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Exam ALTAM Study Manual

4th Edition

Abraham Weishaus, Ph.D., FSA, CFA, MAAA

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Contents

Preface

Welcome to Exam ALTAM!

On Exam FAM, you learned the basics of life insurance mathematics; mortality models, insurances, annuities, premiums, and reserves. On ALTAM, we discuss more general long-term insurances. Sometimes insurance covers more than one life. Sometimes the benefit depends on how one died. And there are other long-term products to analyze, such as disability income and long term care. We discuss all of these products, and also discuss pricing and analysis of profit.

Syllabus

According to the syllabus, the exam is 3 hours and will consist of 60 points of written answer questions. The topics this exam will cover are:

1. Multistate models
2. Multiple decrement models
3. Multiple life models
4. Pensions
5. Profit tests for traditional products
6. Universal life
7. Variable annuities

The textbook for the course is *Actuarial Mathematics for Life Contingent Risks* third edition. This is a college-style textbook. It is oriented towards practical application rather than exam preparation. Almost all exercises require use of spreadsheets or derivation of formulas. The syllabus also includes a study note on variable annuities.

The syllabus splits the material into 7 topics, not quite the same as the list above, with the following weights:

Topic	Weight	Lessons in this manual
Survival Models for Contingent Cash Flows	10–20%	2–3, 5–6, 9–14
Premium and Policy Valuation for Long-Term State-Dependent Coverage	12–20%	4, 7–8, 15
Joint Life Insurance and Annuities	8–16%	17–24
Profit Analysis	10–20%	28–29
Pension Plans and Retirement Benefits	10–18%	26–27
Universal Life Insurance	10–18%	30
Embedded Options in Life Insurance and Annuity Products	10–18%	31–36

The exam will have 6 questions, each of them with 3 or more parts. One of the questions will be a worksheet question. The 6 questions are worth 60 points in total. Thus at least one of the topics will not appear, unless a single question spans two topics.

Here is the distribution of number of points for the released ALTAM exams:

Topic	Lessons	Spring 2023	Fall 2023	Spring 2024	Fall 2024
Alive-dead model	1–4	0	0	0	0
Markov chains	5–9	20	21	20	11
Multi-decrement models	10–16	0	0	0	8
Multiple life models	17–25	11	4	9	2
Retirement benefits	26–27	9	10	10	8
Profit tests, traditional products	28–29	0	8	10	11
Universal life	30	10	8	4.5	11
Variable annuities	31–36	10	9	6.5	9
Total		60	60	60	60

Notes on this table:

- The Spring 2024 exam's question 6 combined Universal Life and Variable Annuity into the first four parts of the question, so I split the point value of those four parts between the two topics. Since the number of points was odd, that resulted in assigning halves of points.
- Thiele's equation in a single-state setting has not been tested. Thiele's equation is part of Learning Outcome 2e, "Identify and apply Thiele's differential equation in a *single state* or multiple state setting." (emphasis mine), which is a part of "Multiple State Premium and Policy Valuation".
- There are typically *two* questions on Markov chains. This is consistent with the syllabus, which assigns 10–20% weight to Multiple State Survival Models and 12–20% weight to Multiple State Insurances and Annuities. Fall 2024 is not an exception to this rule, since multi-decrement models are part of the Markov chains topic in the syllabus.

Downloads from the SOA site

The syllabus is at:

`https://www.soa.org/4aec83/globalassets/assets/files/edu/2025/spring/syllabi/2025-spring-exam-altam-syllabus.pdf`

You will receive the life tables as a worksheet at the exam. It is at

`https://www.soa.org/globalassets/assets/files/edu/2023/2023-07-altam-excel-workbook-tables.xlsx`

You may use this worksheet to obtain normal functions if needed. For the five non-worksheet questions, you may also use the worksheet to do your calculations, as long as you explain what you did. This worksheet will have most of the functions of Excel, but will not have VBA or Solver. For the worksheet question, you must use this worksheet and write your answers on it.

You will get the following formula sheet:

`https://www.soa.org/globalassets/assets/files/edu/2023/2023-07-altam-formula-sheet.pdf`

Thus you need not memorize Woolhouse's formula or the Black-Scholes put option formula.

The notation and terminology study note is at

`https://www.soa.org/globalassets/assets/files/edu/2023/2023-07-altam-notation-term.pdf`

This manual follows the conventions of this note, but after you've finished the manual, you may want to read through this note.

