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# HARDY, CHAPTER 9, RISK MEASURES

## I. Introduction

A. A risk measure is a method of encapsulating the riskiness of a distribution in a single number or in a real-valued function.
B. Regulators use risk measures as a succinct way of quantifying risk.
C. A risk measure is functional, mapping a distribution to the real numbers.

## II. The Quantile Risk Measure (QRM)

### A. Introduction

1. $L_0$ = PV of losses discounted at risk-free rate of interest.
2. QRM is $H(L_0) = V_\alpha = \inf \{V : \Pr [L_0 \le V] \ge \alpha\}$.
3. $V_\alpha$ is the $100\alpha$ percentile of the loss distribution.
4. It is the smallest sum to hold in risk-free assets in order that at maturity, the probability of having a sufficient amount to pay the guarantee G is at least $\alpha$.
5. The probability distribution used is the real-world measure or P-measure.
6. Q-measure is only used for pricing or determining the hedge portfolio.

### B. Simulation

1. QRM is very easy to estimate when liability distribution is constructed by stochastic simulation.
2. The estimated $\alpha$-quantile risk measure is the $N\alpha^{th}$ value of ordered liability values.
3. It is useful to calculate the standard error of the estimate $L_{0(N\alpha)}$.
4. A $100\beta\%$ confidence interval for the $\alpha$-quantile is $(L_{0(N\alpha-A)}, L_{0(N\alpha+A)})$.
5. $A = \Phi^{-1} [(1 + \beta) / 2] \sqrt{N\alpha(1 - \alpha)}$.

### C. Exact Calculation

1. Assume $L_0 = (G - F_n) e^{-rn}$ for $G \ge F_n$ and 0 for $G < F_n$.
2. Assume $F_k = S_k (1 - m)^k$ for integer k and assume $\varepsilon = \Pr [L_0 = 0]$.
3. Assume that stock returns follow a lognormal process.
4. Then $\varepsilon = 1 - \Phi \{\log G/S_0 - n [\mu + \log (1 - m)] / \sqrt{n\sigma}\}$.
5. Quantile risk measure is $V_\alpha$, the smallest amount satisfying $\Pr [F_n + V_\alpha e^{rn} > G] \ge \alpha$.
6. Assume $F_n$ is a continuous random variable then $V_\alpha = [G - F^{-1}_{Fn}(1 - \alpha)] e^{-rn}$.
7. Assume $z_p = \Phi^{-1} (p)$ then $V_\alpha = \{G - F_0 \exp [- z_\alpha \sqrt{n\sigma} + n (\mu + \log (1 - m))]\} e^{-rn}$.
8. It is also possible to calculate QRMs for other distributions analytically.
9. Effect of calibration brings results closer together.

III. The Conditional Tail Expectation (CTE) Risk Measure

A. Introduction

1. Expected value of loss given that loss falls in the upper $(1 - \alpha)$ tail of distribution.

2. $CTE_\alpha (L) = E [L_0 \mid L_0 > V_\alpha]$.

3. Find $\beta' = \max \{\beta : V_\alpha = V_\beta\}$.

4. $CTE_\alpha (L) = \{(1 - \beta') E [X \mid X > V_\alpha] + (\beta' - \alpha) V_\alpha\} / (1 - \alpha)$.

B. Simulation

1. Start by ordering the simulated losses.

2. Provided $N\alpha$ is an integer, $\tilde{CTE}_\alpha (L_0) = \Sigma L_{0(j)} / N (1 - \alpha)$, for $j = N\alpha + 1$ to N.

3. Estimate will have uncertainty attached from sampling variability.

4. This may be quantified by standard error or by confidence interval.

5. SD $(L_{(j)} : j > N\alpha) / \sqrt{N (1 - \alpha)}$, where SD () denotes standard deviation of the L's.

C. Exact Calculation

1. It is possible to calculate the CTE risk measure for a plain vanilla GMMB, with no allowance for margin offset and no dynamic hedging.
2. As with QRMs effect of calibration is to bring tail measures closer.

