollar-denominated call option, the premium in dollars is ($0.04)(0.9091) = $0.03636. Dividing by the spot rate, the premium in euros is $\frac{0.03636}{1.05} = \boxed{\text{€}\mathbf{0.03463}}$ □

Let's generalize the example. Let the domestic currency be the one the option is denominated in, the one in which the price is expressed. Let the foreign currency be the underlying asset of the option. Given spot rate x_0 expressed as units of domestic currency per foreign currency, strike price K of the domestic currency, and call premium $C(x_0, K, T)$ of the domestic currency, the corresponding put option denominated in the foreign currency will have strike price $1/K$ expressed in the foreign currency. The exchange rate expressed in the foreign currency is $1/x_0$ units of the foreign currency per 1 unit of the domestic currency. The call option allows the payment of K domestic units for 1 foreign unit. The put option allows the payment of $1/K$ foreign units for 1 domestic unit. So K foreign-denominated puts produce the same payment as 1 domestic-denominated call. Let P_d be the price of the put option expressed in the domestic currency, and C_d the price of the call option expressed in the domestic currency. Then

$$KP_d\left(\frac{1}{x_0}, \frac{1}{K}, T\right) = C_d(x_0, K, T)$$

Since the put option's price should be expressed in the foreign currency, the left side must be multiplied by x_0, resulting in

$$Kx_0P_f\left(\frac{1}{x_0}, \frac{1}{K}, T\right) = C_d(x_0, K, T)$$

where P_f is the price of the put option in the foreign currency.

Note that if settlement is through cash rather than through actual exchange of currencies, then the put options $Kx_0P_f(1/x_0, 1/K, T)$ may not have the same payoff as the call option $C_d(x_0, K, T)$. The put options $Kx_0P_f(1/x_0, 1/K, T)$ pay off in the foreign currency while the call option $C_d(x_0, K, T)$ pays off in the domestic currency. The payoffs are equal based on exchange rate x_0, but the exchange rate x_T at time T may be different from x_0.

Quiz 1-8 The spot rate for yen denominated in pounds sterling is 0.005£/¥. A 3-month pound-denominated put option has strike 0.0048£/¥ and costs £0.0002.

Determine the premium in yen for an equivalent 3-month yen-denominated call option with a strike of ¥$208\frac{1}{3}$.

Exercises

Put-call parity for stock options

1.1. [**CAS8-S03:18a**] A four-month European call option with a strike price of 60 is selling for 5. The price of the underlying stock is 61, and the annual continuously compounded risk-free rate is 12%. The stock pays no dividends.

Calculate the value of a four-month European put option with a strike price of 60.

1.2. For a nondividend paying stock, you are given:

(i) Its current price is 30.
(ii) A European call option on the stock with one year to expiration and strike price 25 costs 8.05.
(iii) The continuously compounded risk-free interest rate is 0.05.

Determine the premium of a 1-year European put option on the stock with strike 25.

1.3. A nondividend paying stock has price 30. You are given:

(i) The continuously compounded risk-free interest rate is 5%.
(ii) A 6-month European call option on the stock costs 3.10.
(iii) A 6-month European put option on the stock with the same strike price as the call option costs 5.00.

Determine the strike price.

1.4. A stock pays continuous dividends proportional to its price at rate δ. You are given:

(i) The stock price is 40.
(ii) The continuously compounded risk-free interest rate is 4%.
(iii) A 3-month European call option on the stock with strike 40 costs 4.10.
(iv) A 3-month European put option on the stock with strike 40 costs 3.91.

Determine δ.

1.5. For a stock paying continuous dividends proportional to its price at rate $\delta = 0.02$, you are given:

(i) The continuously compounded risk-free interest rate is 3%.
(ii) A 6-month European call option with strike 40 costs 4.10.
(iii) A 6-month European put option with strike 40 costs 3.20.

Determine the current price of the stock.

Exercises continue on the next page . . .

1.6. A stock's price is 45. Dividends of 2 are payable quarterly, with the next dividend payable at the end of one month. You are given:

(i) The continuously compounded risk-free interest rate is 6%.
(ii) A 3-month European put option with strike 50 costs 7.32.

Determine the premium of a 3-month European call option on the stock with strike 50.

1.7. A dividend paying stock has price 50. You are given:

(i) The continuously compounded risk-free interest rate is 6%.
(ii) A 6-month European call option on the stock with strike 50 costs 2.30.
(iii) A 6-month European put option on the stock with strike 50 costs 1.30.

Determine the present value of dividends paid over the next 6 months on the stock.

1.8. You are given the following values for 1-year European call and put options at various strike prices:

Strike Price	Call Premium	Put Premium
40	8.25	1.12
45	5.40	P_2
50	4.15	6.47

Determine P_2.

(A) 2.60 (B) 2.80 (C) 3.00 (D) 3.20 (E) 3.40

1.9. Consider European options on a stock expiring at time t. Let $P(K)$ be a put option with strike price K, and $C(K)$ be a call option with strike price K. You are given

(i) $P(50) - C(55) = -2$
(ii) $P(55) - C(60) = 3$
(iii) $P(60) - C(50) = 14$

Determine $C(60) - P(50)$.

1.10. [**CAS8-S00:26**] You are given the following:

(i) Stock price = \$50
(ii) The risk-free interest rate is a constant annual 8%, compounded continuously
(iii) The price of a 6-month European call option with an exercise price of \$48 is \$5.
(iv) The price of a 6-month European put option with an exercise price of \$48 is \$3.
(v) The stock pays no dividends

There is an arbitrage opportunity involving buying or selling one share of stock and buying or selling puts and calls.

Calculate the profit after 6 months from this strategy.

Exercises continue on the next page . . .

Synthetic assets

1.11. You are given:

(i) The price of a stock is 43.00.
(ii) The continuously compounded risk-free interest rate is 5%.
(iii) The stock pays a dividend of 1.00 three months from now.
(iv) A 3-month European call option on the stock with strike 44.00 costs 1.90.

You wish to create this stock synthetically, using a combination of 44-strike options expiring in 3 months and lending.

Determine the amount of money you should lend.

1.12. You are given:

(i) The price of a stock is 95.00.
(ii) The continuously compounded risk-free rate is 6%.
(iii) The stock pays quarterly dividends of 0.80, with the next dividend payable in 1 month.

You wish to create this stock synthetically, using 1-year European call and put options with strike price K, and lending 96.35.

Determine the strike price of the options.

1.13. You are given:

(i) A stock index is 22.00.
(ii) The continuous dividend rate of the index is 2%.
(iii) The continuously compounded risk-free interest rate is 5%.
(iv) A 90-day European call option on the index with strike 21.00 costs 1.90.
(v) A 90-day European put option on the index with strike 21.00 costs 0.75.

You wish to create an equivalent synthetic stock index using a combination of options and lending.

Determine the amount of money you should lend.

1.14. You are given:

(i) The stock price is 40.
(ii) The stock pays continuous dividends proportional to its price at a rate of 1%.
(iii) The continuously compounded risk-free interest rate is 4%.
(iv) A 182-day European put option on the stock with strike 50 costs 11.00.

You wish to create a synthetic 182-day Treasury bill with maturity value 10,000.

Determine the number of shares of the stock you should purchase.

1.15. You wish to create a synthetic 182-day Treasury bill with maturity value 10,000. You are given:

(i) The stock price is 40.
(ii) The stock pays continuous dividends proportional to its price at a rate of 2%.
(iii) A 182-day European put option on the stock with strike price K costs 0.80.
(iv) A 182-day European call option on the stock with strike price K costs 5.20.
(v) The continuously compounded risk-free interest rate is 5%.

Determine the number of shares of the stock you should purchase.

Exercises continue on the next page . . .

1.16. You are given:

(i) The price of a stock is 100.
(ii) The stock pays discrete dividends of 2 per quarter, with the first dividend 3 months from now.
(iii) The continuously compounded risk-free interest rate is 4%.

You wish to create a synthetic 182-day Treasury bill with maturity value 10,000, using a combination of the stock and 6-month European put and call options on the stock with strike price 95.

Determine the number of shares of the stock you should purchase.

Put-call parity for exchange options

1.17. For two stocks, S_1 and S_2:

(i) The price of S_1 is 30.
(ii) S_1 pays continuous dividends proportional to its price. The dividend yield is 2%.
(iii) The price of S_2 is 75.
(iv) S_2 pays continuous dividends proportional to its price. The dividend yield is 5%.
(v) A 1-year call option to receive a share of S_2 in exchange for 2.5 shares of S_1 costs 2.50.