Sample questions

The SOA's sample questions are at

https://www.soa.org/globalassets/assets/files/edu/2023/spring/2023-07-altam-sample-questions.pdf

and their solutions are at

https://www.soa.org/globalassets/assets/files/edu/2023/spring/2023-07-altam-sample-solutions.pdf

Their sample worksheet questions are at

https://www.soa.org/globalassets/assets/files/edu/2024/spring/questions/2024-spring-altam-sample-questions.xlsx

and their solutions are at

https://www.soa.org/globalassets/assets/files/edu/2024/spring/questions/2024-spring-altam-sample-solutions.xlsx

At the end of each lesson there is a list of relevant sample questions. Most sample questions cover several lessons; the reference to the sample question is at the end of the last lesson covered by the question.

Exercises and old exam questions in this manual

There are about 480 original exercises in the manual and about 250 old exam questions. Even though the exam is written answer, I have retained these short answer and multiple choice questions. You will get a lot of good practice doing these questions; they are not a waste of time. To a large extent, each multipart written answer question on the exam is a series of short answer questions related to each other.

Questions from old exams are marked xxx:yy, where xxx is the time the exam was given, with S for spring and F for fall followed by a 2-digit year, and yy is the question number. There was a period in the 1990s when the SOA, while it allowed use of its old exam questions, did not want people to reveal which exam they came from. As a result, I sometimes had study notes for old exams in this period and could not identify the exam they came from. In such a case, I mark the question aaa-bb-cc:yy, where aaa-bb-cc is the study note number and yy is the question number. Generally aaa is the exam number (like 150), and cc is the 2-digit year the study note was published.

The exam, however, may ask certain questions for which you will have to study separately because the exercises don't cover them:

1. Questions asking you to derive formulas. Most of the formula derivations are provided in this manual. Make sure you can reproduce them.

2. Questions asking you for a list, or for some other fact. Like "Give 6 reasons that employers offer pension plans" or "List the 6 ADLs." Much of this information is in the manual. Some of it is in chapter 1 of the textbook.

3. Questions asking you for reasons. Like "Explain why A is less than B", or "Without doing computation, state with explanation whether A is less than, equal to, or greater than B".

To help you prepare for worksheet questions on the exam, there are 30 worksheet exercises. These exercises are not quite as large as exam worksheet questions, and concentrate on the topic covered in the lesson whereas exam questions often integrate several topics. Still, they imitate the style of exam worksheet questions and give you practice. The worksheet question is usually the last question in each lesson. Even if you skip other questions, I recommend working out all the worksheet questions.

Table 1: 9.5 Week Study Schedule for Exam ALTAM

Subject	Lessons	Study Period	Hard/Long Lessons	Easy/Short Lessons
Alive-dead Model	1–4	0.5 weeks		2
Markov Chains	5–9	2 weeks	8	
Multiple Decrements	10–15	1 week	13	12
Multiple Lives	17–24	1.5 weeks		17,20
Pension	26–27	2 weeks		
Profit Tests—Traditional	28–29	1 week		29
Universal Life	30	0.5 weeks		
Variable Life	31–36	1.5 weeks		

Change in Syllabus for 2025

For 2025, the SOA removed Retiree Health Benefits from the syllabus. They have never asked a written answer question on this topic, and since ALTAM has no multiple choice questions, they have never asked a question on this topic since 2023. They added the following three topics:

1. Characteristics of the mortality curve.
2. Heterogeneity of mortality.
3. Estimating mortality.

The first two of these are sections of chapter 3 of the textbook that are not on FAM. They are short topics that almost surely will not generate calculation questions. The third topic used to be on Exam LTAM.

Study schedule

Although this manual is large, much of it is exercises and practice exams. You do not have to do every exercise; do enough to gain confidence with the material. With intense studying, you should be able to cover all the material in 3 months.

It is up to you to set up a study schedule. Different students will have different speeds and different constraints, so it's hard to create a study schedule useful for everybody. However, I offer a sample 10.5-week study schedule, Table 1, as a guide. This study schedule omits the first lesson. which is review.

The study schedule lists lessons that are either long or hard, as well as those that are short or easy or just background, so that you may better allocate your study time within the study periods provided for each subject.