IV. Quantile and CTE Measures Compared

A. Both are very simple to work with.
B. CTE > QRM until maximum value of $L_0$ is reached, when they will be equal.
C. If distribution of $L_0 \mid L_0 > V_\alpha$ is uniform then $CTE_\alpha (L_0) = V_{(1+\alpha)/2}$.
D. For most GMMB, GMDB and GMAB contracts, $CTE_\alpha (L_0) > V_{(1+\alpha)/2}$.
E. Coherence of risk measures: H [X] is said to be coherent if it is

1. Bounded above by maximum loss: $H [X] \leq \max (X)$.
2. Bounded below by the mean loss: $H [X] \geq E [X]$.
3. Scalar additive and multiplicative: $H [a X + b] = a H [X] + b$, for $a, b > 0$.
4. Subadditive: $H [X + Y] \leq H [X] + H [Y]$.

F. Quantile risk measures fail both property 2. and property 4. above.
G. CTE satisfies all criteria for coherence.
H. Quantile measure is determined by one point on loss distribution.
I. CTE uses all of loss distribution to the right of the quantile.
J. Robustness under simulation: Sensitivity to sampling variability

1. Quantile takes a single ordered outcome to determine risk measure.
2. CTE takes an average of set of largest outcomes: less sensitive.

K CTE is used for both reserve and solvency capital calculations for segregated fund contracts in Canada

1. Recommended by CIA and accepted by OSFI.
2. For reserves, use $\alpha$ around 80% and for total solvency capital, use $\alpha = 95\%$.

V. Risk Measures for GMAB Liability: Examples

A. CTE and Quantile Risk Measure for Actuarially Managed GMAB

1. Quantile falls below mean at all values of $\alpha <$ around 60%.
2. CTE curve lies above quantile curve until maximum value is reached.
3. CTE curve is somewhat smoother than the quantile curve.
4. Significant tail risk arises from offering reset option (+ 3% of fund value).
5. Restricting reset option to 2 shouts, or resets per year does not help a lot.

B. Comparison of Actuarial and Hedging Approaches to Risk Management of GMAB

1. Risk measure using dynamic hedging includes cost of the hedge.
2. $V_{.9}^{act} = 1.29\%$ of fund while $V_{.9}^{dh} = 1.06\%$ of fund.
3. These results indicate very similar risk under 2 approaches (deceptive).
4. $CTE_{.9}^{act} = 5.92\%$ of fund while $CTE_{.9}^{dh} = 1.74\%$ of fund.
5. These results indicate more of a difference.
6. Difference also increases as the $\alpha$ parameter increases.

VI. Risk Measures for VA Death Benefits: Examples

A. Many VA contracts carry only a death benefit guarantee.
B. Mortality and withdrawals are treated deterministically.
C. Costs are fairly low and simulation evidence supports this.
D. Risk measures for VA-type GMDB benefits, 30-year contract, as % of initial fund value

1. Fixed GMDB carries relatively little risk.
2. Slight tail risk using actuarial risk management but this is damped by using hedging strategy, that virtually eliminates the risk.
3. Increasing GMDB is a more substantial risk.
4. Hedging strategy significantly reduces the tail risk.

# SECTION Q

## ***Review Questions***

**Introductory Note**

**This section of the study manual contains an array of review questions covering the entire syllabus. These questions were written to serve as an aid in assessing your understanding of the material after you have completely covered it through your studies. It is unlikely that you would see questions of this type on the actual exam, since those questions are developed with an eye toward application of multiple parts of the syllabus in actual job situations.**

While these questions were not developed as possible exam questions by themselves, it is entirely possible that you could see some of these questions as parts of actual exam questions.

This section of the manual is relatively new. We would welcome your feedback on it, both as it is presented here and any suggestions you have for improvement.

**Source**: **Hardy, Chapter 13**

**Question 2**
(55 Points)

A 7-year EIA contract is indexed to the S&P 500 index. If the guaranteed payout is found by accumulating 95% of premium at 4% per year interest, the payout is twice the guaranteed amount, the floor rate is 0% and the values of the index over the 7-year term are the following

| t | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $S_t$ | 100 | 125 | 150 | 120 | 120 | 150 | 240 | 300 |

(a) What is the participation rate of the contract under a point-to-point (PTP) indexing?

(b) If the participation rate is 80% of that in (a), what is the ratio of the payout under the compound annual ratchet (CAR) indexing to that in (a)?

(c) If the participation rate is 80% of that in (a), what is the ratio of the payout under the simple annual ratchet (SAR) indexing to that in (a)?