Determine the premium of a 1-year call option to receive 1 share of S_1 in exchange for 0.4 shares of S_2.

1.18. S_1 is a stock with price 30 and quarterly dividends of 0.25. The next dividend is payable in 3 months.

S_2 is a nondividend paying stock with price 40.

The continuously compounded risk-free interest rate is 5%.

Let x be the premium of an option to give S_1 in exchange for receiving S_2 at the end of 6 months, and let y be the premium of an option to give S_2 in exchange for receiving S_1 at the end of 6 months.

Determine $x - y$.

1.19. For the stocks of Sohitu Autos and Flashy Autos, you are given:

(i) The price of one share of Sohitu is 180.
(ii) The price of one share of Flashy is 90.
(iii) Sohitu pays quarterly dividends of 3 on Feb. 15, May 15, Aug. 15, and Nov. 15 of each year.
(iv) Flashy pays quarterly dividends of 1 on Jan. 31, Apr. 30, July 31, and Oct. 31 of each year.
(v) The continuously compounded risk-free interest rate is 0.06.
(vi) On Dec. 31, an option expiring in 6 months to get x shares of Flashy for 1 share of Sohitu costs 4.60.
(vii) On Dec. 31, an option expiring in 6 months to get 1 share of Sohitu for x shares of Flashy costs 7.04.

Determine x.

1.20. Stock A is a nondividend paying stock. Its price is 100.

Stock B pays continuous dividends proportional to its price. The dividend yield is 0.03. Its price is 60.

An option expiring in one year to buy x shares of A for 1 share of B costs 2.39.

An option expiring in one year to buy $1/x$ shares of B for 1 share of A costs 2.74.

Determine x.

Exercises continue on the next page . . .

Put-call parity for currency options

Use the following information for questions 1.21 and 1.22:

You are given:

(i) The spot exchange rate is 1.5\$/£.
(ii) The continuously compounded risk-free rate in dollars is 6%.
(iii) The continuously compounded risk-free rate in pounds sterling is 3%.
(iv) A 6-month dollar-denominated European put option on pounds with a strike of 1.5\$/£ costs \$0.03.

1.21. Determine the premium in pounds of a 6-month pound-denominated European call option on dollars with a strike of (1/1.5)£/\$.

1.22. Determine the premium in pounds of a 6-month pound-denominated European put option on dollars with a strike of (1/1.5)£/\$.

1.23. You are given

(i) The spot exchange rate is 95¥/\$1.
(ii) The continuously compounded risk-free rate in yen is 1%.
(iii) The continuously compounded risk-free rate in dollars is 5%.
(iv) A 1-year dollar denominated European call option on yen with strike \$0.01 costs \$0.0011.

Determine the premium of a 1-year dollar denominated European put option on yen with strike \$0.01.

1.24. The spot exchange rate of dollars for euros is 1.2\$/€. A dollar-denominated put option on euros has strike price \$1.3.

Determine the strike price of the corresponding euro-denominated call option to pay a certain number of euros for one dollar.

1.25. You are given:

(i) The spot exchange rate of dollars for euros is 1.2\$/€.
(ii) A one-year dollar-denominated European call option on euros with strike price \$1.3 costs 0.05.
(iii) The continuously compounded risk-free interest rate for dollars is 5%.
(iv) A one-year dollar-denominated European put option on euros with strike price \$1.3 costs 0.20.

Determine the continuously compounded risk-free interest rate for euros.

1.26. You are given:

(i) The spot exchange rate of yen for euros is 110¥/€.
(ii) The continuously compounded risk-free rate for yen is 2%.
(iii) The continuously compounded risk-free rate for euros is 4%.
(iv) A one year yen-denominated call on euros costs ¥3.
(v) A one year yen-denominated put on euros with the same strike price as the call costs ¥2.

Determine the strike price in yen.

Exercises continue on the next page . . .

1.27. You are given:

(i) The continuously compounded risk-free interest rate for dollars is 4%.
(ii) The continuously compounded risk-free interest rate for pounds is 6%.
(iii) A 1-year dollar-denominated European call option on pounds with strike price 1.6 costs $0.05.
(iv) A 1-year dollar-denominated European put option on pounds with strike price 1.6 costs $0.10.

Determine the spot exchange rate of dollars per pound.

1.28. You are given:

(i) The continuously compounded risk-free interest rate for dollars is 4%.
(ii) The continuously compounded risk-free interest rate for pounds is 6%.
(iii) A 6-month dollar-denominated European call option on pounds with strike price 1.45 costs $0.05.
(iv) A 6-month dollar-denominated European put option on pounds with strike price 1.45 costs $0.02.

Determine the 6-month forward exchange rate of dollars per pound.

Additional released exam questions: Sample:1, CAS3-S07:3,4,13, SOA MFE-S07:1, CAS3-F07:14,15,16,25, MFE/3F-S09:9

Solutions

1.1. By put-call parity,

$$\begin{aligned}P(61,60,1/3) &= C(61,60,1/3)+60e^{-r(1/3)}-61e^{-\delta(1/3)}\\ &= 5+60e^{-0.04}-61=\boxed{\mathbf{1.647}}\end{aligned}$$

1.2. We want $P(25,1)$. By put-call parity:

$$\begin{aligned}C(25,1)-P(25,1) &= S-Ke^{-r}\\ 8.05-P(25,1) &= 30-25e^{-0.05}=6.2193\\ P(25,1) &= 8.05-6.2193=\boxed{\mathbf{1.8307}}\end{aligned}$$

1.3. We need K, the strike price. By put-call parity:

$$\begin{aligned}C(K,0.5)-P(K,0.5) &= S-Ke^{-0.5r}\\ 3.10-5.00 &= 30-Ke^{-0.5(0.05)}\\ -31.90 &= -Ke^{-0.025}\\ K &= 31.90e^{0.025}=\boxed{\mathbf{32.7076}}\end{aligned}$$

1.4. By put-call parity,

$$\begin{aligned}C(40,0.25)-P(40,0.25) &= Se^{-0.25\delta}-Ke^{-0.25r}\\ 4.10-3.91 &= 40e^{-0.25\delta}-40e^{-0.01}\\ &= 40e^{-0.25\delta}-39.6020\\ 0.19+39.6020 &= 40e^{-0.25\delta}\end{aligned}$$

$$\begin{aligned}
39.7920 &= 40e^{-0.25\delta}\\
e^{-0.25\delta} &= \frac{39.7920}{40} = 0.9948\\
0.25\delta &= -\ln 0.9948 = 0.005214\\
\delta &= \frac{0.005214}{0.25} = \boxed{\mathbf{0.02086}}
\end{aligned}$$

1.5. By put-call parity,

$$\begin{aligned}
C(40,0.5) - P(40,0.5) &= Se^{-0.5\delta} - Ke^{-0.5r}\\
4.10 - 3.20 &= Se^{-0.01} - 40e^{-0.015}\\
Se^{-0.01} &= 0.9 + 39.40448 = 40.30448\\
S &= 40.30448e^{0.01} = \boxed{\mathbf{40.7095}}
\end{aligned}$$

1.6. The value of the prepaid forward on the stock is

$$F^P_{0.25} = S - \text{PV(Div)} = 45 - 2e^{-0.06(1/12)} = 45 - 1.9900 = 43.0100$$

By put-call parity,

$$\begin{aligned}
C(50,0.25) - P(50,0.25) &= 43.0100 - Ke^{-0.25(0.06)}\\
C(50,0.25) - 7.32 &= 43.0100 - 49.2556 = -6.2456\\
C(50,0.25) &= 7.32 - 6.2456 = \boxed{\mathbf{1.0744}}
\end{aligned}$$

1.7. By put-call parity,

$$\begin{aligned}
C(50,0.5) - P(50,0.5) &= S - \text{PV(Divs)} - Ke^{-r(0.5)}\\
2.30 - 1.30 &= 50 - \text{PV(Divs)} - 50e^{-0.03}\\
\text{PV(Divs)} &= 50 - 2.30 + 1.30 - 50(0.970446) = \boxed{\mathbf{0.4777}}
\end{aligned}$$

1.8. By put-call parity,

$$\begin{aligned}
40e^{-r} - Se^{-\delta} &= 1.12 - 8.25 = -7.13\\
50e^{-r} - Se^{-\delta} &= 6.47 - 4.15 = 2.32
\end{aligned}$$

Subtracting the first from the second

$$10e^{-r} = 7.13 + 2.32 = 9.45$$

By put-call parity at 45,

$$\begin{aligned}
45e^{-r} - Se^{-\delta} &= P_2 - 5.40\\
5e^{-r} + (40e^{-r} - Se^{-\delta}) &= P_2 - 5.40\\
\frac{9.45}{2} + (-7.13) &= P_2 - 5.40\\
P_2 &= \frac{9.45}{2} - 7.13 + 5.40 = \boxed{\mathbf{2.995}} \qquad \textbf{(C)}
\end{aligned}$$

1.9. By put-call parity, adding up the three given statements

$$P(50)-C(50)+P(55)-C(55)+P(60)-C(60)=(50+55+60)e^{-rt}-3Se^{-\delta t}=-2+3+14=15$$

Then $P(60)-C(60)+P(50)-C(50)=110e^{-rt}-2Se^{-\delta t}=(2/3)(15)=10$. Since $P(60)-C(50)=14$, it follows that $P(50)-C(60)=10-14=-4$, and $C(60)-P(50)=\boxed{4}$.