Acknowledgements

I would like to thank the SOA and CAS for allowing me to use their old exam questions. I'd also like to thank Harold Cherry for suggesting this manual and for providing three of the pre-2000 SOA exams and all of the pre-2000 CAS exams I used.

The creators of TEX, LATEX, and its multitude of packages all deserve thanks for making possible the professional typesetting of this mathematical material.

I would like to thank all readers who sent in errata. Here is a partial list: Steve Guo, Professor Wafaa Shaban.

Errata

Please report all errors you find in these notes to the author. You may send them to the publisher at mail@studymanuals.com or directly to me at errata@aceyourexams.net. Please identify the manual and edition the error is in. This is the 4^{th} edition of the SOA Exam ALTAM manual.

An errata list will be posted at errata.aceyourexams.net. Check this errata list frequently.

Flashcards

Many students find flashcards a useful tool for learning key formulas and concepts. ASM flashcards, available from the same distributors that sell this manual, contain the formulas and concepts from this manual in a convenient deck of cards. The cards have cross references, usually by page, to the manual.

Lesson 2

Other Topics in Mortality

Reading: *Actuarial Mathematics for Life Contingent Risks* 3rd edition 3.4, 3.10

This lesson summarizes two sections of the textbook added to the syllabus for Spring 2025. This material for the most part does not lend itself to calculation questions.

2.1 Characteristics of Life Tables

It is interesting to look at national life tables. The textbook provides summary information regarding q_x from three national tables: Australian 2010–2012, Engligh 2010–2012, and US Life Tables 2013. The most recently released U.S. Life Table at this writing is from 2021, so we will look at that.[1] Figure 2.1 shows q_x from U.S. Life Table 2021 on a log scale for males and females.

Note the following characteristics of mortality:

1. q_0 is very high. Mortality immediately following birth is called *perinatal mortality*.
2. Mortality decreases until age 10 and then increases.
3. Male mortality is higher than female mortality at almost all ages. Interestingly, male mortality is lower than female mortality at ages 10 and 11 in the U.S. Life Table 2021. It was close, but not lower, in the textbook's graph, which was based on U.S. Life Table 2013.
4. Male mortality has a hump in the 20s. This is called the *accident bump*.[2]
5. In the log-scale graph of mortality, q_x is approximately linear starting at age 40, or even starting earlier for females, whose mortality is not impacted by the accident bump. Thus Gompertz's model works well for them. Figure 2.2 compares U.S. Life Table 2021 male mortality to fitted Gompertz, and the fit is fairly good.[3]

2.2 Heterogeneity in Mortality

Mortality differs among insureds in many ways. Insurance companies develop and use distinct mortality tables for different groups.

Some factors that distinguish mortality among insureds are:

1. **Sex**. Male and female mortality differ in shape and level, with male mortality being higher at all ages, as much as twice as high in the 20s and 30s but with lower ratios at the ages below 20 and above 40. Most companies use different mortality tables for male and female if it doesn't violate anti-discrimination laws.
2. **Smoking status**. Most companies use separate smoker and nonsmoker tables.
3. **Product**. Mortality varies by product.

[1] National tables are dated based on the years the data is from. This is unlike actuarial tables, which are usually based on several years of data and are dated based on the year in which the table was released; for example, the 2017 CSO table is based on mortality experience from 2002–2009.

[2] A couple of decades ago, in some mortality tables male mortality decreased at ages around 27–29, but current tables have it increasing slowly at those ages.

[3] The textbook does this for U.S. Life Table 2015 and gets an even better fit.

