Mahler's Guide to Stochastic Models

Information in bold or sections whose title is in bold are more important for passing the exam. Larger bold type indicates it is extremely important. Information presented in italics (and sections whose titles are in italics) should not be needed to directly answer exam questions and should be skipped on first reading. It is provided to aid the reader's overall understanding of the subject, and to be useful in practical applications.

Solutions to problems are given at the end.[1] Highly Recommended problems are double underlined. Recommended problems are underlined.

[1] Note that problems include both some written by me and some from past exams. The latter are copyright by the Society of Actuaries and the Casualty Actuarial Society and are reproduced here solely to aid students in studying for exams. The solutions and comments are solely the responsibility of the author; the SOA and CAS bear no responsibility for their accuracy. While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams.

CAS Exam 3 versus SOA Exam MLC:

The following are on SOA Exam M and not on CAS Exam 3:

Probability Models, Section 5.4.3

Therefore, my section on Mixing Poisson Processes is on SOA Exam M, but not on CAS Exam 3. Nevertheless, the CAS Exam 3 has had questions on this material.

Probability Models, Section 5.4.2: bottom of page 342 and pages 324-325.

Nevertheless, students taking CAS Exam 3 should know the how to compute the variance of Compound Processes and how to add Poisson Processes.

Probability Models, Section 5.3.5: Proposition 5.3 and proceeding paragraph and Example 5.18.

Therefore, my section on Time Dependent Classification of Poisson Processes is on SOA Exam M, but not on CAS Exam 3. This is not an important idea in any case.

Loss Models, Section 8.1.1

While it not a bad idea to read this section of Loss Models, it contains no material that is not already covered in Probability Models by Ross.

The following are on CAS Exam 3 and not on SOA Exam M:

Probability Models, Section 5.3.4: Example 5.13, Example 5.15 and beyond.

Therefore, my section on **Comparing Poisson Processes is on CAS Exam 3, but not SOA Exam M**.

Probability Models, Section 5.4.1: Example 5.23 and following.

Therefore, the density of a Nonhomogeneous Poisson Process is on CAS Exam 3, but not SOA Exam M. This is not an important idea in any case.

Course 3 Exam Questions by Section of this Study Aid[2]

							CAS	SOA
Section	Sample	5/00	11/00	5/01	11/01	11/02	11/03	11/03
1								
2					10 11		32	26
3								
4	23	2	23 29			9, 20	31	11
5							12, 13	
6			6					
7								
8								
9		10		4	19	15	30	20
10								
11								
12				37				
13								
14						30		
15	26				29			24
16								
17								
18								
19								

Sections 17-19 on Nonhomogeneous Markov Chains cover material that was added to the syllabus in 2005.

The CAS/SOA did not release the 5/02 and 5/03 exams.
From 5/00 to 5/03, the Course 3 Exam was jointly administered by the CAS and SOA.
Starting in 11/03, the CAS and SOA gave separate exams.

[2] Excluding any questions that are no longer on the syllabus.

	CAS	CAS	SOA	CAS	SOA M	CAS	SOA M	CAS	CAS	SOA M
Section	5/04	11/04	11/04	5/05	5/05	11/05	11/05	5/06	11/06	11/06
1										
2		18, 19		39		28			26	
3									27	8
4	31	17		7, 11, 13	5, 24	29, 31				9
5				17	39					
6				12		25		34		
7										
8					25					
9					6	27	7, 40			
10	15, 27		26	14		26		33	28	
11										10
12										
13										
14	18, 23	15		36	11		23	29		14
15			14			39		24	38	
16					12		6			
17				34		23	4, 5		21, 22	
18										15
19										

The SOA did not release its 5/04 and 5/06 exams.

Section 1, Introduction

The concepts in Introduction to Probability Models by Sheldon M. Ross and "Multi-State Transition Models with Actuarial Applications" by James W. Daniel are demonstrated.

Assume we let X(t) = the surplus of the Sari Insurance Company at time t.
Then as time changes the surplus changes randomly.
This is an example of a **Stochastic Process**.
For each time t, X(t) is a random variable.

If we only look at the surplus at each year end, then this would be a **discrete-time** stochastic process. If instead we were able to examine the surplus at any point in time, this would be a **continuous-time** stochastic process. Generally, continuous-time processes can be approximated by discrete-time processes, by taking very small time intervals in a discrete time process. For example, the difference between being able to examine the surplus of the Sari Insurance Company at the end of each day or at any time is unlikely to be of any practical importance.

A **stochastic process {X(t), t $\in$ T} is a collection of random variables.**
T is the index set of the stochastic process.

So if one can only look at the surplus at each year end, we would have T = {1, 2, 3,...} in units of years. We would have corresponding random variables X(1), X(2), X(3) , ..., the observed amounts of surplus at years end. If instead we were able to examine the surplus at any point in time, we would have T = {t >0} and X(t), t >0.

In the Surplus example, $-\infty < X(t) < \infty$, since the surplus can (in theory) be any real number.[3] *The set of possible values for the random variables is called the state space of the stochastic process. For the surplus example, the state space is the set of real numbers.*[4]

Exercise: The price of stock at the close of business each day is P(t). What type of stochastic process is this? What is the state space? What is the index set?
[Solution: This is a discrete-time process. The state space is the positive real numbers.
The index set is the positive integers (in units of days.)]

[3] Small or negative amounts of Surplus would indicate an insurer in serious trouble or insolvent.
[4] Ignoring as usual, that the smallest unit of currency is $.01.

There are three types of stochastic processes covered on CAS Exam 3:[5]

1. Poisson Processes, including the nonhomogeneous and compound cases.
2. Markov Chains.
3. Brownian Motion[6]

Poisson Processes and Brownian Motion are continuous time models.
Markov Chains are discrete time models.

Poisson Distribution:

Support: $x = 0,1,2,3...$ Parameters: $\lambda > 0$

D. f. : $F(x) = 1 - \Gamma(x+1 ; \lambda)$ *Incomplete Gamma Function*

P. d. f. : $\mathbf{f(x) = \lambda^x e^{-\lambda} / x!}$

Mean = λ

Variance = λ

Mode = largest integer in λ (if λ is an integer then $f(\lambda)=f(\lambda-1)$ and both λ and λ-1 are modes.)

The sum of two independent variables each of which is Poisson with parameters λ_1 and λ_2 is also Poisson, with parameter $\lambda_1 + \lambda_2$.

Exponential Distribution:

$\mathbf{F(x) = 1 - e^{-x/\theta}}$ $\mathbf{f(x) = e^{-x/\theta} / \theta,\ x > 0.}$

Mean = θ   **Variance = θ^2**   Second Moment = $2\theta^2$

When an Exponential Distribution is truncated and shifted from below, in other words when looks at the nonzero payments excess of a deductible, one gets the same Exponential Distribution, due to its memoryless property.

[5] Other stochastic processes not covered on this exam include continuous time Markov Processes.

[6] Brownian Motion is discussed in Derivatives Markets by Robert McDonald, and is covered in "Mahler's Guide to Financial Economics."
The material in Derivatives Markets by Robert McDonald is on SOA Exam M-FE, not SOA Exam M-LC.

Gamma Distribution:

$F(x) = \Gamma(\alpha; x/\theta)$ $\qquad$ $f(x) = \theta^{-\alpha} x^{\alpha-1} e^{-x/\theta} / \Gamma(\alpha)$, $x > 0$.

Mean = $\alpha\theta$ $\qquad$ **Variance =** $\alpha\theta^2$

$$E[X^n] = \theta^n \prod_{i=0}^{n-1} (\alpha+i).$$

The sum of n independent identically distributed variables which are Gamma with parameters α and θ is a Gamma distribution with parameters $n\alpha$ and θ.

For α = a positive integer, the Gamma distribution is the sum of α independent variables each of which follows an Exponential distribution.
For α =1 you get the Exponential.

For very large α, the Gamma distribution approaches a symmetric Normal Distribution.

Normal Distribution:

Support: $\infty > x > -\infty$ $\qquad$ Parameters: $\infty > \mu > -\infty$ (location parameter)
$\sigma > 0$ (scale parameter)

D. f. : $\qquad$ **$F(x) = \Phi((x-\mu)/\sigma)$**

P. d. f. : $\qquad$ **$f(x) = \phi((x-\mu)/\sigma) = \exp[-(x-\mu)^2/(2\sigma^2)] / (\sigma\sqrt{2\pi})$.**

Mean = μ $\qquad$ **Variance =** σ^2

Skewness = 0 (distribution is symmetric)

Mode = μ $\qquad$ Median = μ

In general, let μ be the mean of the frequency distribution, while σ is the standard deviation of the frequency distribution, then the chance of observing at least i claims and not more than j claims is approximately: $\Phi[\{(j+.5)-\mu\} /\sigma] - \Phi[\{(i-.5)-\mu\} /\sigma]$.

Attached to the exam is a table of the Standard Normal Distribution, with $\mu = 0$ and $\sigma = 1$.
The table attached to SOA Exam M is somewhat more detailed than that attached to CAS 3.[7]

Normal Distribution Table, SOA Exam M

Entries represent the area under the standardized normal distribution from $-\infty$ to z, Pr(Z < z).
The value of z to the first decimal place is given in the left column.
The second decimal is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Table continued on the next page

[7] The Normal Table attached to CAS 3 is shown in "Mahler's Guide to Statistics."

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z < z).
The value of z to the first decimal place is given in the left column.
The second decimal is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

	Values of z for selected values of Pr(Z < z)						
z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z < z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

Section 2, Poisson Processes[8]

A (homogeneous) **Poisson Process has a constant claims intensity λ with independent increments. The number of claims observed in time interval (0, T) is given by the Poisson Distribution with mean Tλ.**[9][10]

For example, assume we have a Poisson Process on (0,5) with λ = .03. Then the total number of claims is given by a Poisson Distribution with mean (5)(.03) = 0.15.