1.10. First use put-call parity to determine whether the put is underpriced or overpriced.

$$\begin{aligned}P(50,48,0.5)&=C(50,48,0.5)+48e^{-rt}-50\\&=5+48e^{-0.04}-50=1.1179\end{aligned}$$

Since the put has price 3, it is overpriced and you should sell it. This means you buy a call and sell a put. In 6 months, you must buy a share of stock, so sell one short right now. The cash flow of this strategy is

Short one share of stock	50
Buy a call	−5
Sell a put	3
	48

After 6 months, 48 will grow to $48e^{0.04}=49.96$ and you will pay 48 for the stock, for a net gain of $49.959-48=\boxed{1.959}$.

1.11. By put-call parity,

$$\begin{aligned}C(S,44,0.25)-P(S,44,0.25)&=S-\text{PV(Divs)}-Ke^{-0.05(0.25)}\\S&=C(S,44,0.25)-P(S,44,0.25)+Ke^{-0.0125}+\text{PV(Divs)}\\&=C(S,44,0.25)-P(S,44,0.25)+44e^{-0.0125}+e^{-0.0125}\end{aligned}$$

So the amount to lend is $45e^{-0.0125}=\boxed{44.4410}$.

1.12. The present value of the dividends is $0.8(e^{-0.005}+e^{-0.02}+e^{-0.035}+e^{-0.05})=3.1136$. By formula (1.8), the amount to lend is $Ke^{-rT}+PV(\text{dividends})$, or

$$\begin{aligned}Ke^{-0.06}+3.1136&=96.35\\K=(96.35-3.1136)e^{0.06}&=\boxed{99.00}\end{aligned}$$

We did not need the stock price for this exercise.

1.13. Did you verify that these option prices satisfy put-call parity? Not that you have to for this exercise.

By put-call parity,

$$\begin{aligned}C(S,21,0.25)-P(S,21,0.25)&=Se^{-(0.02)(0.25)}-Ke^{-(0.05)(0.25)}\\S&=\frac{C(S,21,0.25)-P(S,21,0.25)+21e^{-0.0125}}{e^{-0.005}}\\&=\Big(C(S,21,0.25)-P(S,21,0.25)\Big)e^{0.005}+21e^{-0.0075}\end{aligned}$$

Thus we need to lend $21e^{-0.0075}=\boxed{20.84}$. It can then be verified that buying $e^{0.005}$ calls and selling $e^{0.005}$ puts costs 1.16 for a total investment of 22, the price of the index.

1.14. By put-call parity,

$$\begin{aligned}P(S,50,0.5)-C(S,50,0.5)&=Ke^{-0.04(0.5)}-Se^{-0.01(0.5)}\\Ke^{-0.04(0.5)}&=P(S,50,0.5)-C(S,50,0.5)+Se^{-0.005}\end{aligned}$$

Since we want a maturity value of 10,000, we need the left hand side to be $10{,}000e^{-0.02}$, so we multiply the equation by $\frac{10{,}000}{K}=200$. Thus we need to buy $200e^{-0.005}=\boxed{199.0025}$ shares of stock.

1.15. We will back out the strike price K using put-call parity.

$$\begin{aligned}
P(S,K,0.5)-C(S,K,0.5) &= Ke^{-0.05(0.5)}-Se^{-0.02(0.5)}\\
0.80-5.20 &= Ke^{-0.025}-40e^{-0.01}\\
-4.40 &= Ke^{-0.025}-40e^{-0.01}\\
K &= \left(40e^{-0.01}-4.40\right)e^{0.025}\\
&= 40e^{0.015}-4.40e^{0.025}\\
&= 40.60452-4.51139=36.0931
\end{aligned}$$

By put-call parity (using the 3rd equation above)

$$Ke^{-0.025}=Se^{-0.01}+P(S,K,0.5)-C(S,K,0.5)$$

We multiply by $\frac{10{,}000}{K}$ to get a maturity value of 10,000. The number of shares of stock needed is

$$\left(\frac{10{,}000}{K}\right)e^{-0.01}=\frac{10{,}000}{36.0931}(0.99005)=\boxed{\mathbf{274.30}}$$

1.16. Using equation (1.10), the maturity value of the Treasury for every share purchased is

$$K+\text{CumValue(dividends)}=95+2e^{0.04(0.25)}+2=99.0201$$

Therefore, the number of shares of stock is $10{,}000/99.0201=\boxed{\mathbf{100.99}}$.

1.17. By put-call parity,

$$C(S_1,0.4S_2,1)-C(0.4S_2,S_1,1)=F^P_{0,1}(S_1)-F^P_{0,1}(0.4S_2)$$

A call for 0.4 shares of S_2 in return for a share of S_1 is 0.4 of the call in (v), so it is worth $(0.4)(2.50)=1.00$. Then

$$\begin{aligned}
C(S_1,0.4S_2,1)-1.00 &= 30e^{-0.02}-(0.4)(75)e^{-0.05}\\
&= 29.40596-28.53688=0.86908\\
C(S_1,0.4S_2,1) &= 0.86908+1.00=\boxed{\mathbf{1.86908}}
\end{aligned}$$

1.18. Let Divs be the dividends on S_1. By put-call parity, $x-y$ is the difference in prepaid forward prices for the two stocks. x is a call on S_2 and y is a put on S_2, so

$$x-y=S_2-\left(S_1-\text{PV(Divs)}\right)$$

Let's compute the present value of dividends.

$$\text{PV(Divs)}=0.25e^{-0.05(0.25)}+0.25e^{-0.05(0.5)}=0.490722$$

So $x-y$ is

$$x-y=40-(30-0.490722)=\boxed{\mathbf{10.490722}}$$

1.19. The present value of 6 months of dividends is $3(e^{-0.06(1/8)}+e^{-0.06(3/8)})=5.9108$ for Sohitu and $e^{-0.005}+e^{-0.02}=1.9752$ for Flashy. By put-call parity

$$\begin{aligned}
7.04-4.60 &= 180-5.9108-x(90-1.9752)\\
2.44 &= 174.0892-88.0248x\\
x &= \frac{171.6492}{88.0248}=\boxed{\mathbf{1.95}}
\end{aligned}$$

1.20. Consider the option to buy x shares of A for 1 share of B as the put in put-call parity. Then the option to buy $1/x$ shares of B for 1 share of A is $1/x$ times an option to sell x shares of A for 1 share of B, which would correspond to the call in put-call parity. So

$$\begin{aligned} 2.39-2.74x &= 100x-60e^{-0.03}\\ 102.74x &= 60.6176\\ x &= \boxed{\mathbf{0.59}} \end{aligned}$$

1.21. The dollar-denominated put option is equivalent to a pound-denominated call option on dollars paying \$1.5 per £1. To reduce this to one paying \$1 per £(1/1.5), divide by 1.5, so the price in dollars is 0.02 and the price in pounds is £$\frac{0.02}{1.5}=\boxed{\textbf{£0.01333}}$.