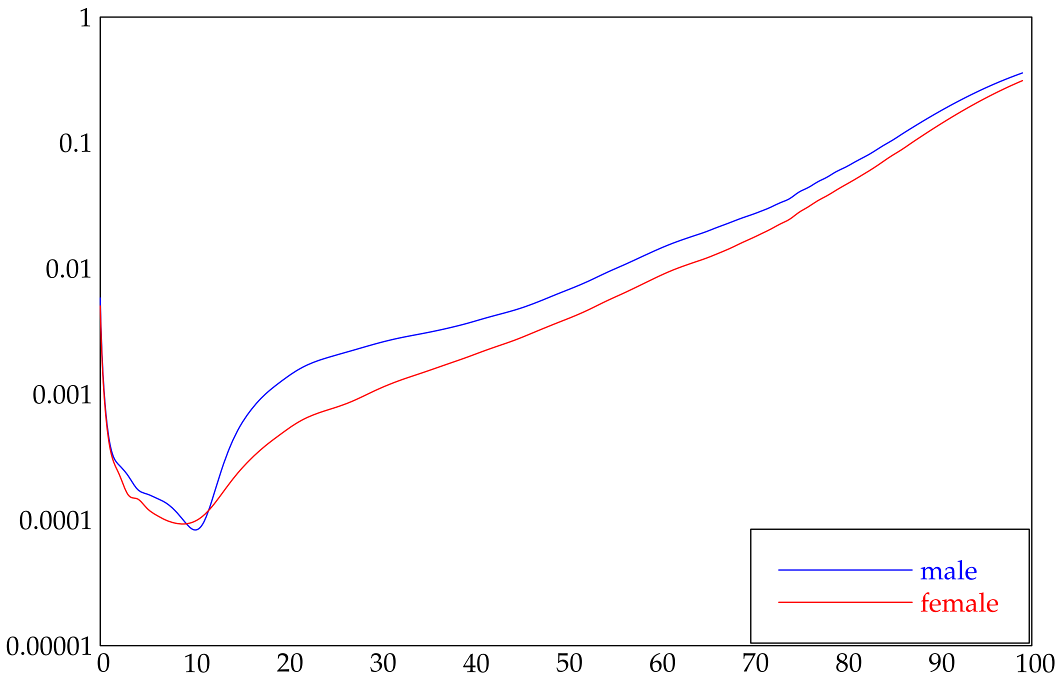

Figure 2.1: q_x from U.S. Life Table 2021 male (Table 2) and female (Table 3), on a log scale

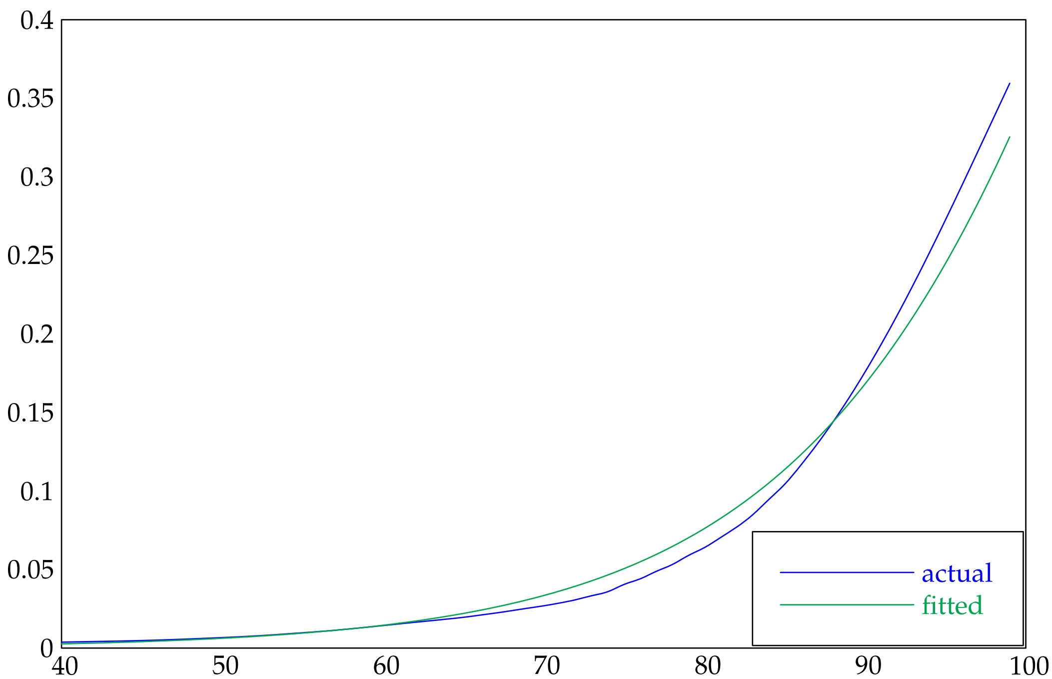

Figure 2.2: U.S. Life Table 2021 male q_x compared to Gompertz fitted q_x

Three general principles apply to mortality:

1. Wealthier individuals have lower mortality than others.
2. Due to adverse selection, mortality of annuitants is lower than mortality of insurance policyholders. Although insurance policyholders are underwritten, not everything can be caught through underwriting. A person who buys an annuity expects to live longer.
3. The more rigorous the underwriting, the lower the mortality. Guaranteed issue policies have higher mortality than underwritten policies. Group policies provided by employers have little or no underwriting, but the fact the insured is working may make the insured a better risk.