Exercise: For a Poisson Process with λ = .7, what is the chance of exactly 3 claims by time 2?
[Solution: Poisson Distribution with mean: (2)(.7) = 1.4. $f(3) = 1.4^3 e^{-1.4}/3! = 11.3\%$.]

Thus if you understand the Poisson frequency distribution[11], you understand the (homogeneous) Poisson Process. However, in the case of a (homogeneous) Poisson Process one normally keeps track of both the total number of claims and the times they each occurred. If one had three claims, one would also want to know the three times at which they occurred.

Mathematical Assumptions:

The specific mathematical assumptions for a Poisson Process on the interval (0, T) with claims intensity λ, are:

1. The process starts at time = 0; at time = zero there are a total of zero claims.

2. The number of claims occurring in disjoint subintervals of (0, T) are independent.[12]

3. The mean number of claims expected in any subinterval of (0, T) is λ times the width of the subinterval.[13]

4. As a subinterval gets very small, the chance of more than one claim occurring in that subinterval quickly goes to zero.
Prob[more than one claim in interval of length h] / h, goes to zero as h goes to zero.[14]

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[8] See Section 5.3 of an Introduction to Probability Models by Sheldon M. Ross.
Also see Section 8.1.1 of Loss Models, not on the syllabus of CAS Exam 3.
[9] Ross refers to "events." I will usually refer to "claims", which is the most common application of these ideas for actuaries.
[10] *Note that by changing the time scale and therefore the claims intensity, one can always reduce to a mathematically equivalent situation where the interval is (0,1) .*
[11] See "Mahler's Guide to Frequency Distributions."
[12] Increments are independent.
[13] Increments are stationary, with claims intensity lambda.
[14] $P[N(h) \geq 2] = o(h)$.

Counting Processes:[15]

A stochastic process N(t) on t≥0 is a counting process if represents the total number of events by time t. N(t) must satisfy:
1. N(t) ≥ 0.
2. N(t) is integer valued.
3. If s < t, then N(s) ≤ N(t).
4. For s < t, N(t) - N(s) equals the number of events that have occurred in the interval (s, t].

Thus the state space of a counting process is the nonnegative integers.
A counting process is nondecreasing with time.

A **counting process is what an actuary would call a claims frequency process.** A Poisson Process is an example of a counting process. N(t) is the number of claims[16] that have occurred by time t. N(t) - N(s) is the number of claims that have occurred in the interval (s, t], is the increment to the process.

A claims frequency process has independent increments if the number of claims in two disjoint periods of time are independent of each other.

For example, for practical purposes the number of dental claims a large insurer gets this week are probably independent of the number of dental claims it gets next week. Thus we could model this frequency process with independent increments. However, the number of claims for the flu this week might be correlated with the number of claims for the flu last week.

A claims frequency has stationary increments if the distribution of the increment is a function of the width of the interval t - s, rather than s or t individually.[17]

Stationary Increments ⇔ the distribution of N(s + Δ) - N(s) is independent of s.

A (homogeneous) Poisson Process is a counting process with stationary and independent increments.

If exposure levels are changing, then we would be unlikely to have stationary increments and therefore unlikely to have a (homogeneous) Poisson Process. Rather one would have a Nonhomogeneous Poisson Process, to be discussed subsequently.

[15] See Section 5.3.1 of an Introduction to Probability Models by Sheldon M. Ross.
[16] Ross uses "events." In actuarial applications an event is usually, but not always, a claim.
[17] See the top of page 298 of an Introduction to Probability Models by Sheldon M. Ross.

o(h)

o(h) is a mathematical property a function may have.
A function f(x) is said to be o(h), if the limit as h approaches zero of f(h)/h is zero.[18]

In other words as h approaches zero, f(h) goes to zero more quickly than h.
For example, $f(x) = x^{1.5}$ is o(h). Thus x is of lower order of magnitude than $x^{1.5}$.
On the other hand, $\sqrt{x}$ is <u>not</u> o(h).

For a Poisson Process, the chance of having more than 1 claim in an interval of length h, is o(h). In other words, as the length of an interval approaches zero, the chance of having more than one claim in that interval also approaches zero, faster than the length of that interval approaches zero.
Prob[more than one claim in an interval of length h]/h, goes to zero as h goes to zero.

For a Poisson Process, the chance of having 1 claim in an interval of length h, is
λh + o(h). In other words, as h goes to zero,
Prob[one claim in interval of length h]/h, goes to λ as h goes to zero.

A function f(x) is equivalent to x as x goes to zero, f(x) ~ x, if the limit as h approaches zero of f(h)/h is one. For example, sin(x) ~ x, as x approaches zero.

A function f(x) is said to be O(h), if there is a positive constant K, such that for all sufficiently small h, $|f(h)/h| \le K$. For example, $f(x) = x(1 + a/x)^x$ is O(h), since the limit as x goes to zero of $(1 + a/x)^x$ is e^a. If a function is either o(h) or equivalent to h as h goes zero, then it is also O(h).

<u>A Definition of a Poisson Process</u>:[19]

A Poisson Process is a counting process, N(t), such that:
N(0) = 0.
Stationary and independents increments.
Prob[N(h) = 1] = λh + o(h).
Prob[N(h) $\ge$ 2] = o(h).

The assumption of stationary and independent increments is basically equivalent to asserting that at any point in time the process probabilistically restarts itself. In other words, the process has no memory.[20]

[18] See Definition 5.2 of an <u>Introduction to Probability Models</u> by Sheldon M. Ross.
[19] See Definition 5.3 of an <u>Introduction to Probability Models</u> by Sheldon M. Ross.
[20] See the remark at the bottom of page 293 of an <u>Introduction to Probability Models</u> by Sheldon M. Ross.

Problems:

2.1 (1 point) A Poisson Process has a claims intensity of 0.3.
What is the probability of exactly 2 claims from time 0 to time 10?
A. 22% B. 24% C. 26% D. 28% E. 30%

2.2 (1 point) A Poisson Process has a claims intensity of 0.3.
Given that there have been 2 claims from time 0 to time 10, what is the probability of exactly 2 claims from time 10 to time 20?
A. 22% B. 24% C. 26% D. 28% E. 30%

2.3 (3 points) A Poisson Process has a claims intensity of 0.3.
Given that there have been 2 claims from time 0 to time 10, what is the probability of exactly 2 claims from time 6 to time 16?
A. 19% B. 21% C. 23% D. 25% E. 27%

2.4 (1 point) A Poisson Process has $\lambda = 0.6$.
What is the probability of exactly 3 claims from time 5 to time 9?
A. 19% B. 21% C. 23% D. 25% E. 27%

2.5 (3 points) A Poisson Process $\lambda = 0.6$.
If there are three claims from time 2 to time 8, what is the probability of exactly 3 claims from time 5 to time 9?
A. 19% B. 21% C. 23% D. 25% E. 27%

2.6 (2 points) A Poisson Process has $\lambda = 0.6$.
What is the probability of 2 claims from time 0 to time 4 and 5 claims from time 0 to time 10?
A. 4.0% B. 4.5% C. 5.0% D. 5.5% E. 6.0%

2.7 (3 points) A Poisson Process has $\lambda = 0.6$. What is the probability of at least 1 claim from time 0 to time 4 and at least 3 claims from time 0 to time 6?
A. 69% B. 71% C. 73% D. 75% E. 77%

2.8 (2 points) Claims are given by a Poisson Process with $\lambda = 7$. What is probability that the number of claims between time 5 and 15 is at least 60 but no more than 80?
Use the Normal Approximation.
(A) 73% (B) 75% (C) 77% (D) 79% (E) 81%

2.9 (3 points) Claims are given by a Poisson Process with $\lambda = 7$. What is probability that the number of claims between time 5 and 11 is greater than the number of claims from time 11 to 15? Use the Normal Approximation.

(A) 87% (B) 89% (C) 91% (D) 93% (E) 95%

Use the following information for the next two questions:
The android Data is stranded on the planet Erehwon.

- Data uses energy uniformly at a rate of 10 gigajoules per year.
- If Data's stored energy reach 0, he ceases to function.
- Data gets his energy from dilithium crystals.
- Data gets 6 gigajoules of energy from each dilithium crystal.
- Data finds dilithium crystals at a Poisson rate of 2 per year.
- Data can store dilithium crystals without limit until needed.
- Data currently has 8 gigajoules of energy stored.

2.10 (4 points) What is the probability that Data ceases to function within the next 2.5 years?

(A) 30% (B) 33% (C) 36% (D) 39% (E) 42%

2.11 (3 points) What is the expected number of gigajoules of energy found by Data in the next 2.5 years?

(A) 20 (B) 22 (C) 24 (D) 26 (E) 28

2.12 (3 points) Claims are given by a Poisson Process with $\lambda = 5$. What is probability that the number of claims between time 2 and 10 is greater than the number of claims from time 7 to 16? Use the Normal Approximation.

(A) 17% (B) 19% (C) 21% (D) 23% (E) 25%

2.13 (1 point) A Poisson Process has a claims intensity of 0.4 per day. How many whole number of days do we need to observe, in order to have at least a 95% probability of seeing at least one claim?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

2.14 (2 points) A Poisson Process has a claims intensity of 0.4 per day. How many whole number of days do we need to observe, in order to have at least a 95% probability of seeing at least two claims?