1.22. We'll use the answer to the previous exercise and put-call parity.

$$\begin{aligned} C(\$,£,0.5)-P(\$,£,0.5) &= e^{-(0.06)(0.5)}x_0-e^{-(0.03)(0.5)}K\\ 0.01333-P(\$,£,0.5) &= 0.67e^{-0.03}-0.67e^{-0.015}\\ 0.01333-P(\$,£,0.5) &= -0.00983\\ P(\$,£,0.5) &= 0.01333+0.00983=\boxed{\mathbf{0.02316}} \end{aligned}$$

1.23. The spot exchange rate for yen in dollars is $\frac{1}{95}$\$/¥. By put-call parity,

$$\begin{aligned} C(¥,\$,1)-P(¥,\$,1) &= x_0e^{-r_{¥}}-Ke^{-r_{\$}}\\ 0.0011-P(¥,\$,1) &= \frac{1}{95}e^{-0.01}-0.01e^{-0.05}\\ &= 0.0104216-0.0095123=0.0009093\\ P(¥,\$,1) &= 0.0011-0.0009093=\boxed{\mathbf{\$0.0001907}} \end{aligned}$$

1.24. The put option allows giving a euro and receiving \$1.3. The call option allows receiving \$1 and giving euros. The number of euros would have to be $\frac{1}{1.3}=\boxed{\textbf{€0.7692}}$. The spot exchange rate isn't relevant.

1.25. By put-call parity,

$$\begin{aligned} C(€,\$,1)-P(€,\$,1) &= x_0e^{-r_{€}}-Ke^{-r_{\$}}\\ 0.05-0.20 &= 1.2e^{-r_{€}}-1.3e^{-0.05}\\ 1.2e^{-r_{€}} &= -0.15+1.236598=1.086598\\ e^{-r_{€}} &= \frac{1.086598}{1.2}=0.90550\\ r_{€} &= -\ln 0.90550=\boxed{\mathbf{0.09927}} \end{aligned}$$

1.26. By put-call parity,

$$\begin{aligned} C(€,¥,1)-P(€,¥,1) &= x_0e^{-r_{€}}-Ke^{-r_{¥}}\\ ¥3-¥2 &= ¥110e^{-0.04}-Ke^{-0.02}\\ ¥1 &= ¥110(0.960789)-K(0.980199)\\ K &= \frac{¥110(0.960789)-1}{0.980199}=\boxed{\textbf{¥106.802}} \end{aligned}$$

1.27. By put-call parity,

$$\begin{aligned}C(£,\$,1)-P(£,\$,1)&=x_0e^{-0.06}-Ke^{-0.04}\\ 0.05-0.10&=x_0e^{-0.06}-1.6e^{-0.04}\\ x_0e^{-0.06}&=-0.05+1.6(0.960789)=1.48726\\ x_0&=1.48726e^{0.06}=1.48726(1.061837)=\boxed{\mathbf{1.57923}}\end{aligned}$$

1.28. If x_0 is the spot exchange rate, the prepaid forward exchange rate is $x_0e^{-r_£t}$ and the forward exchange rate is $x_0e^{(r_\$-r_£)t}$, since you have to pay interest for time t on the prepaid forward exchange rate. So we need $x_0e^{(0.04-0.06)(0.5)}=x_0e^{-0.01}$.

Using put-call parity,

$$\begin{aligned}C(£,\$,0.5)-P(£,\$,0.5)&=x_0e^{-0.03}-Ke^{-0.02}\\ 0.05-0.02&=x_0e^{-0.03}-1.45e^{-0.02}\\ x_0e^{-0.03}&=0.03+1.45(0.980199)=1.45129\\ x_0e^{-0.01}&=1.45129e^{0.02}=\boxed{\mathbf{1.4806}}\end{aligned}$$

Quiz Solutions

1-1. The accumulated value of the dividend at the end of three months (which is accumulated for three months to the end of the six month term) is

$$100(0.20)e^{(0.25)(0.05)}=20e^{0.0125}=20.25$$

The accumulated value of the dividend at the end of six months is $100(0.20)=20$. Therefore, the forward rate is

$$\begin{aligned}F_{0,0.5}&=S_0e^{rT}-\text{CumValue(Div)}\\ &=100(50)e^{0.05(0.5)}-20.25-20\\ &=5126.58-40.25=\boxed{\mathbf{5086.33}}\end{aligned}$$

1-2. Now dollars are being purchased (instead of euros) and yen are the denomination (instead of dollars). The formula gives, for \$1,

$$F_{0,0.25}=e^{0.02(0.25)-0.05(0.25)}(110)=¥109.1781$$

For \$100, the forward price is $100(109.1781)=\boxed{\mathbf{¥10{,}917.81}}$.

1-3. Using equation (1.5),

$$\begin{aligned}C(K,T)-P(K,T)&=S_0-\text{PV}_{0,T}(\text{Divs})-Ke^{-rT}\\ 1.62-P(K,T)&=50-2e^{-0.04/3}-50e^{-0.04(0.5)}\\ &=50-2(0.9867552)-50(0.9801987)=-0.9834\\ P(K,T)&=1.62+0.9834=\boxed{\mathbf{2.6034}}\end{aligned}$$

1-4. Using equation (1.6),

$$\begin{aligned} C(K,T)-P(K,T) &= S_0e^{-\delta T}-Ke^{-rT} \\ C(K,T)-4.46 &= 57e^{-(0.03)(0.25)}-55e^{-(0.05)(0.25)} \\ &= 57(0.992528)-55(0.987578)=2.2573 \\ C(K,T) &= 4.46+2.2573=\boxed{\mathbf{6.7173}} \end{aligned}$$

1-5. The stock price is irrelevant.
We have

$$\begin{aligned} C(K,T)-P(K,T) &= S_0e^{-\delta T}-Ke^{-rT} \\ S_0 &= e^{0.01(0.5)}\Big(C(45,0.5)-P(45,0.5)\Big)+45e^{(-0.04+0.01)(0.5)} \end{aligned}$$

For one share of stock, the first summand indicates the puts to sell and calls to buy, and the second summand indicates the investment in Treasuries. Therefore, the amount to invest in Treasuries to synthesize 100 shares of stock is

$$100\left(45e^{(-0.04+0.01)(0.5)}\right)=4500e^{-0.015}=\boxed{\mathbf{4433}}.$$

1-6. A trick question. The option to receive 1 share of Stock A for 2 shares of Stock B is the same as the option to sell 2 shares of Stock B for 1 share of Stock A, the put option mentioned in the example, so the premium for this call option is $\boxed{\mathbf{11.50}}$.

1-7. The prepaid forward price for dollars is

$$x_0e^{-r_fT}=90e^{-0.05(0.5)}=¥87.7779$$

The prepaid forward price for the strike asset, yen, is

$$Ke^{-r_dT}=92e^{-0.01(0.5)}=¥91.5411$$

By put-call parity,

$$\begin{aligned} P(90,92,0.5)-C(90,92,0.5) &= 91.5411-87.7779=3.7632 \\ P(90,92,0.5) &= 0.75+3.7632=\boxed{\mathbf{4.5132}} \end{aligned}$$

1-8. We divide by Kx_0 to obtain $0.0002\big/\big((0.005)(0.0048)\big)=\boxed{\mathbf{¥8\tfrac{1}{3}}}$.

Practice Exam 1

1. For American put options on a stock with identical expiry dates, you are given the following prices:

Strike price	Put premium
30	2.40
35	6.40

For an American put option on the same stock with the same expiry date and strike price 38, which of the following statements is correct?

(A) The lowest possible price for the option is 8.80.
(B) The highest possible price for the option is 8.80.
(C) The lowest possible price for the option is 9.20.
(D) The highest possible price for the option is 9.20.
(E) The lowest possible price for the option is 9.40.

2. A company has 100 shares of ABC stock. The current price of ABC stock is 30. ABC stock pays no dividends.

The company would like to guarantee its ability to sell the stock at the end of six months for at least 28.

European call options on the same stock expiring in 6 months with exercise price 28 are available for 4.10.

The continuously compounded risk-free interest rate is 5%.

Determine the cost of the hedge.

(A) 73 (B) 85 (C) 99 (D) 126 (E) 141

3. You are given the following prices for a stock:

Time	Price
Initial	39
After 1 month	39
After 2 months	37
After 3 months	43

A portfolio of 3-month Asian options, each based on monthly averages of the stock price, consists of the following:

(i) 100 arithmetic average price call options, strike 36.
(ii) 200 geometric average strike call options.
(iii) 300 arithmetic average price put options, strike 41.

Determine the net payoff of the portfolio after 3 months.

(A) 1433 (B) 1449 (C) 1464 (D) 1500 (E) 1512

Questions continue on the next page . . .

4. The price of a 6-month futures contract on widgets is 260.

A 6-month European call option on the futures contract with strike price 256 is priced using Black's formula.

You are given:

(i) The continuously compounded risk-free rate is 0.04.
(ii) The volatility of the futures contract is 0.25.