The textbook provides an example that shows that in a group with smokers and nonsmokers, the majority of the deaths may come from nonsmokers even though nonsmokers are the minority of the group. Here is a similar example.

EXAMPLE 2A A group of insureds age 45 is 50% male, 50% female. Mortality for males follows the Standard Ultimate Mortality Model. Mortality for females is 60% of male mortality.

None of the insureds lapse their policies within the next 5 years.

Calculate the proportion of females in the group at age 50.

SOLUTION: For males,

$$ {}_5p_{45} = \frac{l_{50}}{l_{45}} = \frac{98{,}576.4}{99{,}033.9} = 0.995380 $$

For females, we have to calculate the 5-year survival probability by multiplying p_x for $x = 45, 46, 47, 48, 49$. The following table shows the calculation:

x	q_x^{SUMM}	$1 - 0.6q_x^{\text{SUMM}}$
45	0.000771	0.999537
46	0.000839	0.999496
47	0.000916	0.999450
48	0.001003	0.999338
49	0.001100	0.999340

The product of the last column is 0.997226. Assuming we start with 50 males and 50 females, at the end of 5 years, there will be 50(0.995380) = 49.7690 males and 50(0.997226) = 49.8613 females, so females will be **0.5005** of the group. □

Exercises

These questions are provided for review. They involve no calculation.

2.1. Define "accident bump".

2.2. Define "perinatal mortality".

Exercises continue on the next page . . .

2.3. You are given the following excerpt from a mortality table:

x	l_x
15	98,689
16	98,658
17	98,618
18	98,570
19	98,512
20	98,444
21	98,365
22	98,275
23	98,173
24	98,061
25	97,941
26	97,815
27	97,684
28	97,546
29	97,401
30	97,246

Determine whether this table is more likely to represent male or female mortality.

2.4. State the minimum number of mortality tables that a life insurance company should have.

Solutions

2.1. The steep rise in male mortality in the 20s.

2.2. Mortality occurring immediately after birth.

2.3. Differencing the l_xs to obtain d_x, we see that there is no accident bump; the d_xs increase steadily. So this table is probably female. In fact, it is an excerpt from U.S. Life Tables, Table 15, Life table for Black, non-Hispanic females. In contrast, in Table 14, Life table for Black, non-Hispanic males, d_x increases rapidly from ages 14-22, quite slowly from ages 24–28, and then have a regular rate of increase for higher ages.

2.4. For insurance, we need 4 tables to take care of smoker/nonsmoker and male/female. For annuities, which are not underwritten, we only need male/female. So a total of 6 tables are needed. Of course many more may be needed, for example unisex for work-related insurance where one cannot discriminate by sex.

Lesson 20

Multiple Lives: Common Shock Probabilities

Reading: *Actuarial Mathematics for Life Contingent Risks* 3rd edition 10.7

The **common shock model** allows for both lives dying at the same time, for example through a car accident. Its Markov chain is Figure 5.7, repeated here for convenience.

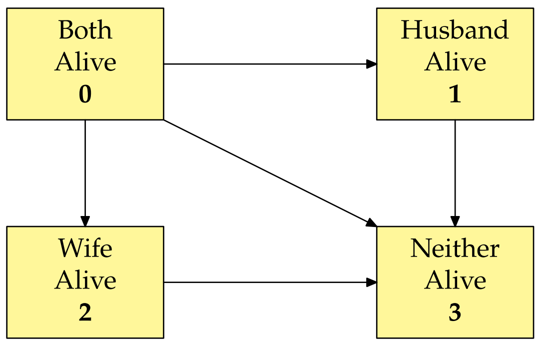

Figure 5.7, repeated for convenience

The probability of the joint status failing by time t is

$$ {}_tp_{xy}^{0\bullet} = {}_tp_{xy}^{01} + {}_tp_{xy}^{02} + {}_tp_{xy}^{03} $$

In other words, the probability of the joint status failing is the sum of the probabilities of each status failing for reasons other than common shock and the probability of both failing simultaneously.