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

2.15 (1 point) A Poisson Process has a claims intensity of 0.04 per day. If there is at least 1 claim during a week, what is the probability that there are at least 2 claims during that week?
(A) 7% (B) 9% (C) 11% (D) 13% (E) 15%

2.16 (3 points) A Poisson Process has a claims intensity of 0.5.
What is the probability that the third claim occurs between time 5 and time 8?
(A) 10% (B) 15% (C) 20% (D) 25% (E) 30%

***2.17** (3 points) Data the Android gets new Emails at a Poisson rate of 15 per hour. Data checks for new Emails every x hours, where x has distribution $F(x) = 1 - 1/(27x^3)$, $x > 1/3$.*
What is the variance of the number of new Emails Data finds when he checks?
(A) 18 (B)20 (C) 22 (D) 24 (E) 26

2.18 (3 points) Joe has four homework assignments: Math, English, History, and Chemistry.
He randomly chooses one of his assignments, and works on it until he completes it.
When Joe completes an assignment, he randomly chooses one of the remaining assignments, and works on it until completed.
Joe completes assignments at a Poisson rate of 1 assignment per 40 minutes.
Calculate the probability that Joe has completed his English assignment within 1 hour of starting his homework.
A. Less than 25%
B. At least 25%, but less than 30%
C. At least 30%, but less than 35%
D. At least 35%, but less than 40%
E. At least 40%

2.19 (3 points) For a certain company, losses follow a Poisson process with $\lambda = 3$ per year.
The amount of every loss is 100.
An insurance policy covers all losses in a year, subject to an annual aggregate deductible of 100.
There have been no payments for this insurance policy during the first half of the year.
Calculate the expected claim payments for this insurance policy during the second half of the year.
(A) 90 (B) 100 (C) 110 (D) 120 (E) 130

2.20 (3 points) One has a Poisson Process with $\lambda = 25$.
Let A = the number of events that occur from time 1 to time 6.
Let B = the number of events that occur from time 3 to time 10.
Determine the correlation of A and B.
A. 30% B. 40% C. 50% D. 60% E. 70%

Use the following information for 3, 11/01 questions 10 and 11:
For a tyrannosaur with 10,000 calories stored:
(i) The tyrannosaur uses calories uniformly at a rate of 10,000 per day.
If his stored calories reach 0, he dies.
(ii) The tyrannosaur eats scientists (10,000 calories each) at a Poisson rate of 1 per day.
(iii) The tyrannosaur eats only scientists.
(iv) The tyrannosaur can store calories without limit until needed.

2.21 (3, 11/01, Q.10) (2.5 points)
Calculate the probability that the tyrannosaur dies within the next 2.5 days.
(A) 0.30 (B) 0.40 (C) 0.50 (D) 0.60 (E) 0.70

2.22 (3, 11/01, Q.11) (2.5 points)
Calculate the expected calories eaten in the next 2.5 days.
(A) 17,800 (B) 18,800 (C) 19,800 (D) 20,800 (E) 21,800

2.23 (CAS3, 11/03, Q.32) (2.5 points) Ross, in *Introduction to Probability Models*, identifies four requirements that a counting process N(t) must satisfy.
Which of the following is NOT one of them?
A. N(t) must be greater than or equal to zero.
B. N(t) must be an integer.
C. If s<t, then N(s) must be less than or equal to N(t).
D. The number of events that occur in disjoint time intervals must be independent.
E. For s<t, N(t)-N(s) must equal the number of events that have occurred in the interval (s,t].

2.24 (SOA3, 11/03, Q.26) (2.5 points) A member of a high school math team is practicing for a contest. Her advisor has given her three practice problems: #1, #2, and #3.
She randomly chooses one of the problems, and works on it until she solves it. Then she randomly chooses one of the remaining unsolved problems, and works on it until solved.
Then she works on the last unsolved problem.
She solves problems at a Poisson rate of 1 problem per 5 minutes.
Calculate the probability that she has solved problem #3 within 10 minutes of starting the problems.
(A) 0.18 (B) 0.34 (C) 0.45 (D) 0.51 (E) 0.59

2.25 (CAS3, 11/04, Q.18) (2.5 points) Justin takes the train to work each day.
It takes 10 minutes for Justin to walk from home to the train station.
In order to get to work on time, Justin must board the train by 7:50 a.m.
Trains arrive at the station at a Poisson rate of 1 every 8 minutes.
What is the latest time he must leave home each morning so that he is on time for work at least 90% of the time?

A. 7:21 a.m. B. 7:22 a.m. C. 7:31 a.m. D. 7:32 a.m. E. 7:41 a.m.

2.26 (CAS3, 11/04, Q.19) (2.5 points) XYZ Insurance introduces a new policy and starts a sales contest for 1,000 of its agents. Each agent makes a sale of the new product at a Poisson rate of 1 per week. Once an agent has made 4 sales, he gets paid a bonus of $1,000. The contest ends after three weeks.
Assuming 0% interest, what is the expected cost of the contest?

A. $18,988 B. $57,681 C. $168,031 D. $184,737 E. $352,768

2.27 (CAS3, 5/05, Q.39) (2.5 points) Longterm Insurance Company insures 100,000 drivers who have each been driving for at least five years.
Each driver gets "violations" at a Poisson rate of 0.5/year.
Currently, drivers with 1 or more violations in the past three years pay a premium of 1000.
Drivers with 0 violations in the past three years pay 850.
Your marketing department wants to change the pricing so that drivers with 2 or more accidents in the past five years pay 1,000 and drivers with zero or one violations in the past five years pay X.
Find X so that the total premium revenue for your firm remains constant when this change is made.

A. Less than 900
B. At least 900, but less than 925
C. At least 925, but less than 950
D. At least 950, but less than 975
E. 975 or more

2.28 (CAS3, 11/05, Q.28) (2.5 points) Big National Bank has 3 teller windows open for customer service. Each teller services customers at a Poisson rate of 6 customers per hour.
There is a single line to wait for the next available teller and all tellers are currently serving customers.
If there are 2 people in line when the next customer arrives, calculate the probability that he must wait more than 10 minutes for the next available teller.

A Less than 30%
B. At least 30%, but less than 40%
C. At least 40%, but less than 50%
D. At least 50%, but less than 60%
E. At least 60%

2.29 (CAS3, 11/06, Q.26) (2.5 points) Which of the following is/are true?

1. A counting process is said to possess independent increments if the number of events that occur between time s and t is independent of the number of events that occur between time s and t+u for all $u > 0$.
2. All Poisson processes have stationary and independent increments.
3. The assumption of stationary and independent increments is essentially equivalent to asserting that at any point in time the process probabilistically restarts itself.

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. 2 and 3 only

Section 3, Interarrival Times, Poisson Processes

Exercise: What is the probability that the waiting time until the first claim is less than or equal to 10 for a Poisson Process with $\lambda = .03$?
[Solution: The number of claims by time 10 is Poisson with mean: (10)(.03) = 0.3.
Waiting time is $\leq 10 \Leftrightarrow$ At least one claim by t = 10. Prob[0 claims by time 10] = $e^{-0.3}$.
Prob[1 or more claims by time 10] = 1 - $e^{-0.3}$ = 25.9%.]

In this case, the distribution function of the waiting time until the first claim is:
$F(t) = 1 - e^{-.03t}$, an Exponential Distribution with mean 1/.03. In general, for a Poisson Process with claims intensity λ, the **waiting time until the first claim has an Exponential Distribution with mean 1/λ.** $F(t) = 1 - e^{-\lambda t}$.[21]

Interarrival Times:

The **interarrival times** are the times between claims. T_1 is the waiting time until the first claim. T_2 is the time from the first claim until the second claim. T_3 is the time from the second claim until the third claim, etc.

Exercise: What is the probability that the waiting time from the third claim to the fourth claim is less than or equal to 10 for a Poisson Process with $\lambda = .03$?
[Solution: Interarrival time from 3rd to 4th claim $\leq 10 \Leftrightarrow$ # claims ≥ 1 in interval of length 10.
The number of claims over a time interval of length 10 is Poisson with mean: (10)(.03) = 0.3.
Prob[0 claims] = $e^{-0.3}$. Prob[1 or more claims] = 1 - $e^{-0.3}$ = 25.9%.]

In general, the interarrival time between claim n and n+1 has an Exponential Distribution with mean 1/λ. Due to the constant, independent claims intensity, we can start a new Poisson Process, with claims intensity λ, when the nth claim occurs; therefore, the wait until the next claim is Exponential with mean 1/λ. This interarrival time is independent of what happened before. For a Poisson Process with claims intensity λ, the **interarrival times are independent Exponential Distributions each with mean 1/λ.**[22]

For example, assume we have a Poisson Process with $\lambda = .03$, with an unrestricted time horizon. Then the waiting time to the first claim is Exponential, with $\theta = 1/.03 = 33.33$.

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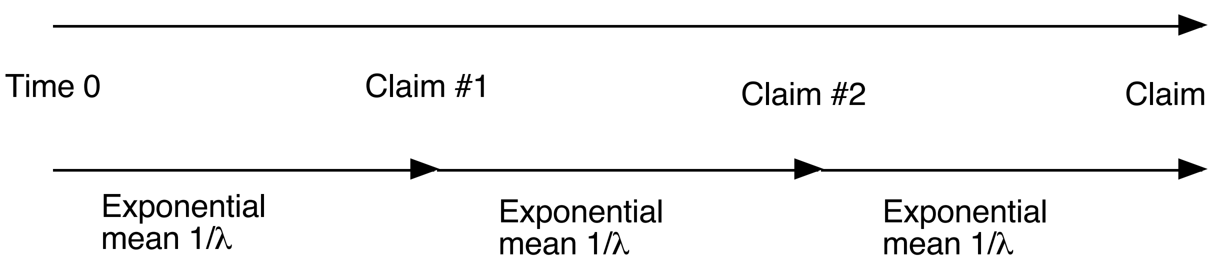
[21] The survival function is $\exp[-\lambda t]$, the same as that for a constant force of mortality λ. As long as we wait for the first claim, the mathematics is the same as the time until death for a constant force of mortality λ.
[22] Conversely, if the interarrival times are independent, identically distributed Exponentials with mean θ, then the counting process is Poisson with hazard rate $1/\theta$.

The second interarrival time is an independent Exponential, with θ = 33.33.
Each interarrival time is an independent Exponential, with θ = 33.33.
The interarrival times for a Poisson Distribution are those of the corresponding Poisson Process. If claims follow a Poisson Distribution with mean annual frequency of .02, then each interarrival time is an independent Exponential, with θ = 1/.02 = 50.