Determine the price of the option.

(A) 19.84 (B) 20.16 (C) 20.35 (D) 20.57 (E) 20.74

5. Which of the following statements is true?

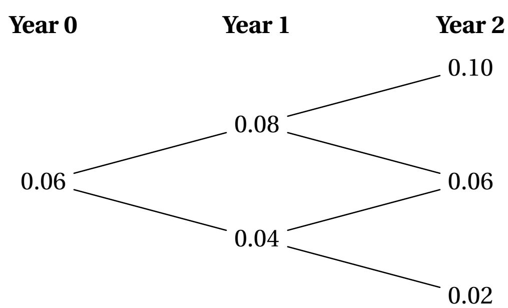
I. If an asset's gamma is zero, the asset can be replicated with a static hedging strategy.

II. If a stock's price follows the Black-Scholes framework, then the price of an unexercised American option on that stock satisfies the Black-Scholes equation.

III. If a stock's price follows the Black-Scholes framework, then the price of an Asian option on that stock satisfies the Black-Scholes equation.

(A) None (B) I only (C) II only (D) III only
(E) The correct answer is not given by (A) , (B) , (C) , or (D) .

6. You are given the following binomial tree for continuously compounded interest rates:



The probability of an up move is 0.5.

Calculate the continuously compounded interest rate on a default-free 3-year zero-coupon bond.

(A) 0.0593 (B) 0.0594 (C) 0.0596 (D) 0.0597 (E) 0.0598

7. An asset's price at time t, $X(t)$, satisfies the stochastic differential equation

$$dX(t) = 0.5\,dt + 0.9\,dZ(t)$$

You are given that $X(0) = 10$.

Determine $\Pr\big(X(2) > 12\big)$.

(A) 0.17 (B) 0.22 (C) 0.26 (D) 0.29 (E) 0.32

Questions continue on the next page . . .

8. For a delta-hedged portfolio, you are given

(i) The stock price is 40.
(ii) The stock's volatility is 0.2.
(iii) The option's gamma is 0.02.

Estimate the annual variance of the portfolio if it is rehedged every half-month.

(A) 0.001 (B) 0.017 (C) 0.027 (D) 0.034 (E) 0.054

9. You own 100 shares of a stock whose current price is 42. You would like to hedge your downside exposure by buying 100 6-month European put options with a strike price of 40. You are given:

(i) The Black-Scholes framework is assumed.
(ii) The continuously compounded risk-free interest rate is 5%.
(iii) The stock pays no dividends.
(iv) The stock's volatility is 22%.

Determine the cost of the put options.

(A) 121 (B) 123 (C) 125 (D) 127 (E) 129

10. You are given the following information for a European call option expiring at the end of three years:

(i) The current price of the stock is 66.
(ii) The strike price of the option is 70.
(iii) The continuously compounded risk-free interest rate is 0.05.
(iv) The continuously compounded dividend rate of the stock is 0.02.

The option is priced using a 1-period binomial tree with $u = 1.3$, $d = 0.7$.

A replicating portfolio consists of shares of the underlying stock and a loan.

Determine the amount borrowed in the replicating portfolio.

(A) 14.94 (B) 15.87 (C) 17.36 (D) 17.53 (E) 18.43

11. You are given the following weekly stock prices for six consecutive weeks:

50.02 51.11 50.09 48.25 52.06 54.18

Using the method in the McDonald textbook that does not assume expected return is zero, estimate the annual volatility of the stock.

(A) 0.11 (B) 0.12 (C) 0.29 (D) 0.33 (E) 0.34

Questions continue on the next page . . .

12. For European options on a stock having the same expiry and strike price, you are given:

(i) The stock price is 85.
(ii) The strike price is 90.
(iii) The continuously compounded risk free rate is 0.04.
(iv) The continuously compounded dividend rate on the stock is 0.02.
(v) A call option has premium 9.91.
(vi) A put option has premium 12.63.

Determine the time to expiry for the options.

(A) 3 months (B) 6 months (C) 9 months (D) 12 months (E) 15 months

13. You are given the following stochastic differential equations for two geometric Brownian motion processes for the prices of nondividend paying stocks:

$$\mathrm{d}S_1(t) = 0.10S_1(t)\,\mathrm{d}t + 0.06S_1(t)\,\mathrm{d}Z(t)$$
$$\mathrm{d}S_2(t) = 0.15S_2(t)\,\mathrm{d}t + 0.10S_2(t)\,\mathrm{d}Z(t)$$

Determine the continuously compounded risk-free rate.

(A) 0.02 (B) 0.025 (C) 0.03 (D) 0.035 (E) 0.04

14. Which of statements (A)–(D) is *not* a weakness of the lognormal model for stock prices?

(A) Volatility is constant.
(B) Large stock movements do not occur.
(C) Projected stock prices are skewed to the right.
(D) Stock returns are not correlated over time.
(E) (A)–(D) are all weaknesses.

15. The price of euros expressed in dollars, $x(t)$, follows the process

$$\frac{\mathrm{d}x(t)}{x(t)} = 0.02\,\mathrm{d}t + 0.10\,\mathrm{d}Z(t)$$

You are given:

(i) $x(0) = 1.50$
(ii) The continuously compounded risk-free rate for dollars is 0.04.

Calculate the 2-year prepaid forward price in euros of \$100.

(A) €60 (B) €61 (C) €62 (D) €63 (E) €64

Questions continue on the next page . . .

16. For a put option on a stock:

(i) The premium is 2.56.
(ii) Delta is -0.62.
(iii) Gamma is 0.09.
(iv) Theta is -0.02 per day.

Calculate the delta-gamma-theta approximation for the put premium after 3 days if the stock price goes up by 2.

(A) 1.20 (B) 1.32 (C) 1.44 (D) 1.56 (E) 1.62

17. S_t is the price of a stock at time t, with t expressed in years. You are given:

(i) S_t/S_0 is lognormally distributed.
(ii) The continuously compounded expected annual return on the stock is 5%.
(iii) The annual σ for the stock is 30%.
(iv) The stock pays no dividends.

Determine the probability that the stock will have a positive return over a period of three years.

(A) 0.49 (B) 0.51 (C) 0.54 (D) 0.59 (E) 0.61

18. All zero-coupon bonds have a 5% continuously compounded yield to maturity. To demonstrate an arbitrage, you buy one 5-year zero-coupon bond with maturity value 1000 and duration-hedge by buying N 2-year zero-coupon bonds with maturity value 1000. You finance the position by borrowing at the short-term rate.

Determine the amount you borrow.

(A) -1168 (B) -524 (C) 0.389 (D) 524 (E) 1168

19. For an at-the-money European call option on a nondividend paying stock:

(i) The price of the stock follows the Black-Scholes framework
(ii) The option expires at time t.
(iii) The option's delta is 0.5832.

Calculate delta for an at-the-money European call option on the stock expiring at time $2t$.

(A) 0.62 (B) 0.66 (C) 0.70 (D) 0.74 (E) 0.82

20. An insurance company offers a contract that pays a floating interest rate at the end of each year for 2 years. The floating rate is the 1-year-bond interest rate prevailing at the beginning of each of the two years. A rider provides that a minimum of 3% effective will be paid in each year.

You are given that the current interest rate is 5% effective for 1-year bonds and 5.5% effective for 2-year zero-coupon bonds. The volatility of a 1-year forward on a 1-year bond is 0.12.

Using the Black formula, calculate the value of the rider for an investment of 1000.

(A) 31 (B) 32 (C) 33 (D) 58 (E) 60

Questions continue on the next page . . .

21. Gap options on a stock have six months to expiry, strike price 50, and trigger 49. You are given:

(i) The stock price is 45.
(ii) The continuously compounded risk free rate is 0.08.
(iii) The continuously compounded dividend rate of the stock is 0.02.

The premium for a gap call option is 1.68.

Determine the premium for a gap put option.

(A) 4.20 (B) 5.17 (C) 6.02 (D) 6.96 (E) 7.95

22. The time-t price of a stock is $S(t)$. The stock's rate of price appreciation is 0.10, and its volatility is 0.2.

A claim on a stock pays $C(t)=\sqrt{S(t)}$.

The Itô process followed by C is of the form

$$\mathrm{d}C(t)=aC(t)\,\mathrm{d}t+bC(t)\,\mathrm{d}Z(t)$$

Determine a.