(d) Same as (b) with a cap rate of 15%?

(e) Same as (c) with a cap rate of 10%?

(f) If the participation rate is 80% of that in (a), what is the ratio of the payout under the high water mark indexing to that in (a)?

**Source**: **Marino, Chapter 3**

**Question 6**
(14 Points)

With respect to the taxation of life insurance policies, describe the disposition of an interest in a life insurance policy through a transfer of ownership or absolute assignment.

**Source**: **Hardy, Chapter 13**

**Solution to Question 2**

| **Statement** | | | | | | | | | **Points** |
|---|---|---|---|---|---|---|---|---|---|
| 1. $G = (0.95)\ P\ (1.04)^{7} = 1.250\ P.$ | | | | | | | | | **3** |
| 2. Total increase of index: 300%. | | | | | | | | | **3** |
| 3. Annual increases of the index are | t | 1 | 2 | 3 | 4 | 5 | 6 | 7 | **7** |
| | % | 25% | 20% | -20% | 0% | 25% | 60% | 25% | |

**(a) PTP indexing** **7**

1. Payout is: $P\ (1 + \alpha\ (3.00 - 1)) = 2.50\ P.$ 3
2. $P\ (1 + 2.00\ \alpha) = 2.50\ P$ 2
3. $\alpha = 1.50 / 2.00 = 0.75.$ 2

**(b) CAR indexing with α = 80% x 0.75 = 0.60** **7**

1. Payout is: $P\ (1+.15)(1+.12)(1+.15)(1+.36)(1+.15) = 2.32\ P.$ 4
2. Ratio is 2.32 P / 2.50 P = 92.8%. 3

**(c) SAR indexing with α = 80% x 0.75 = 0.60** **7**

1. Payout is: $P\ (1+ (.15 + .12 + .15 + .36 + .15)) = 1.93\ P.$ 4
2. Ratio is 1.93 P / 2.50 P = 77.2%. 3

**(d) CAR indexing with α = 80% x 0.75 = 0.60 and cap rate of 15%** **7**

1. Payout is: $P\ (1+.15)(1+.12)(1+.15)(1+.15)(1+.15) = 1.96\ P.$ 4
2. Ratio is 1.96 P / 2.50 P = 78.4%. 3

**(e) SAR indexing with α = 80% x 0.75 = 0.60 and cap rate of 10%** **7**

1. Payout is: $P\ (1+ (.10 + .10 + .10 + .10 + .10)) = 1.50\ P.$ 4
2. Ratio is 1.50 P / 2.50 P = 60.0%. 3

**(f) High water mark indexing with α = 80% x 0.75 = 0.60** **7**

1. Payout is: $P\ (1 + 0.60\ (3.00 - 1)) = 2.20\ P.$ 4
2. Ratio is 2.20 P / 2.50 P = 88.0%. 3

**TOTAL POINTS** **55**

**Source**: **Marino, Chapter 3**

**Solution to Question 6**

| **Statement** | **Points** |
|---|---|
| 1. General rule: Policy gain = Transfer price – ACB. | 1 |
| 2. Specific rules apply to certain transactions | 1 |
| a. Transfer by way of gift or bequest. | 1 |
| b. Distribution from a corporation. | 1 |
| c. Disposition by way of operation of law only. | 1 |
| d. Disposition to any non-arm's length person. | 1 |
| 3. Value of interest for these special cases is usually CSV – policy loan. | 1 |
| 4. Tax-free rollovers (Proceeds = ACB) | 1 |
| a. Generation transfers and intergeneration transfers. | 2 |
| b. Inter vivos transfer to spouse or common-law partner. | 2 |
| c. Transfer to spouse or common-law partner at death. | 2 |
| **TOTAL POINTS** | **14** |

**Solution to Question 7**

| **Statement** | **Points** |
|---|---|
| 1. CSV of the policy. | 1 |
| 2. Policy's loan value. | 1 |
| 3. Face value of policy. | 1 |
| 4. State of health of insured and his/her life expectancy. | 1 |
| 5. Conversion privileges under the policy. | 1 |
| 6. Other policy terms, such as term riders, double indemnity provisions. | 1 |
| 7. Replacement value of the policy. | 1 |
| **TOTAL POINTS** | **7** |