Waiting Times:

The Poisson Process is a random process. Sometimes you have to wait a long time for the next claim, and sometimes the next claim shows up right away.

Waiting Time until the third claim is a Gamma Distribution: α = 3, θ = 1/λ.



The interarrival times are independent, identically distributed Exponential Distributions with mean 1/λ. The waiting time until the nth claim is the sum on n independent Exponential Variables, each with mean 1/λ. Therefore, **the waiting time until the nth claim follows a Gamma Distribution with parameters α = n and θ = 1/λ.**[23]

The mean time until the nth claim is n/λ. Note that if we restrict the Poisson Process to (0,T), then there may be fewer than n claims; in other words the time until the nth claim could be greater than T.

For example, assume we have a Poisson Process with λ = .03, with an unrestricted time horizon. The waiting time until the 4th claim is given by a Gamma Distribution, as per Loss Models, with parameters α = 4 and θ = 33.33. The mean waiting time until the 4th claim is the mean of the Gamma Distribution, $\alpha\theta$ = (4)(33.33) = 133.33.

Exercise: Assume we have a Poisson Process with λ = .03. What is the probability that we have observed at least 4 claims by time 100?

[23] Gamma Distribution as per Appendix A of Loss Models.

[Solution: The waiting time until the fourth claim is given by a Gamma Distribution with $\alpha = 4$ and $\theta = 33.33$. Thus F(100) = Γ[4; 100/33.33] = Γ[4; 3] = 0.352768, by use of a computer.
Alternately, the number of claims in the time period from 0 to 100 is Poisson Distributed with mean (.03)(100) = 3. Therefore, the chance of zero claims in this interval is e^{-3}. The chance of one claim in this time interval is $3e^{-3}$. The chance of two claims in this time interval is $3^2e^{-3}/2$. The chance of three claims in this time interval is $3^3e^{-3}/6$. Thus the chance of at least 4 claims in the time interval is:
$1 - \{e^{-3} + 3e^{-3} + 3^2e^{-3}/2 + 3^3e^{-3}/6\} = 1 - .647232 = 0.352768$.]

In general, assume claims are given by a Poisson Process with claims intensity λ. Then the claims in the interval from (0, t) are Poisson Distributed with mean $t\lambda$. One can calculate the chance that there are least n claims in two different ways. First, the chance of at least n claims is a sum of Poisson densities:

$$1 - F(n-1) = 1 - \sum_{i=0}^{n-1} e^{-\lambda t} \lambda t^i / i! = \sum_{i=n}^{\infty} e^{-\lambda t} \lambda t^i / i!.$$

On the other hand, the waiting time until the nth claim is a Gamma Distribution with $\alpha = n$ and $\theta = 1/\lambda$. Thus the waiting time until the nth claim has distribution $\Gamma[n ; \lambda t]$.

A Formula for the Incomplete Gamma Function;

Comparing the two results when t = 1, the Incomplete Gamma Function with integer shape parameter can be written in terms of a sum of Poisson densities:[24]

$$\Gamma[n ; \lambda] = 1 - \sum_{i=0}^{n-1} e^{-\lambda} \lambda^i / i! = \sum_{i=n}^{\infty} e^{-\lambda} \lambda^i / i!.$$

For example, $\Gamma[3; 3.5] = 1 - \{e^{-3.5} + 3.5e^{-3.5} + 3.5^2e^{-3.5}/2\} = 1 - .320847 = .679153$.

We have also established a formula for the Distribution Function of a Poisson with mean λ, in terms of the Incomplete Gamma Function:

$$F(x) = \sum_{i=0}^{x} e^{-\lambda} \lambda^i / i! = 1 - \Gamma[x+1 ; \lambda].$$

[24] See Theorem A.1 in Appendix A of Loss Models.

Since the Chi-Square Distribution is a special case of the Gamma Distribution, one can use a Chi-Square Table in order to estimate the Distribution Function of a Poisson. A Chi-Square Distribution with ν degrees of freedom is a Gamma Distribution, as per Loss Models, with parameters $\alpha = \nu/2$ and $\theta = 2$.

Exercise: Use the following Chi-Square Table in order to estimate F(5) for a Poisson with mean 10.5.

Degrees	*Significance Levels*				
of Freedom	*0.100*	*0.050*	*0.025*	*0.010*	*0.005*
12	*18.55*	*21.03*	*23.34*	*26.22*	*28.30*

[Solution: $F(5) = 1 - \Gamma(5+1; 10.5) = 1 - \Gamma(6; 21/2) =$

1 - Chi-Square Distribution for 12 degrees of freedom at 21 $\cong$ .05.]

In general, for a Poisson with mean λ, F(x) = 1 - Chi-Square Distribution with (2x+2) degrees of freedom at 2λ = significance level of 2λ for a Chi-Square with (2x+2) degrees of freedom.

Problems:

3.1 (1 point) A Poisson Process has a claims intensity of 0.05.
What is the mean time until the first claim?
A. 5 B. 10 C. 15 D. 20 E. 25

3.2 (1 point) A Poisson Process has a claims intensity of 0.05.
What is the mean time until the tenth claim?
A. 50 B. 100 C. 200 D. 300 E. 400

3.3 (1 point) A Poisson Process has a claims intensity of 0.05.
What is the probability that the time until the first claim is greater than 35?
A. Less than 14%
B. At least 14%, but less than 15%
C. At least 15%, but less than 16%
D. At least 16%, but less than 17%
E. At least 17%

3.4 (1 point) A Poisson Process has a claims intensity of 0.05.
What is the probability that the time from the fifth claim to the sixth claim is less than 10?
A. Less than 36%
B. At least 36%, but less than 38%
C. At least 38%, but less than 40%
D. At least 40%, but less than 42%
E. At least 42%

3.5 (2 points) A Poisson Process has a claims intensity of 0.05.
What is the probability that the time from the eighth claim to the tenth claim is greater than 50?
A. Less than 28%
B. At least 28%, but less than 30%
C. At least 30%, but less than 32%
D. At least 32%, but less than 34%
E. At least 34%

<u>3.6</u> (2 points) For a claim number process you are given that the elapsed times between successive claims are mutually independent and identically distributed with distribution function:

$F(t) = 1 - e^{-t/2}$, $t \geq 0$.

Determine the probability of exactly 3 claims in an interval of length 7.

A. Less than 23%

B. At least 23%, but less than 25%

C. At least 25%, but less than 27%

D. At least 27%, but less than 29%

E. At least 29%

3.7 (2 points) Claims follow a Poisson Process. The average time between claims is 5.

What is the probability that we have observed at least 2 claims by time 9?

A. Less than 53%

B. At least 53%, but less than 55%

C. At least 55%, but less than 57%

D. At least 57%, but less than 59%

E. At least 59%

3.8 (2 points) Claims follow a Poisson Process. The average time between claims is 5.

What is the probability that we have observed exactly 2 claims by time 9?

A. Less than 23%

B. At least 23%, but less than 25%

C. At least 25%, but less than 27%

D. At least 27%, but less than 29%

E. At least 29%

3.9 (2 points) Use the following information:

- Customers arrive at a subway token booth in accordance with a Poisson process with mean 4 per minute.
- There is one clerk and his service time is exponentially distributed with a mean of 10 seconds.

When Joe arrives at the token booth, no other customers are there.

What is the probability that Joe is done being served before another customer arrives?

(A) 40% (B) 45% (C) 50% (D) 55% (E) 60%

Use the following information for the next three questions:

- You and your friend George go together to the subway platform.
- George is waiting for the uptown train and you are waiting for the downtown train.
- Uptown trains arrive via a Poisson Process at a rate of 5 per hour.
- Downtown trains arrive via a Poisson Process at a rate of 5 per hour.
- The arrival of downtown and uptown trains are independent.

3.10 (2 points) What is the average time until the first one of you catches his train?
(A) 4 minutes (B) 6 minutes (C) 8 minutes (D) 10 minutes (E) 12 minutes

3.11 (2 points) What is the average time until the last one of you catches his train?
(A) 15 minutes (B) 18 minutes (C) 21 minutes (D) 24 minutes (E) 27 minutes

3.12 (2 points) What is the average time you wait on the platform without George?
(A) 6 minutes (B) 8 minutes (C) 10 minutes (D) 12 minutes (E) 14 minutes

3.13 (1 point) Claims occur via a homogeneous Poisson Process. The expected waiting time until the first claim is 770 hours. If the claims intensity had been 5 times as large, what would have been the expected waiting time until the first claim?
A. 154 hours B. 765 hours C. 770 hours D. 3850 hours E. None of the above.

3.14 (2 points) Assume a driver's claim frequency is given by a Poisson distribution, with an average annual claim frequency of 8%.
What is the probability that it will be more than 25 years until this driver's first claim?
A. less than 10%
B. at least 10% but less than 11%
C. at least 11% but less than 12%
D. at least 12% but less than 13%
E. at least 13%

3.15 (2 points) For a Poisson Process with $\lambda = 5$, what is the probability that we have observed at least 3 claims by time 1.2?
A. Less than 93%
B. At least 93%, but less than 95%
C. At least 95%, but less than 97%
D. At least 97%, but less than 99%
E. At least 99%

3.16 (3 points) Use the following information:

- In the late afternoon, customers arrive at a bagel shop via a Poisson Process at a rate of 20 per hour.
- Each customer first has his order filled and then pays for that order.
- The time to fill an order is exponentially distributed with a mean of 90 seconds.
- The time to pay for an order is exponentially distributed with a mean of 10 seconds.
- The times to fill an order and to pay for that order are independent.