(A) 0.040 (B) 0.045 (C) 0.050 (D) 0.055 (E) 0.060

23. A 1-year American pound-denominated put option on euros allows the sale of €100 for £90. It is modeled with a 2-period binomial tree based on forward prices. You are given

(i) The spot exchange rate is £0.8/€.
(ii) The continuously compounded risk-free rate in pounds is 0.06.
(iii) The continuously compounded risk-free rate in euros is 0.04.
(iv) The volatility of the exchange rate of pounds to euros is 0.1.

Calculate the price of the put option.

(A) 8.92 (B) 9.36 (C) 9.42 (D) 9.70 (E) 10.00

24. For a 1-year call option on a nondividend paying stock:

(i) The price of the stock follows the Black-Scholes framework.
(ii) The current stock price is 40.
(iii) The strike price is 45.
(iv) The continuously compounded risk-free interest rate is 0.05.

It has been observed that if the stock price increases 0.50, the price of the option increases 0.25.

Determine the implied volatility of the stock.

(A) 0.32 (B) 0.37 (C) 0.44 (D) 0.50 (E) 0.58

25. The Itô process $X(t)$ satisfies the stochastic differential equation

$$\frac{\mathrm{d}X(t)}{X(t)}=0.1\,\mathrm{d}t+0.2\,\mathrm{d}Z(t)$$

Determine $\Pr\left(X(2)^3>X(0)^3\right)$.

(A) 0.63 (B) 0.65 (C) 0.67 (D) 0.69 (E) 0.71

Questions continue on the next page . . .

26. A market-maker writes a 1-year call option and delta-hedges it. You are given:

(i) The stock's current price is 100.
(ii) The stock pays no dividends.
(iii) The call option's price is 4.00.
(iv) The call delta is 0.76.
(v) The call gamma is 0.08.
(vi) The call theta is -0.02 per day.
(vii) The continuously compounded risk-free interest rate is 0.05.

The stock's price rises to 101 after 1 day.

Estimate the market-maker's profit.

(A) -0.04 (B) -0.03 (C) -0.02 (D) -0.01 (E) 0

27. You are simulating one value of a lognormal random variable with parameters $\mu = 1$, $\sigma = 0.4$ by drawing 12 uniform numbers on $[0, 1]$. The sum of the uniform numbers is 5.

Determine the generated lognormal random number.

(A) 1.7 (B) 1.8 (C) 1.9 (D) 2.0 (E) 2.1

28. A nondividend paying stock satisfies the stochastic differential equation

$$\frac{\mathrm{d}S(t)}{S(t)} = 0.15\,\mathrm{d}t + 0.3\,\mathrm{d}Z(t)$$

You are given

(i) The price of the stock is 50.
(ii) The Sharpe ratio is 0.35.

Calculate the price of a European at-the-money call option on the stock with two years to expiry.

(A) 9.62 (B) 10.05 (C) 10.11 (D) 10.29 (E) 10.37

29. You are given:

(i) The price of a stock is 40.
(ii) The continuous dividend rate for the stock is 0.02.
(iii) Stock volatility is 0.3.
(iv) The continuously compounded risk-free interest rate is 0.06.

A 3-month at-the-money European call option on the stock is priced with a 1-period binomial tree. The tree is constructed so that the risk-neutral probability of an up move is 0.5 and the ratio between the prices on the higher and lower nodes is $e^{2\sigma\sqrt{h}}$, where h is the amount of time between nodes in the tree.

Determine the resulting price of the option.

(A) 3.11 (B) 3.16 (C) 3.19 (D) 3.21 (E) 3.28

Questions continue on the next page . . .

30. For a portfolio of call options on a stock:

Number of shares of stock	Call premium per share	Delta
100	11.4719	0.6262
100	11.5016	0.6517
200	10.1147	0.9852

Calculate delta for the portfolio.

(A) 0.745 (B) 0.812 (C) 0.934 (D) 297.9 (E) 324.8

Solutions to the above questions begin on page 675.

Appendix A. Solutions for the Practice Exams

Answer Key for Practice Exam 1

1	A	11	D	21	B
2	E	12	E	22	B
3	B	13	B	23	E
4	A	14	C	24	B
5	B	15	D	25	E
6	D	16	C	26	B
7	B	17	B	27	B
8	D	18	A	28	E
9	E	19	A	29	B
10	B	20	C	30	E

Practice Exam 1

1. [**Section 2.4**] Options are convex, meaning that as the strike price increases, the rate of increase in the put premium does not decrease. The rate of increase from 30 to 35 is $(6.40-2.40)/(35-30)=0.80$, so the rate of increase from 35 to 38 must be at least $(38-35)(0.80)=2.40$, making the price at least $6.40+2.40=8.80$. Thus **(A)** is correct.

2. [**Subsection 1.2.1**] By put-call parity,

$$\begin{aligned} P &= C + Ke^{-rt} - Se^{-\delta t} \\ &= 4.10 + 28e^{-0.025} - 30 = 1.4087 \end{aligned}$$

For 100 shares, the cost is $100(1.4087) = \boxed{\mathbf{140.87}}$. **(E)**

3. [**Section 13.1**] The monthly arithmetic average of the prices is

$$\frac{39+37+43}{3} = 39.6667$$

The monthly geometric average of the prices is

$$\sqrt[3]{(39)(37)(43)} = 39.5893$$

The payments on the options are:

- The arithmetic average price call options with strike 36 pay $39.6667-36=3.6667$.
- The geometric average strike call options pay $43-39.5893=3.4107$.
- The arithmetic average price put options with strike 41 pay $41-39.6667=1.3333$.

The total payment on the options is $100(3.6667)+200(3.4107)+300(1.3333) = \boxed{\mathbf{1448.8}}$. **(B)**

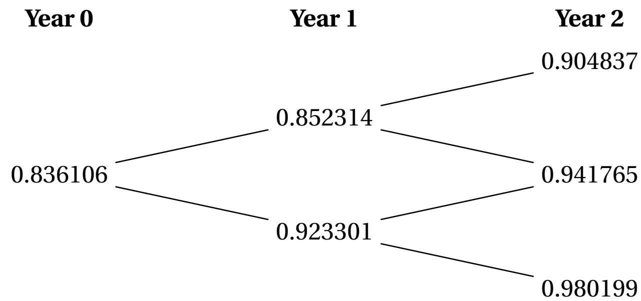


Figure A.1: Zero-coupon bond prices in the solution for question 6

4. [**Section 9.3**] By Black's formula,

$$\begin{aligned}
d_1 &= \frac{\ln(260/256)+0.5(0.25^2)(0.5)}{0.25\sqrt{0.5}} = 0.17609\\
d_2 &= 0.17609 - 0.25\sqrt{0.5} = -0.00068\\
N(d_1) &= N(0.17609) = 0.56989\\
N(d_2) &= N(-0.00068) = 0.49973\\
C &= 260e^{-0.02}(0.56989) - 256e^{-0.02}(0.49973) = \boxed{\mathbf{19.84}} \qquad \textbf{(A)}
\end{aligned}$$

5. [**Lesson 19**] I is true, since delta never changes, so one can buy delta of the option and borrow the cost.
II is false. An American option that should be exercised early does not satisfy the Black-Scholes equation if it is unexercised.
III is false. The equation must be modified to take into account the average stock price.
(B)

6. [**Section 24.1**] The prices of bonds at year 2 are $e^{-0.1} = 0.904837$, $e^{-0.06} = 0.941765$, and $e^{-0.02} = 0.980199$ at the 3 nodes. Pulling back, the price of a 2-year bond at the upper node of year 1 is

$$0.5e^{-0.08}(0.904837 + 0.941765) = 0.852314$$

and the price of a 2-year bond at the lower node of year 1 is

$$0.5e^{-0.04}(0.941765 + 0.980199) = 0.923301$$

The price of a 3-year bond initially is

$$0.5e^{-0.06}(0.852314 + 0.923301) = 0.836106$$

The yield is $-(\ln 0.836106)/3 = \boxed{\mathbf{0.059667}}$. **(D)** The binomial tree of bond prices is shown in Figure A.1.