A special case of the common shock model has the following relationships between forces of transition:

1. μ^{03} is constant
2. $\mu_{x+t}^{13} = \mu_{x+t:y+t}^{02} + \mu^{03}$
3. $\mu_{y+t}^{23} = \mu_{x+t:y+t}^{01} + \mu^{03}$

The latter two conditions say that the rate of death for each life does not change as a result of the other one dying. This simplifies the calculation of last survivor probabilities, as the next example shows.

Example 20A In a common shock model, you are given

(i) All forces of transition are constant.
(ii) $\mu^{01} = u$
(iii) $\mu^{23} = u + w$
(iv) $\mu^{02} = v$
(v) $\mu^{13} = v + w$
(vi) $\mu^{03} = w$

(a) Calculate the probability of both lives dying at the same time.

(b) Calculate the probability that at least one life survives t years.

Solution:

(a) This is ${}_{\infty}p_{xy}^{03\text{direct}}$, which we can calculate with the usual Markov chain formula:

$$ {}_{\infty}p_{xy}^{03\text{direct}} = \int_0^{\infty} {}_tp_{xy}^{00}\,\mu^{03}\,\mathrm{d}t = \int_0^{\infty} e^{-(u+v+w)t} w\,\mathrm{d}t = \boxed{\frac{w}{u+v+w}} $$

(b) We need the sum of the probabilities of transition to states 0, 1, or 2.

$$ \begin{aligned} {}_tp_{xy}^{00} &= e^{-(u+v+w)t} \\ {}_tp_{xy}^{01} &= \int_0^t e^{-(u+v+w)s}\, u(e^{-(v+w)(t-s)})\mathrm{d}s \\ &= u e^{-(v+w)t} \int_0^t e^{-us}\mathrm{d}s \\ &= e^{-(v+w)t} - e^{-(u+v+w)t} \\ {}_tp_{xy}^{02} &= e^{-(u+w)t} - e^{-(u+v+w)t} \end{aligned} $$

The answer is the sum of these three expressions, or

$$ {}_tp_{\overline{xy}} = \boxed{e^{-(u+w)t} + e^{-(v+w)t} - e^{-(u+v+w)t}} $$

However, a simpler way to work this out is to take advantage of the fact that the force of mortality for each life does not change when the other person dies, and is constant.

$$ \begin{aligned} {}_tp_x &= e^{-(v+w)t} \\ {}_tp_y &= e^{-(u+w)t} \\ {}_tp_{xy} &= e^{-(u+v+w)t} \\ {}_tp_{\overline{xy}} &= {}_tp_x + {}_tp_y - {}_tp_{xy} = e^{-(u+w)t} + e^{-(v+w)t} - e^{-(u+v+w)t} \end{aligned} $$

□

The next example is an example of the more general common shock model.

Example 20B For two lives (x) and (y) in a common shock model of the form of Figure 5.7,

(i) $\mu_{x+t:y+t}^{01} = \mu_{x+t}^{23} = 0.03t^2 - 0.01$

(ii) $\mu_{x+t:y+t}^{02} = \mu_{y+t}^{13} = 1/(50-t) - 0.01$, $t < 50$

(iii) $\mu_{x+t:y+t}^{03} = 0.01$

Calculate the probability that the joint status fails during the fifth year.

Solution: Let's calculate ${}_4p_{xy}^{00}$ and ${}_5p_{xy}^{00}$.

$$ \begin{aligned} {}_tp_{xy}^{00} &= \exp\left(-\int_0^t (0.03s^2 + 1/(50-s) - 0.01)\mathrm{d}s\right) \\ &= \exp\left(-0.01s^3 + \ln(50-s) + 0.01s\right)\Big|_0^t \\ &= (e^{-0.01t^3})\left(\frac{50-t}{50}\right)(e^{0.01t}) \\ {}_4p_{xy}^{00} &= e^{-0.64}\left(\frac{46}{50}\right)e^{0.04} = 0.504907 \\ {}_5p_{xy}^{00} &= e^{-1.25}\left(\frac{45}{50}\right)e^{0.05} = 0.271075 \end{aligned} $$

The answer is $0.504907 - 0.271075 = \boxed{0.233832}$. □

Exercises

20.1. In a common shock model for two lives age 50 and 55 of the form of Figure 5.7,

(i) $\mu^{01}_{50+t:55+t} = 1/(70-t),\ t < 70$
(ii) $\mu^{02}_{50+t:55+t} = 1/(65-t),\ t < 65$
(iii) $\mu^{03}_{50+t:55+t} = 0.01$

Calculate ${}_{20}p_{50:55}$.