When Mary arrives at the bagel shop, no other customers are there.
What is the probability that Mary is done getting her order and paying for it before another customer arrives?
(A) 63% (B) 65% (C) 67% (D) 69% (E) 71%

<u>3.17</u> (2 points) Buses arrive via a Poisson Process with λ = 1/6 hour. Sandy arrives at 9:02. Sandy is still waiting for a bus at 9:05.
What is the probability that the next bus arrives by 9:10?
A. 35% B. 40% C. 45% D. 50% E. 55%

3.18 (1 point) Debbie receives phone calls at work via a Poisson Process at a rate of 5 per hour. Right after finishing a phone call, Debbie leaves her desk for 10 minutes in order to get a snack. Determine the probability that Debbie received at least one call while away from her desk.
(A) 57% (B) 59% (C) 61% (D) 63% (E) 65%

3.19 (2 points) One has a counting process N(t) with stationary and independent increments.
$N(0) = 0$. $\text{Prob}[N(x) = 1] = 8x + o(x)$. $\text{Prob}[N(x) \geq 2] = o(x)$.
What is the variance of the time between the fourth and fifth events?
A. 1/64 B. 1/8 C. 1/4 D. 1/2 E. 1

Use the following information for the next two questions:
Bonnie and Clyde arrive at the bus terminal and each wait for their bus to leave.
Bonnie is going to Brighton and Clyde is going to Chelsea.
Brighton buses leave the terminal via a Poisson Process at a rate of 5 per hour.
Chelsea buses leave the terminal via a Poisson Process at a rate of 15 per hour.
Brighton buses and Chelsea buses are independent.

3.20 (2 points) On average how many Chelsea buses leave while Bonnie is at the terminal?
A. 1 B. 2 C. 3 D. 4 E. 5

3.21 (3 points) Determine the average time Bonnie waits if Clyde's bus leaves first.
A. 12 minutes B. 13 minutes C. 14 minutes D. 15 minutes E. 16 minutes

3.22 (2 points) Lucky Tom finds coins on his way to work at a Poisson rate of 0.5 coins/minute. What is the probability that the first two coins Lucky Tom finds during his one-hour walk today are found within one minute of each other?

A. 30% B. 35% C. 40% D. 45% E. 50%

3.23 (2 points) Trains arrive via a Poisson Process with $\lambda = 1/10$. Conditional on the first train arriving before time 15, what is the expected waiting time until the first train?

A. 5.7 B. 5.9 C. 6.1 D. 6.3 E. 6.5

Use the following information for the next two questions:

- Cautious Clarence wants to cross a road.
- Vehicles pass the spot where Clarence is waiting via a Poisson Process at a rate of one every 4 seconds.
- Clarence will wait until he can see that no vehicle will come by in the next 10 seconds.

3.24 (1 point) What is the probability that Clarence does not have to wait?

A. 6% B. 8% C. 10% D. 12% E. 14%

3.25 *(3 points) Calculate Clarence's average waiting time.*

A. 20 B. 25 C. 30 D. 35 E. 40

Use the following information for the next two questions:

- Mistakes in cell division occur via a Poisson Process with $\lambda = 2$ per year.
- An individual dies when 150 such mistakes have occurred.

3.26 (2 points) What is the variance of the lifetime of an individual?

A. Less than 30

B. At least 30, but less than 35

C. At least 35, but less than 40

D. At least 40, but less than 45

E. At least 45

3.27 (2 points) Using the Normal Approximation, estimate the probability that an individual survives to age 85.

A. 3% B. 5% C. 7% D. 9% E. 11%

3.28 (3 points) A bus leaves at 10 minute intervals.
Passengers arrive via a Poisson Process with λ = 3 per minute.
Let X be the total time in minutes spent waiting by all of the passengers who board a single bus.
Determine the variance of X.
A. 250 B. 500 C. 750 D. 1000 E. 1250

3.29 (2, 5/90, Q.43) (1.7 points) Customers arrive randomly and independently at a service window, and the time between arrivals has an exponential distribution with a mean of 12 minutes.
Let X equal the number of arrivals per hour. What is P[X = 10]?
A. $10e^{-12}/10!$ B. $10^{12}e^{-10}/10!$ C. $12^{10}e^{-10}/10!$ D. $12^{10}e^{-12}/10!$ E. $5^{10}e^{-5}/10!$

3.30 (Course 151 Sample Exam #2, Q.7) (0.8 points)
For a claim number process $\{N(t), t \geq 0\}$ you are given that the elapsed times between successive claims are mutually independent and identically distributed with distribution function

$F(t) = 1 - e^{-3t}, t \geq 0.$

Determine the probability of exactly 4 claims in an interval of length 2.
(A) 0.11 (B) 0.13 (C) 0.15 (D) 0.17 (E) 0.19

3.31 (1, 11/00, Q.34) (1.9 points) The number of days that elapse between the beginning of a calendar year and the moment a high-risk driver is involved in an accident is exponentially distributed. An insurance company expects that 30% of high-risk drivers will be involved in an accident during the first 50 days of a calendar year.
What portion of high-risk drivers are expected to be involved in an accident during the first 80 days of a calendar year?
(A) 0.15 (B) 0.34 (C) 0.43 (D) 0.57 (E) 0.66

3.32 (IOA 101, 4/01, Q.3) (2.25 points) Suppose that the occurrence of events which give rise to claims in a portfolio of automobile insurance policies can be modeled as follows: the events occur through time at random, at rate μ per hour. Then the number of events which occur in a given period of time has a Poisson distribution (you are given this).
Show that the time between two consecutive events occurring has an exponential distribution with mean $1/\mu$ hours.

3.33 (1, 5/01, Q.22) (1.9 points) The waiting time for the first claim from a good driver and the waiting time for the first claim from a bad driver are independent and follow exponential distributions with means 6 years and 3 years, respectively.
What is the probability that the first claim from a good driver will be filed within 3 years and the first claim from a bad driver will be filed within 2 years?

(A) $(1 - e^{-2/3} - e^{-1/2} + e^{-7/6})/18$

(B) $e^{-7/6}/18$

(C) $1 - e^{-2/3} - e^{-1/2} + e^{-7/6}$

(D) $1 - e^{-2/3} - e^{-1/2} + e^{-1/3}$

(E) $1 - e^{-2/3}/3 - e^{-1/2}/6 + e^{-7/6}/18$

3.34 (IOA 101, 9/01, Q.6) (3.75 points)
(i) (1.5 points) The occurrence of claims in a group of 200 policies is modeled such that the probability of a claim arising in the next year is 0.015 independently for each policy.
Each policy can give rise to a maximum of one claim.
Calculate an approximate value for the probability that more than 10 claims arise from this group of policies in the next year by approximating via a Poisson.
Leave your answer in terms of an Incomplete Gamma Function.
(ii) (2.25 points) The occurrence of claims in a group of 2000 policies is modeled such that the probability of a claim arising in the next year is 0.015 independently for each policy.
Each policy can give rise to a maximum of one claim.
Using the Normal Approximation, calculate an approximate value for the probability that more than 40 claims arise from this group of policies in the next year.

3.35 (CAS3, 11/06, Q.27) (2.5 points)
A customer service operator accepts calls continuously throughout the work day.
The length of each call is exponentially distributed with an average of 3 minutes.
Calculate the probability that at least one call will be completed in the next 2 minutes.
A. Less than 0.50
B. At least 0.50, but less than 0.55
C. At least 0.55, but less than 0.60
D. At least 0.60, but less than 0.65
E. At least 0.65

3.36 (SOA M, 11/06, Q.8) (2.5 points)
The time elapsed between claims processed is modeled such that V_k represents the time elapsed between processing the k-1th and kth claim.
(V_1 = time until the first claim is processed).
You are given:
(i) V_1, V_2,... are mutually independent.
(ii) The pdf of each V_k is $f(t) = 0.2\ e^{-0.2t}$, $t > 0$, where t is measured in minutes.
Calculate the probability of at least two claims being processed in a ten minute period.
(A) 0.2 (B) 0.3 (C) 0.4 (D) 0.5 (E) 0.6

Section 4, Thinning and Adding Poisson Processes

One can thin or add Poisson Processes, just as with Poisson frequency distributions.[25]

Thinning a Poisson Process:

If we select at random a fraction of the claims from a Poisson Process, we get a new Poisson Process, with smaller claims intensity. This is called **thinning a Poisson Process**.

For example, assume we have a Poisson Process on (0,5) with $\lambda = .03$. If one accepts at random[26] 1/3 of the claims from this first Poisson Process, then one has a new Poisson Process, with $\lambda = .03/3 = .01$. Also, the remaining 2/3rds of the original claims are also a Poisson Process, with $\lambda = .03(2/3) = .02$. Also these two Poisson Processes are independent.[27]

If claims are from a Poisson Process, and one divides these claims into subsets in a manner independent of the frequency process, then the claims in each subset are <u>independent</u> Poisson Processes.

Exercise: Assume claims are given by a Poisson Process with claims intensity $\lambda = 10$.
Assume frequency and severity are independent. 30% of claims are of size less than $10,000, 50% of claims are of size between $10,000 and $25,000, and 20% of the claims are of size greater than $25,000. What are the frequency processes for the claims of different sizes?
[Solution: There are three independent Poisson Processes. Claims of a size less than $10,000, have a claims intensity of 3. Claims of size between $10,000 and $25,000 have a claims intensity of 5. Claims of size greater than $25,000 have a claims intensity of 2.]

Exercise: In the prior exercise, what is the probability that by time 0.7 we will have had at least 2 claims of size less than $10,000?

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[25] See "Mahler's Guide to Frequency Distributions."
[26] The key thing is that the selection process can not depend in any way on the frequency process. For example, if severity is independent of frequency, we may select only the large losses. As another example, we may select only those claims from insureds with middle initial C.
[27] The two (or more) smaller Poisson Processes that result from thinning can be added up to recover the original Poisson Process. This situation bears some similarity to that involving Multiple Decrements (see <u>Actuarial Mathematics</u>.) Here after thinning we have multiple independent claims intensities.