7. [**Lesson 17**] This is arithmetic Brownian motion with $\mu = 0.5$, $\sigma = 0.9$. For time $t = 2$, $\mu t = (0.5)(2) = 1$, $\sigma\sqrt{t} = 0.9\sqrt{2}$. The movement is $12 - 10 = 2$.

$$\Pr\bigl(X(2) > 12\bigr) = 1 - N\left(\frac{2-1}{0.9\sqrt{2}}\right) = 1 - N(0.78567) = 1 - 0.78397 = \boxed{\mathbf{0.21603}} \qquad \textbf{(B)}$$

8. [**Section 12.4**] By the Boyle-Emanuel formula, with period $\frac{1}{24}$ of a year, the variance of annual returns is

$$\operatorname{Var}(R_{1/24,1}) = \tfrac{1}{2}\Big((40^2)(0.20^2)(0.02)\Big)^2\Big/24 = \boxed{\mathbf{0.0341}} \qquad \textbf{(D)}$$

9. [**Lesson 9**] For one share, Black-Scholes formula gives:

$$d_1 = \frac{\ln(42/40) + \Big(0.05 - 0 + 0.5(0.22^2)\Big)(0.5)}{0.22\sqrt{0.5}} = 0.55212$$
$$d_2 = 0.55212 - 0.22\sqrt{0.5} = 0.39656$$
$$N(-d_2) = N(-0.39656) = 0.34585$$
$$N(-d_1) = N(-0.55212) = 0.29043$$
$$P = 40e^{-0.05(0.5)}(0.34585) - 42(0.29043) = 1.2944$$

The cost of 100 puts is $100(1.2944) = \boxed{\mathbf{129.44}}$. **(E)**

Note that this question has nothing to do with delta hedging. The purchaser is merely interested in guaranteeing that he receives at least 40 for each share, and does not wish to give up upside potential. A delta hedger gives up upside potential in return for keeping loss close to zero.

10. [**Lesson 3**] $C_d = 0$ and $C_u = 1.3(66) - 70 = 15.8$. By equation (3.2),

$$B = e^{-rt}\left(\frac{uC_d - dC_u}{u-d}\right) = e^{-0.15}\left(\frac{-0.7(15.8)}{0.6}\right) = -15.87$$

$\boxed{\mathbf{15.87}}$ is borrowed. **(B)**

11. [**Lesson 8**] First calculate the logarithms of ratios of consecutive prices

t	S_t	$\ln(S_t/S_{t-1})$
0	50.02	
1	51.11	0.02156
2	50.09	−0.02016
3	48.25	−0.03743
4	52.06	0.07600
5	54.18	0.03991

Then calculate the sample standard deviation.

$$\frac{0.02156 - 0.02016 - 0.03743 + 0.07600 + 0.03991}{5} = 0.01598$$
$$\frac{0.02156^2 + 0.02016^2 + 0.03743^2 + 0.07600^2 + 0.03991^2}{5} = 0.001928$$
$$\tfrac{5}{4}(0.001928 - 0.01598^2) = 0.002091$$
$$\sqrt{0.002091} = 0.04573$$

Then annualize by multiplying by $\sqrt{52}$

$$0.04573\sqrt{52} = \boxed{\mathbf{0.3298}} \qquad \textbf{(D)}$$

12. [**Subsection 1.2.1**] By put-call parity

$$12.63-9.91=90e^{-0.04t}-85e^{-0.02t}$$
$$90e^{-0.04t}-85e^{-0.02t}-2.72=0$$

Let $x=e^{-0.02t}$ and solve the quadratic for x.

$$x=\frac{85+\sqrt{85^2+4(90)(2.72)}}{2(90)}=\frac{175.577}{180}=0.975428$$

The other solution to the quadratic leads to $x<0$, which is impossible for $x=e^{-0.02t}$. Now we solve for t.

$$e^{-0.02t}=0.975428$$
$$0.02t=-\ln 0.975428=0.024879$$
$$t=50(0.024879)=\boxed{\mathbf{1.244}}\qquad\textbf{(E)}$$

13. [**Lesson 20**] The Sharpe ratios must be equal, so

$$\frac{0.10-r}{0.06}=\frac{0.15-r}{0.10}$$
$$0.01-0.1r=0.009-0.06r$$
$$0.04r=0.001$$
$$r=\boxed{\mathbf{0.025}}\qquad\textbf{(B)}$$

14. [**Subsection 6.2.1**] **(C)** is not a weakness, since one would expect that the multiplicative change in stock price, rather than the additive change, is symmetric.

15. [**Lesson 22**] We want $100F^P_{0,2}\big(1/x(2)\big)$. We can apply formula (22.2) with $S(0)=1.5$, $a=-1$, $r=r_\$=0.04$, and $\delta=r_€=0.02$ to obtain

$$F_{0,2}\big(y(2)\big)=\frac{1}{1.5}e^{[-1(0.04-0.02)+0.5(-1)(-2)0.1^2]2}=\frac{1}{1.5}e^{-0.02}$$

Formula (22.3) is not appropriate, since it assumes payment is in dollars, and discounts at the dollar rate, but we want to pay in euros. Instead, we discount using the euro discount rate of 0.02 to obtain the final answer:

$$100F^P_{0,2}\big(1/x(2)\big)=\frac{100}{1.5}e^{-2(0.02)}e^{-0.02}=\boxed{\mathbf{62.78}}\qquad\textbf{(D)}$$

16. [**Section 12.2**] Theta is expressed per day of decrease, so we just have to multiply it as given by 3. Thus the change in price is

$$\Delta\epsilon+0.5\Gamma\epsilon^2+\theta h=-0.62(2)+0.5(0.09)(2^2)-0.02(3)=-1.12$$

The new price is $2.56-1.12=\boxed{\mathbf{1.44}}$. **(C)**

17. [**Section 7.2**] We are given that the average return $\alpha=0.05$, so the parameter of the associated normal distribution is $\mu=0.05-0.5(0.3^2)=0.005$. For a three year period, $m=\mu t=0.015$ and $v=\sigma\sqrt{t}=0.3\sqrt{3}=0.5196$. For a positive return, we need the normal variable with these parameters to be greater than 0. The probability that an $\mathcal{N}(0.015,0.5196^2)$ variable is greater than 0 is $N(0.015/0.5196)=N(0.02887)=\boxed{\mathbf{0.51151}}$. **(B)**

18. [**Subsection 26.1**] The price of a 5-year bond with maturity value 1 is $e^{-5(0.05)} = 0.778801$. The price of a 2-year bond with maturity value 1 is $e^{-2(0.05)} = 0.904837$. Letting $P(0,T)$ be the price of a T-year zero-coupon bond, the duration-hedge ratio is

$$N = -\frac{5P(0,5)}{2P(0,2)} = -\frac{5(0.778801)}{2(0.904837)} = -2.15177$$

Thus you buy a 5-year bond for 1000 and sell 2.15177 2-year bonds for 1000 at a cost of

$$778.80 - 2.15177(904.83) = \boxed{\mathbf{-1168}} \qquad \textbf{(A)}$$

19. [**Section 10.1**] Delta is $e^{-\delta t}N(d_1)$, or $N(d_1)$ for a nondividend paying stock. Since the option is at-the-money,

$$d_1 = \frac{(r+0.5\sigma^2)t}{\sigma\sqrt{t}} = \frac{r+0.5\sigma^2}{\sigma}\sqrt{t}$$

So doubling time multiplies d_1 by $\sqrt{2}$.

$$\begin{aligned} N(d_1) &= 0.5832 \\ d_1 &= N^{-1}(0.5832) = 0.2101 \\ d_1\sqrt{2} &= (0.2101)(1.4142) = 0.2971 \\ N(0.2971) &= \boxed{\mathbf{0.6168}} \qquad \textbf{(A)} \end{aligned}$$

20. [**Section 25.2**] This is a floorlet in the second year. The value of a 1-year forward on a 1-year bond with maturity value 1 is

$$F_{0,1}\big(P(1,2)\big) = \frac{P(0,2)}{P(0,1)} = \frac{1.05}{1.055^2} = 0.943375$$

The strike price is $1/(1+K_R) = 1/1.03 = 0.970874$, and this is $1+K_R = 1.03$ calls. The Black formula gives

$$\begin{aligned} d_1 &= \frac{\ln(0.943375/0.970874) + 0.5(0.12^2)}{0.12} = -0.17944 \\ d_2 &= -0.17944 - 0.12 = -0.29944 \\ N(d_1) &= N(-0.17944) = 0.42880 \\ N(d_2) &= N(-0.29944) = 0.38230 \\ C &= \frac{1}{1.05}\Big(0.943375(0.42880) - 0.970874(0.38230)\Big) = 0.03176 \end{aligned}$$

Multiplying by 1000, the answer is $1000\Big(1.03(0.03176)\Big) = \boxed{\mathbf{32.71}}$. **(C)**