20.2. **[SOA3-F04:20]** The mortality of (x) and (y) follows a common shock model with states:

State 0—both alive
State 1—only (x) alive
State 2—only (y) alive
State 3—both dead

You are given

(i) $\mu_{x+t} = \mu^{02}_{x+t:y+t} + \mu^{03}_{x+t:y+t} = \mu^{13}_{x+t} = g$, a constant, $0 \le t \le 5$
(ii) $\mu_{y+t} = \mu^{01}_{x+t:y+t} + \mu^{03}_{x+t:y+t} = \mu^{23}_{y+t} = h$, a constant, $0 \le t \le 5$
(iii) $p_{x+t} = 0.96,\ 0 \le t \le 4$
(iv) $p_{y+t} = 0.97,\ 0 \le t \le 4$
(v) $\mu^{03}_{x+t:y+t} = 0.01,\ 0 \le t \le 5$

Calculate the probability that both (x) and (y) survive 5 years.

(A) 0.65 (B) 0.67 (C) 0.70 (D) 0.72 (E) 0.74

20.3. **[MLC-S07:20]** For professional athletes Derek and A-Rod:

(i) Pairs of professional athletes are subject to a common shock model with the following states:
State 0 = both alive
State 1 = first one alive
State 2 = second one alive
State 3 = neither alive
(ii) Professional athletes on the same team are subject to a constant force of mortality due to crashes of the team airplane.
(iii) For professional athletes on the same team, $\mu^{01} = \mu^{02} = 0.0008$, $\mu^{13} = \mu^{23} = 0.001$, and $\mu^{03} = 0.0002$.
(iv) Professional athletes not on the same team have independent survival rates, so $\mu^{01} = \mu^{02} = \mu^{13} = \mu^{23} = 0.001$ and $\mu^{03} = 0$.
(v) Derek and A-Rod are on the same team now, but after one year will play for different teams.

Calculate the probability that both Derek and A-Rod survive two years.

(A) 0.9958 (B) 0.9960 (C) 0.9962 (D) 0.9964 (E) 0.9966

Exercises continue on the next page . . .

20.4. The mortality of (x) and (y) follows a common shock model of the form of Figure 5.7, with constant forces of transition and the following states:

State 0—both alive
State 1—only (x) alive
State 2—only (y) alive
State 3—both dead

You are given

(i) $\mu^{01}_{x+t:y+t} = 0.05$
(ii) $\mu^{02}_{x+t:y+t} = 0.04$
(iii) $\mu^{03}_{x+t:y+t} = 0.01$
(iv) $\mu^{13}_{x} = 0.07$
(v) $\mu^{23}_{y} = 0.06$

Calculate the probability that (x) dies within 10 years.

20.5. Mortality for two lives ages x and y follows a common shock model of the form of Figure 5.7. The forces of transition are

(i) $\mu^{01}_{x+t:y+t} = \mu^{02}_{x+t:y+t} = \begin{cases} 0.01 & t \le 20 \\ 0.03 & t > 20 \end{cases}$
(ii) $\mu^{03}_{x+t:y+t} = 0.005$

Calculate the probability that both lives die at the same time.

20.6. The mortality of (x) and (y) follows a common shock model with states:

State 0—both alive
State 1—only (x) alive
State 2—only (y) alive
State 3—both dead

You are given

(i) $\mu^{01}_{x+t:y+t} = 0.03$
(ii) $\mu^{02}_{x+t:y+t} = 0.02$
(iii) $\mu^{03}_{x+t:y+t} = 0.005$
(iv) $\mu^{13}_{x+t} = 0.04$
(v) $\mu^{23}_{y+t} = 0.035$

Calculate the probability that (x) dies before (y) and within 10 years.

20.7. The mortality of (x) and (y) follows a common shock model of the form of Figure 5.7. You are given:

(i) $\mu^{01}_{x+t:y+t} = 1/(50-t)$, $t < 50$
(ii) $\mu^{02}_{x+t:y+t} = 0.05$
(iii) $\mu^{03}_{x+t:y+t} = 0.02$.

Calculate the probability that (x) and (y) die simultaneously within 10 years.

Exercises continue on the next page . . .

20.8. [**Worksheet question**] The workbook can be downloaded here.