[Solution: Since claims of a size less than $10,000 are given by a Poisson Process with a claims intensity of 3, the number of such claims in the time period from 0 to .7 is Poisson Distributed with mean (3)(.7) = 2.1. Therefore, the chance of zero such claims in this interval is $e^{-2.1}$. The chance of one such claims in this time interval is $2.1e^{-2.1}$. Thus the chance of at least 2 claims in the time interval is: $1 - \{e^{-2.1} + 2.1e^{-2.1}\} = 1 - .379615 = 0.620385$.
Alternately, since claims of a size less than $10,000, are given by a Poisson Process with a claims intensity of 3, the waiting time until the second claim of a size less than $10,000 is Gamma Distributed with $\alpha = 2$ and $\theta = 1/3$. Thus the chance that the waiting time until the 2nd such claim is less than or equal to than .7 is : $F(.7) = \Gamma[2; .7/(1/3)] = \Gamma[2; 2.1]$.
Comment: $\Gamma[2; 2.1] = 0.620385$. We have calculated this value of the incomplete Gamma Function for integer shape parameter as per Theorem A.1 in Appendix A of Loss Models.]

Exercise: In the prior exercise, if by time 0.7 we see 3 claims of size greater than $25,000, what is the probability that by time 0.7 we will have had at least 2 claims of size less than $10,000?
[Solution: These are independent processes, so the number of large claims gives us no information about the number of small claims. Therefore the solution is the same as that of the previous exercise: 0.620385.]

Exercise: Assume claims are given by a Poisson Process with claims intensity $\lambda = 10$.
Assume frequency and severity are independent. 30% of claims are of size less than $10,000, 50% of claims are of size between $10,000 and $25,000, and 20% of the claims are of size greater than $25,000. What is the probability that by time 0.7 we will have had at least 2 claims of size less than $10,000, at least 3 claims of size between $10,000 and $25,000, and at least 1 claim of size greater than $25,000?
[Solution: There are three independent Poisson Processes. Claims of a size less than $10,000, have a claims intensity of 3. Claims of size between $10,000 and $25,000 have a claims intensity of 5. Claims of size greater than $25,000 have a claims intensity of 2. The chance of at least 2 small claims in the time interval is: 0.620385, from a previous solution. The number of medium claims is Poisson with mean (.7)(5) = 3.5. The chance of at least 3 medium claims in the time interval is:
$1 - \{e^{-3.5} + 3.5e^{-3.5} + 3.5^2e^{-3.5}/2\} = 0.679153$. The waiting time until the first claim of size greater than $25,000 is Exponentially Distributed with $\theta = 1/2$. Thus $F(.7) = 1 - e^{-.7/(1/2)} = .753403$.
Since the three Poisson Processes are independent, the probability that by time .7 we will have had at least 2 claims of size < $10,000, at least 3 claims of size between $10,000 and $25,000, and at least 1 claim of size > $25,000 is: (.753403)(0.679153)(0.620385) = .317436.]

Adding Poisson Processes:

If one adds two independent Poisson Processes, one gets a new Poisson Process, with claims intensity the sum of the two individual claims intensities. This is the case, because when we add two independent claims intensities, each of which is constant with independent increments, so is their sum.

Exercise: Claims from illness are a Poisson Process with claims intensity 13.
Claims from accident are a Poisson Process with claims intensity 7.
The two processes are independent.
What is the probability of at least 3 claims by time 0.1?
[Solution: Claims are a Poisson Process with $\lambda = 13 + 7 = 20$. thus the number of claims by time 0.1 is Poisson with mean 2. The probability of at least 3 claims is:
$1 - \{e^{-2} + 2e^{-2} + 2^2e^{-2}/2\} = .323$.]

Problems:

4.1 (3 points) Claims are reported according to a Poisson process with intensity 25 per month. The number of claims reported and the claim amounts are independently distributed. Claim amounts are distributed via a Weibull Distribution, $F(x) = 1 - \exp[-(x/\theta)^\tau]$, $x > 0$, with $\tau = 0.7$ and $\theta = 1000$. Calculate the number of complete months of data that must be gathered to have at least a 99.5% chance of observing at least 3 claims each exceeding 3000.
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Use the following information for the next three questions:
As he walks, Clumsy Klem loses coins at a Poisson rate of 0.2 coins/minute.
The denominations are randomly distributed:

(i) 50% of the coins are worth 5;
(ii) 30% of the coins are worth 10; and
(iii) 20% of the coins are worth 25.

4.2 (2 points) Calculate the conditional expected value of the coins Klem loses during his one-hour walk today, given that among the coins he lost exactly three were worth 5 each.
(A) 103 (B) 105 (C) 107 (D) 109 (E) 111

4.3 (2 points) Calculate the conditional variance of the value of the coins Klem loses during his one-hour walk today, given that among the coins he lost exactly three were worth 5 each.
(A) 1800 (B) 1850 (C) 1900 (D) 1950 (E) 2000

4.4 (2 points) Klem goes on a four hour walk for charity. Determine the probability that Klem loses a coin worth 10 within seven minutes subsequent to the time of losing his first coin worth 5.
A. 30% B. 35% C. 40% D. 45% E. 50%

4.5 *(5 points) Hurricanes hitting the State of Windiana follow a Poisson Process, with* $\lambda = 82\%$ *on an annual basis. In 1998, the losses from such a hurricane are given by a Pareto Distribution with* $\alpha = 2.5$ *and* $\theta = 400$ *million.* $F(x) = 1 - (\theta/(\theta+x))^\alpha$, $x > 0$.
Inflation is 5% per year.
Frequency and severity are independent.
Starting in the year 2000, how many complete years must be observed in order to have at least a 90% chance of seeing at least one hurricane with more than \$250 million of loss hitting the State of Windiana?
A. 7 B. 8 C. 9 D. 10 E. 11

Use the following information for the next two questions:

- Number of claims follows a Poisson Process with intensity 5.
- Claim severity is independent of the number of claims and has the following probability density function: $f(x) = 3.5\ x^{-4.5}$, $x > 1$.

4.6 (2 points) What is the average time until the 20th claim of size greater than 3?
A. Less than 150
B. At least 150, but less than 60
C. At least 160, but less than 70
D. At least 170, but less than 80
E. At least 180

4.7 (2 points) What is the probability that the 20th claim of size greater than 3 has occurred by time 200?
Hint: Use the Normal Approximation.
A. Less than 50%
B. At least 50%, but less than 60%
C. At least 60%, but less than 70%
D. At least 70%, but less than 80%
E. At least 80%

4.8 (2 points) You are given the following:

- Number of customers follows a Poisson Process with intensity 0.0125 per minute.
- The amount that a single customer spends has a uniform distribution on [0, 5000].
- Number of customers and the amount each customer spends are independent.

Calculate the probability that it will take more than 1000 minutes for a single customer to spend more than 4000.
(A) 4% (B) 6% (C) 8% (D) 10% (E) 12%

4.9 (2 points) For Broward County, Florida, hurricane season is 24 weeks long. It is assumed that the time between hurricanes is exponentially distributed with a mean of 6 weeks. It is also assumed that 30% of all hurricanes will hit Broward County.
Calculate the probability that in any given hurricane season, there will be three hurricanes of which exactly one hits Broward County.
A. Less than 5%
B. At least 5%, but less than 6%
C. At least 6%, but less than 7%
D. At least 7%, but less than 8%
E. 8% or more

Use the following information for the next 5 questions:

- The number of families who immigrate to Vancougar from Honwan is given by a Poisson Process with intensity per <u>day</u> of 0.8.
- The sizes of these families are distributed as follows:

1	2	3	4	5	6	7	8	9	10
10%	15%	22%	20%	12%	8%	6%	4%	2%	1%

- The number of families and the sizes of the families are independent.

4.10 (1 point) What is the mean number of days until a family of size 5 immigrates?
A. 8 B. 10 C. 12 D. 14 E. 16

<u>4.11</u> (2 points) How many days must one observe to be 95% certain of observing a family of size 8 or more?
A. Less than 35
B. At least 35, but less than 40
C. At least 40, but less than 45
D. At least 45, but less than 50
E. At least 50

4.12 (1 point) Last year there were 66 families of size 4 who immigrated. Estimate the number of families of size 3 or less that immigrated last year.
A. Less than 135
B. At least 135, but less than 140
C. At least 140, but less than 145
D. At least 145, but less than 150
E. At least 150

4.13 (3 points) What is the probability that in the next 100 days at least one family of each size will immigrate?
A. Less than 30%
B. At least 30%, but less than 35%
C. At least 35%, but less than 40%
D. At least 40%, but less than 45%
E. At least 45%

4.14 (3 points) What is the probability that in the next 100 days there are at least three families of size 8, at least two families of size 9, and at least one family of size 10?
A. 12% B. 14% C. 16% D. 18% E. 20%

Use the following information for the next 6 questions:

- Theft losses follow a Poisson Process with $\lambda = 10$.
- The size of Theft losses follows an Exponential Distribution with mean 400.
- Fire losses follow a Poisson Process with $\lambda = 3$.
- The size of Fire losses follows a Pareto Distribution, $F(x) = 1 - (\theta/(\theta+x))^{\alpha}$, $x > 0$, with $\alpha = 3$ and $\theta = 4000$.
- Theft losses and Fire losses are independent of each other.
- An insurance policy that covers Fire losses and Theft losses has a 1000 deductible.

4.15 (1 point) What is the probability of exactly 8 <u>theft</u> losses by time 1?

A. 11% B. 13% C. 15% D. 17% E. 19%

4.16 (1 point) What is the probability of exactly 8 <u>fire</u> losses by time 1?

A. Less than 1%
B. At least 1%, but less than 2%
C. At least 2%, but less than 3%
D. At least 3%, but less than 4%
E. At least 4%

4.17 (1 point) What is the probability of exactly 8 losses by time 1?