21. [**Section 14.2**] For gap options, put-call parity applies with the strike price. If you buy a call and sell a put, if $S_T > K_2$ (the trigger price) you collect S_t and pay K_1, and if $S_t < K_2$ you pay K_1 and collect S_t which is the same as collecting S_t and paying K_1, so

$$C - P = Se^{-\delta t} - K_1 e^{-rt}$$

In this problem,

$$P = C + K_1 e^{-rt} - Se^{-\delta t} = 1.68 + 50e^{-0.04} - 45e^{-0.01} = \boxed{\mathbf{5.167}} \qquad \textbf{(B)}$$

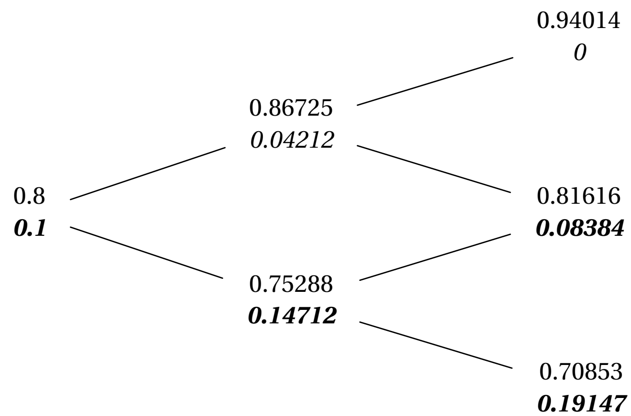


Figure A.2: Exchange rates and option values for put option of question 23

22. [**Lesson 22**] By formula (22.4), the process for $C(t) = S(t)^{1/2}$ is

$$\frac{\mathrm{d}C(t)}{C(t)} = \Big(0.5(\alpha-\delta)+(0.5)(0.5)(-0.5)\sigma^2\Big)\mathrm{d}t + 0.5\sigma\,\mathrm{d}Z(t)$$

Substituting $\alpha - \delta = 0.10$ and $\sigma = 0.2$,

$$\frac{\mathrm{d}C(t)}{C(t)} = \Big((0.5)(0.10) - 0.125(0.2^2)\Big)\mathrm{d}t + 0.5(0.2)\,\mathrm{d}Z(t) = \boxed{\mathbf{0.045}}\,\mathrm{d}t + 0.1\,\mathrm{d}Z(t) \qquad \textbf{(B)}$$

23. [**Section 4.3**] The 6-month forward rate of euros in pounds is $e^{(0.06-0.04)(0.5)} = e^{0.01} = 1.01005$. Up and down movements, and the risk-neutral probability of an up movement, are

$$\begin{aligned} u &= e^{0.01+0.1\sqrt{0.5}} = 1.08406 \\ d &= e^{0.01-0.1\sqrt{0.5}} = 0.94110 \\ p^* &= \frac{1.01005-0.94110}{1.08406-0.94110} = 0.4823 \\ 1-p^* &= 1-0.4823 = 0.5177 \end{aligned}$$

The binomial tree is shown in Figure A.2. At the upper node of the second column, the put value is calculated as

$$P_u = e^{-0.03}(0.5177)(0.08384) = 0.04212$$

At the lower node of the second column, the put value is calculated as

$$P_d^{\text{tentative}} = e^{-0.03}\Big((0.4823)(0.08384) + (0.5177)(0.19147)\Big) = 0.13543$$

but the exercise value $0.9 - 0.75288 = 0.14712$ is higher so it is optimal to exercise. At the initial node, the calculated value of the option is

$$P^{\text{tentative}} = e^{-0.03}\Big((0.4823)(0.04212) + (0.5177)(0.14712)\Big) = 0.09363$$

Since $0.9 - 0.8 = 0.1 > 0.09363$, it is optimal to exercise the option immediately, so its value is 0.10 (which means that such an option would never exist), and the price of an option for €100 is $100(0.10) = \boxed{\mathbf{10}}$. **(E)**

24. [**Section 11.2.1**] Δ is observed to be $0.25/0.50 = 0.5$. In Black-Scholes formula, $\Delta = e^{-\delta t}N(d_1) = N(d_1)$ in our case. Since $N(d_1) = 0.5$, $d_1 = 0$. Then

$$\frac{\ln(S/K) + r + 0.5\sigma^2}{\sigma} = 0$$
$$\ln(40/45) + 0.05 + 0.5\sigma^2 = 0$$
$$0.5\sigma^2 = -\ln(40/45) - 0.05 = 0.11778 - 0.05 = 0.06778$$
$$\sigma^2 = \frac{0.06778}{0.5} = 0.13556$$
$$\sigma = \sqrt{0.13556} = \boxed{\mathbf{0.3682}} \qquad \textbf{(B)}$$

25. [**Section 7.2**] The fraction $X(2)/X(0)$ follows a lognormal distribution with parameters $m = 2\big(0.1 - 0.5(0.2^2)\big) = 0.16$ and $v = 0.2\sqrt{2}$. Cubing does not affect inequalities, so the requested probability is the same as $\Pr\big(\ln X(2) - \ln X(0) > 0\big)$, which is

$$1 - N\left(\frac{-0.16}{0.2\sqrt{2}}\right) = N(0.56569) = \boxed{\mathbf{0.7142}} \qquad \textbf{(E)}$$

26. [**Section 12.2**] By formula (12.3) with $\epsilon = 1$ and $h = 1/365$,

$$\text{Market Maker Profit} = -0.5\Gamma\epsilon^2 - \theta h - rh\big(S\Delta - C(S)\big)$$
$$= -0.5(0.08)(1^2) + 0.02 - \frac{0.05}{365}[(100)(0.76) - 4]$$
$$= -0.04 + 0.02 - 0.00986 = \boxed{\mathbf{-0.02986}} \qquad \textbf{(B)}$$

27. [**Section 15.2**] The sum of the uniform numbers has mean 6, variance 1, so we subtract 6 to standardize it.

$$5 - 6 = -1$$

We then multiply by σ and add μ to obtain a $\mathcal{N}(\mu, \sigma^2)$ random variable.

$$(-1)(0.4) + 1 = 0.6$$

Then we exponentiate.

$$e^{0.6} = \boxed{\mathbf{1.822}} \qquad \textbf{(B)}$$

28. [**Lesson 20**] We back out the risk-free rate from the Sharpe ratio, as defined in equation (20.1):

$$0.35 = \frac{0.15 - r}{0.3}$$
$$r = 0.15 - 0.3(0.35) = 0.045$$

The given Itô process is a geometric Brownian motion. A stock following a geometric Brownian motion is in the Black-Scholes framework, so we use the Black-Scholes formula for the price of the option. Note that $K = S = 50$ since the option is at-the-money.

$$d_1 = \frac{\big(0.045 + 0.5(0.3^2)\big)(2)}{0.3\sqrt{2}} = 0.42427$$

$$\begin{aligned} d_2 &= 0.42427 - 0.3\sqrt{2} = 0 \\ N(d_1) &= N(0.42427) = 0.66431 \\ N(d_2) &= N(0) = 0.5 \\ C(50,50,2) &= 50(0.66431) - 50e^{-0.045(2)}(0.5) = \boxed{\mathbf{10.37}} \qquad \textbf{(E)} \end{aligned}$$

29. [**Lesson 3**] The risk-neutral probability is

$$0.5 = p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{(0.06-0.02)(0.25)} - d}{u-d} = \frac{e^{0.01} - d}{u-d}$$

but $u = d\,e^{2\sigma\sqrt{h}} = d\,e^{2(0.3)(1/2)} = d\,e^{0.3}$, so

$$\begin{aligned} e^{0.01} - d &= 0.5\left(e^{0.3}d - d\right) \\ e^{0.01} &= d\left(0.5(e^{0.3} - 1) + 1\right) = 1.17493d \\ d &= \frac{e^{0.01}}{1.17493} = 0.85967 \\ u &= 0.85967e^{0.3} = 1.16043 \end{aligned}$$

The option only pays at the upper node. The price of the option is

$$C = e^{-rh}p^*(Su - K) = e^{-0.06(0.25)}(0.5)\Big(40(1.16043) - 40\Big) = \boxed{\mathbf{3.1609}} \qquad \textbf{(B)}$$

30. [**Subsection 10.1.7**] Delta for a portfolio of options on a single stock is the sum of the individual deltas of the options.

$$100(0.6262) + 100(0.6517) + 200(0.9852) = \boxed{\mathbf{324.8}} \qquad \textbf{(E)}$$