A. Less than 1%
B. At least 1%, but less than 2%
C. At least 2%, but less than 3%
D. At least 3%, but less than 4%
E. At least 4%

4.18 (2 points) What is the probability of exactly 3 non-zero payments due to <u>theft</u> losses by time 1?

A. 0.5% B. 1% C. 2% D. 3% E. 4%

4.19 (2 points) What is the probability of exactly 3 non-zero payments due to <u>fire</u> losses by time 1?

A. 7% B. 9% C. 11% D. 13% E. 15%

<u>4.20</u> (2 points) What is the probability of exactly 3 non-zero payments by time 1?

A. 19% B. 21% C. 23% D. 25% E. 27%

Use the following information for the next 8 questions:

As he walks, Clumsy Klem loses coins at a Poisson rate of 0.2 coins/minute. The denominations are randomly distributed:

(i) 50% of the coins are worth 5;

(ii) 30% of the coins are worth 10; and

(iii) 20% of the coins are worth 25.

4.21 (2 points) Calculate the probability that the third coin of value 25 has been lost within the first 30 minutes.

(A) 0.12 (B) 0.14 (C) 0.16 (D) 0.18 (E) 0.20

4.22 (2 points) Calculate the probability that in the first ten minutes of his walk he loses 2 coins worth <u>10</u> each, and in the first twenty minutes loses 3 coins worth <u>5</u> each.

(A) 1.8% (B) 2.0% (C) 2.2% (D) 2.4% (E) 2.6%

4.23 (2 points) Calculate the probability that in the first ten minutes of his walk he loses 2 coins worth <u>10</u> each, and in the first fifteen minutes loses 3 coins worth <u>10</u> each.

(A) 1.8% (B) 2.0% (C) 2.2% (D) 2.4% (E) 2.6%

4.24 (2 points) Calculate the probability that in the first ten minutes of his walk he loses at least 2 coins worth <u>5</u> each, and in the next twenty minutes loses at least 4 coins worth <u>5</u> each.

(A) 0.03 (B) 0.04 (C) 0.05 (D) 0.06 (E) 0.07

<u>4.25</u> (3 points) Calculate the probability that in the first ten minutes of his walk he loses at least 2 coins worth <u>10</u> each, and in the first thirty minutes loses at least 4 coins worth <u>10</u> each.

(A) 0.05 (B) 0.06 (C) 0.07 (D) 0.08 (E) 0.09

4.26 (3 points) Calculate the probability that in the first ten minutes of his walk he loses at least 2 coins worth <u>5</u> each, and in the first 25 minutes loses at least 5 coins worth <u>5</u> each.

(A) 0.05 (B) 0.06 (C) 0.07 (D) 0.08 (E) 0.09

4.27 (2 points) Clumsy Klem takes a 30 minute walk. Let A be the number of coins lost during the first ten minutes of that walk. Let B be the number of coins lost during the last ten minutes of that walk. What is the probability that A + B = 5?

(A) 0.12 (B) 0.14 (C) 0.16 (D) 0.18 (E) 0.20

4.28 (3 points) Clumsy Klem takes a 30 minute walk. Let C be the number of coins lost during the first ten minutes of that walk. Let D be the number of coins lost during the first twenty minutes of that walk. What is the probability that C + D = 4?

(A) 0.12 (B) 0.14 (C) 0.16 (D) 0.18 (E) 0.20

Use the following information for the next 2 questions:
Taxicabs leave a hotel with a group of passengers at a Poisson rate $\lambda = 10$ per hour.
The number of people in each group taking a cab is independent and has the following probabilities:

Number of People	Probability
1	0.60
2	0.30
3	0.10

4.29 (2 points) If during an hour no taxicabs leave with 3 passengers each and 8 taxicabs leave with 1 passenger each, estimate how many total people leave via taxicabs during that hour.
(A) 12 or less (B) 13 (C) 14 (D) 15 (E) 16 or more

4.30 (2 points) What is the probability that during the next hour exactly 1 taxicab leaves with 3 passengers, exactly 3 taxicabs leave with 2 passengers each, and exactly 6 taxicabs leave with 1 passenger each?
(A) 0.7% (B) 1.0% (C) 1.3% (D) 1.6% (E) 1.9%

Use the following information for the next two questions:
Arthur the art dealer is trying to sell a valuable painting at his art gallery.
Unfortunately, Arthur is leaving on an extended buying trip to Europe and has to leave his inexperienced niece Cecilia in charge of his art gallery.
Arthur needs to leave Cecilia with very specific instructions that leave no room for judgment.
Offers for the painting will arrive at a Poisson rate of 2 per week.
The size of an offer is independent of the size of any other offer, and follows a Single Parameter Pareto Distribution with $\alpha = 3$ and $\theta = 10{,}000$, $F(x) = 1 - (\theta/x)^{\alpha}$, $x > \theta$.
Cecilia can either accept an offer or reject it and wait for the next offer.
Arthur will instruct Cecilia to accept the first offer greater than a certain amount x.

4.31 (2 points) If Arthur wants there to be only a 5% chance that the painting will be unsold after 10 weeks, what value of x should he select?
A. 15,000 B. 16,000 C. 17,000 D. 18,000 E. 19,000

4.32 (3 points) Arthur incurs 200 in expense each week the painting remains unsold.
If Arthur wants to maximize the expected value of the difference between the price for which the painting is sold and the expenses incurred, what value of x should he select?
A. 50,000 B. 60,000 C. 70,000 D. 80,000 E. 90,000

Use the following information for the next four questions:
Subway trains arrive at a station at a Poisson rate of 15 per hour.
60% of the trains are express and 40% are local.
The type of each train is independent of the types of preceding trains.
Ezra and Fiona work in the same office.
They are waiting at the same station.
An express gets them to work in 20 minutes and a local gets them there in 35 minutes.
Ezra always takes the first express to arrive, while Fiona always take the first train to arrive.
Let E be the time it takes Ezra to arrive at work.
Let F be the time it takes Fiona to arrive at work

4.33 (2 points) Calculate the probability that $F < E$.
(A) 1% (B) 2% (C) 3% (D) 4% (E) 5%

4.34 (2 points) Calculate the expected value of $F - E$, in minutes.
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

4.35 (2 points) Calculate the standard deviation of E, in minutes.
A. Less than 7.0
B. At least 7.0, but less than 7.5
C. At least 7.5, but less than 8.0
D. At least 8.0, but less than 8.5
E. At least 8.5

4.36 (2 points) Calculate the standard deviation of F, in minutes.
A. Less than 7.0
B. At least 7.0, but less than 7.5
C. At least 7.5, but less than 8.0
D. At least 8.0, but less than 8.5
E. At least 8.5

Use the following information for the next two questions:
Subway trains leave the Main Street station at a Poisson rate of 10 per hour.
2/3 of the trains are express and 1/3 are local.
The type of each train is independent of the types of preceding trains.
Ethel and Fred work in the same office.
An express gets them to work in 13 minutes and a local gets them there in 25 minutes.
Ethel always takes the first express to arrive, while Fred always take the first train to arrive.
Ethel arrives at the Main Street station at 7:48.
Fred arrives at the Main Street station at 7:51, and finds Ethel waiting.
Fred takes the local train that leaves at 7:55, leaving Ethel still waiting at the station.
Let E be the time Ethel arrives at work.
Let F be the time Fred arrives at work.

4.37 (2 points) Calculate the expected value of F - E, in minutes.
(A) 0 (B) 1 (C) 3 (D) 7 (E) 10

4.38 (2 points) Calculate the probability that $E < F$.
A. Less than 40%
B. At least 40%, but less than 50%
C. At least 50%, but less than 60%
D. At least 60%, but less than 70%
E. 70% or more

4.39 (4, 5/90, Q.42) (2 points) Suppose the amount of losses due to a single windstorm follows a Weibull distribution, $F(x) = 1 - \exp[-(x/\theta)^\tau]$, $x > 0$, with parameters θ = \$156.25 million and $\tau = 0.5$. Suppose also that the number of windstorms follows a Poisson distribution with constant parameter and that 12 windstorms have exceeded \$5 million (in 1990 dollars) in the past 30 years. If n is the expected number of years until the next windstorm that exceeds \$1 billion (in 1990 dollars), in what range does n fall?
A. $n < 22$ B. $22 \le n < 24$ C. $24 \le n < 26$ D. $26 \le n < 28$ E. $28 \le n$

4.40 (4B, 11/92,Q.30) (2 points) You are given the following:

- The size of loss distribution for damages from a single earthquake follows a Pareto distribution, $F(x) = 1 - (\theta/(\theta+x))^{\alpha}$, $x > 0$, with parameters $\alpha = 4$ and $\theta = \$150,000,000$ in constant 1992 dollars,
- The number of earthquakes in a single year has a Poisson distribution with constant mean.
- In the past 50 years, 10 earthquakes have occurred in which damages exceeded \$30 million in constant 1992 dollars.

Determine the expected number of years between earthquakes having damage in excess of \$100 million in 1992 dollars.

A. Less than 13
B. At least 13 but less than 15
C. At least 15 but less than 17
D. At least 17 but less than 19
E. At least 19

4.41 (4B, 11/95, Q.27) (2 points) You are given the following:

- The number of storms causing losses in excess of \$500,000 (in constant 1995 dollars) in a one-year period follows a Poisson distribution with mean 1.3.
- In constant 1995 dollars, the size of loss for each storm follows a Pareto distribution, $F(x) = 1 - (\theta/(\theta+x))^{\alpha}$, $x > 0$, with parameters $\theta = 7000$ and $\alpha = 1.05$.

Determine the expected number of years between storms causing losses in excess of \$5,000,000 (in constant 1995 dollars).

A. Less than 5.0
B. At least 5.0, but less than 7.5
C. At least 7.5, but less than 10.0
D. At least 10.0, but less than 12.5
E. At least 12.5

4.42 (4B, 5/96, Q.20 & Course 3 Sample Exam, Q.23) (2 points)
You are given the following:

- A loss occurrence in excess of $1 billion (in constant 1996 dollars) may be caused by a hurricane, an earthquake or a fire.
- Hurricanes, earthquakes. and fires occur independently of one another.
- The number of hurricanes causing a loss occurrence in excess of $1 billion (in constant 1996 dollars) in a one-year period follows a Poisson distribution. The expected amount of time between such hurricanes is 2.0 years.
- The number of earthquakes causing a loss occurrence in excess of $1 billion (in constant 1996 dollars) in a one-year period follows a Poisson distribution. The expected amount of time between such earthquakes is 5.0 years.
- The number of fires causing a loss occurrence in excess of $1 billion (in constant 1996 dollars) in a one-year period follows a Poisson distribution. The expected amount of time between such fires is 10.0 years.

Determine the expected amount of time between loss occurrences in excess of $1 billion (in constant 1996 dollars).
A. Less than 1.2 years
B. At least 1.2 years, but less than 1.4 years
C. At least 1.4 years, but less than 1.6 years
D. At least 1.6 years, but less than 1.8 years
E. At least 1.8 years

4.43 (3, 5/00, Q.2) (2.5 points) Lucky Tom finds coins on his way to work at a Poisson rate of 0.5 coins/minute. The denominations are randomly distributed:
(i) 60% of the coins are worth 1;
(ii) 20% of the coins are worth 5; and
(iii) 20% of the coins are worth 10.
Calculate the conditional expected value of the coins Tom found during his one-hour walk today, given that among the coins he found exactly ten were worth 5 each.
(A) 108 (B) 115 (C) 128 (D) 165 (E) 180

4.44 (3, 11/00, Q.23) (2.5 points) Workers' compensation claims are reported according to a Poisson process with mean 100 per month. The number of claims reported and the claim amounts are independently distributed. 2% of the claims exceed 30,000.
Calculate the number of complete months of data that must be gathered to have at least a 90% chance of observing at least 3 claims each exceeding 30,000.
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

4.45 (3, 11/00, Q.29) (2.5 points) Job offers for a college graduate arrive according to a Poisson process with mean 2 per month. A job offer is acceptable if the wages are at least 28,000. Wages offered are mutually independent and follow a lognormal distribution, $F(x) = \Phi[\{\ln(x) - \mu\}/\sigma]$, $x > 0$, with $\mu = 10.12$ and $\sigma = 0.12$. Calculate the probability that it will take a college graduate more than 3 months to receive an acceptable job offer.

(A) 0.27 (B) 0.39 (C) 0.45 (D) 0.58 (E) 0.61

4.46 (3, 11/02, Q.9) (2.5 points) Lucky Tom finds coins on his way to work at a Poisson rate of 0.5 coins/minute. The denominations are randomly distributed:

(i) 60% of the coins are worth 1 each

(ii) 20% of the coins are worth 5 each

(iii) 20% of the coins are worth 10 each.

Calculate the probability that in the first ten minutes of his walk he finds at least 2 coins worth 10 each, and in the first twenty minutes finds at least 3 coins worth 10 each.

(A) 0.08 (B) 0.12 (C) 0.16 (D) 0.20 (E) 0.24

4.47 (3, 11/02, Q.20) (2.5 points) Subway trains arrive at a station at a Poisson rate of 20 per hour. 25% of the trains are express and 75% are local. The types of each train are independent. An express gets you to work in 16 minutes and a local gets you there in 28 minutes. You always take the first train to arrive. Your co-worker always takes the first express. You both are waiting at the same station.

Which of the following is true?

(A) Your expected arrival time is 6 minutes earlier than your co-worker's.

(B) Your expected arrival time is 4.5 minutes earlier than your co-worker's.

(C) Your expected arrival times are the same.

(D) Your expected arrival time is 4.5 minutes later than your co-worker's.

(E) Your expected arrival time is 6 minutes later than your co-worker's.

4.48 (CAS3, 11/03, Q.31) (2.5 points) Vehicles arrive at the Bun-and-Run drive-thru at a Poisson rate of 20 per hour. On average, 30% of these vehicles are trucks.

Calculate the probability that at least 3 trucks arrive between noon and 1:00 PM.

A. Less than 0.80

B. At least 0.80, but less than 0.85

C. At least 0.85, but less than 0.90

D. At least 0.90, but less than 0.95

E. At least 0.95

4.49 (SOA3, 11/03, Q.11) (2.5 points) Subway trains arrive at a station at a Poisson rate of 20 per hour. 25% of the trains are express and 75% are local. The type of each train is independent of the types of preceding trains. An express gets you to the stop for work in 16 minutes and a local gets you there in 28 minutes. You always take the first train to arrive. Your co-worker always takes the first express. You both are waiting at the same station.
Calculate the probability that the train you take will arrive at the stop for work before the train your co-worker takes.
(A) 0.28 (B) 0.37 (C) 0.50 (D) 0.56 (E) 0.75

4.50 (CAS3, 5/04, Q.31) (2.5 points) Coins are tossed into a fountain according to a Poisson process with a rate of one every three minutes.
The coin denominations are independently distributed as follows:

Coin Denomination	Probability
Penny	0.5
Nickel	0.2
Dime	0.2
Quarter	0.1

Calculate the probability that the fourth dime is tossed into the fountain in the first two hours.
A. Less than 0.89
B. At least 0.89, but less than 0.92
C. At least 0.92, but less than 0.95
D. At least 0.95, but less than 0.98
E. At least 0.98

4.51 (CAS3, 11/04, Q.17) (2.5 points) You are given:
- Claims are reported at a Poisson rate of 5 per year.
- The probability that a claim will settle for less than $100,000 is 0.9.

What is the probability that no claim of $100,000 or more is reported during the next 3 years?
A. 20.59% B. 22.31% C. 59.06% D. 60.65% E. 74.08%

4.52 (CAS3, 5/05, Q.7) (2.5 points) An insurance company pays claims at a Poisson rate of 2,000 per year. Claims are divided into three categories: "minor," "major," and "severe," with payment amounts of $1,000, $5,000, and $10,000, respectively. The proportion of "minor" claims is 50%. The total expected claim payments per year is $7,000,000.
What proportion of claims are "severe"?
A. Less than 11%
B. At least 11%, but less than 12%
C. At least 12%, but less than 13%
D. At least 13%, but less than 14%
E. 14% or more

4.53 (CAS3, 5/05, Q.11) (2.5 points) For Broward County, Florida, hurricane season is 24 weeks long. It is assumed that the time between hurricanes is exponentially distributed with a mean of 6 weeks. It is also assumed that 30% of all hurricanes will hit Broward County.
Calculate the probability that in any given hurricane season, Broward County will be hit by more than 1 hurricane.
A. Less than 15%
B. At least 15%, but less than 20%
C. At least 20%, but less than 25%
D. At least 25%, but less than 30%
E. 30% or more

4.54 (CAS3, 5/05, Q.13) (2.5 points) During the hurricane season (August, September, October, and November), hurricanes hit the US coast with a monthly Poisson rate of 1.25 and each hurricane during the period has a 20% chance of being "major." Outside of hurricane season (the other months), hurricanes hit at a Poisson rate of 0.25 per month, and each such hurricane has only a 10% chance of being "major."
Determine the probability that a hurricane selected at random is "major."
A. Less than 14%
B. At least 14%, but less than 15%
C. At least 15%, but less than 16%
D. At least 16%, but less than 17%
E. 17% or more

4.55 (SOA M, 5/05, Q.5) (2.5 points)
Kings of Fredonia drink glasses of wine at a Poisson rate of 2 glasses per day.
Assassins attempt to poison the king’s wine glasses. There is a 0.01 probability that any given glass is poisoned. Drinking poisoned wine is always fatal instantly and is the only cause of death. The occurrences of poison in the glasses and the number of glasses drunk are independent events.
Calculate the probability that the current king survives at least 30 days.
(A) 0.40 (B) 0.45 (C) 0.50 (D) 0.55 (E) 0.60

4.56 (SOA M, 5/05, Q.24) (2.5 points)
Subway trains arrive at your station at a Poisson rate of 20 per hour.
25% of the trains are express and 75% are local.
The types and number of trains arriving are independent.
An express gets you to work in 16 minutes and a local gets you there in 28 minutes.
You always take the first train to arrive.
Your co-worker always takes the first express.
You are both waiting at the same station.
Calculate the conditional probability that you arrive at work before your co-worker, given that a local arrives first.
(A) 37% (B) 40% (C) 43% (D) 46% (E) 49%

4.57 (CAS3, 11/05, Q.29) (2.5 points) ABC Insurance Company estimates that the time between reported claims is exponentially distributed with mean 0.50 years. Times between claims are independent. Each time a claim is reported, a payment is made with probability 0.70. Calculate the probability that no payment will be made on claims reported during the next two years.
A. 0.06 B. 0.14 C. 0.25 D. 0.30 E. 0.50

4.58 (CAS3, 11/05, Q.31) (2.5 points) The Toronto Bay Leaves attempt shots in a hockey game according to a Poisson process with mean 30. Each shot is independent. For each attempted shot, the probability of scoring a goal is 0.10.
Calculate the standard deviation of the number of goals scored by the Bay Leaves in a game.
A. Less than 1.4
B. At least 1.4, but less than 1.6
C. At least 1.6, but less than 1.8
D. At least 1.8, but less than 2.0
E. At least 2.0

4.59 (SOA M, 11/06, Q.9) (2.5 points) A casino has a game that makes payouts at a Poisson rate of 5 per hour and the payout amounts are 1, 2, 3,… without limit.
The probability that any given payout is equal to i is $1/2^i$. Payouts are independent.
Calculate the probability that there are no payouts of 1, 2, or 3 in a given 20 minute period.
(A) 0.08 (B) 0.13 (C) 0.18 (D) 0.23 (E) 0.28