Survivorship for Bill (age 35) and Jill (age 40) follows a common shock model. The states of the model are

0. Both alive
1. Only Bill alive
2. Only Jill alive
3. Neither alive

Forces of transition are (x=Bill, y=Jill)

$$\begin{aligned}
\mu_{xy}^{01} &= 0.0001(1.08^y) \\
\mu_{xy}^{02} &= 0.00015(1.08^x) \\
\mu_{xy}^{03} &= 0.001 \\
\mu_{x}^{13} &= 0.0002(1.08^x) \\
\mu_{y}^{23} &= 0.00016(1.08^y)
\end{aligned}$$

(a) Estimate the state probabilities by using Euler's method with interval 0.1 to solve Kolmogorov's forward equations. Use these probabilities to build a life table l_x for Bill, starting with a radix of 1,000,000 at age 35 and going to age 100.

(b) Calculate the probability of both lives dying at the same time by using approximate integration using Simpson's rule with function values at integers, without using Euler's method.

Additional old SOA Exam LTAM questions: F19:B3(a)–(b)

Additional old SOA Exam ALTAM questions: S23:2

Solutions

20.1.
$$ {}_{20}p_{50:55}^{00} = \exp\left(-\int_0^{20} \sum_{j=1}^{3} \mu_{50+s:55+s}^{0j}\,\mathrm{d}s\right) = \left(\frac{50}{70}\right)\left(\frac{45}{65}\right)e^{-20(0.01)} = \boxed{\mathbf{0.404867}}$$

20.2. Because of (i) and (ii), p_x and p_y are well defined; they don't change at the death of the other. Moreover, due to constant force, the probability of surviving 5 years is the probability of surviving 1 year raised to the 5th power. So

$${}_5p_{xy} = p_{xy}^5$$

Also,

$$\begin{aligned}
p_{xy} = p_{xy}^{00} &= e^{-(\mu^{01}+\mu^{02}+\mu^{03})} \\
&= e^{-(\mu^{01}+\mu^{03})}e^{-(\mu^{02}+\mu^{03})}e^{\mu^{03}} = p_x p_y e^{0.01} \\
&= (0.96)(0.97)e^{0.01} = 0.940559
\end{aligned}$$

So ${}_5p_{xy} = 0.940559^5 = \boxed{\mathbf{0.73609}}$. **(E)**

20.3. p_{xy}^{00} for the first year is

$$e^{-2(0.0008)-0.0002} = e^{-0.0018} = 0.998202.$$

In the second year,

$$p_{x+1:x+1} = e^{-2(0.001)} = e^{-0.002} = 0.998002.$$

The product is ${}_2p_{xy} = (0.998202)(0.998002) = \boxed{\mathbf{0.99620}}$. **(C)**

20.4. We need the probability of $0 \to 2$, $0 \to 3$, and of $0 \to 1 \to 3$. It's easier to calculate $0 \to 1$ and staying in 0, and then take the complement.

$$\begin{aligned}
{}_{10}p_{xy}^{00} &= e^{-(0.05+0.04+0.01)(10)} = 0.367879\\
{}_{10}p_{xy}^{01} &= \int_0^{10} e^{-0.1s}(0.05)e^{-(10-s)0.07}\,\mathrm{d}s\\
&= 0.05e^{-0.7}\int_0^{10} e^{-0.03s}\,\mathrm{d}s\\
&= \frac{0.05e^{-0.7}(1-e^{-0.3})}{0.03} = 0.214510
\end{aligned}$$

The answer is $1 - 0.367879 - 0.214510 = \boxed{0.417611}$.

20.5. First,

$$ {}_tp_{xy}^{00} = \begin{cases} e^{-(0.01+0.01+0.005)t} = e^{-0.025t} & t \le 20\\ e^{-0.025(20)-0.065(t-20)} = e^{-0.065t+0.8} & t \ge 20 \end{cases}$$

Then

$$\begin{aligned}
{}_\infty p_{xy}^{03\text{ direct}} &= \int_0^{20} e^{-0.025t}(0.005)\mathrm{d}t + \int_{20}^{\infty} e^{-0.065t+0.8}(0.005)\mathrm{d}t\\
&= \frac{0.005(1-e^{-0.5})}{0.025} + \frac{0.005e^{-0.5}}{0.065} = \boxed{0.125350}
\end{aligned}$$

As a shortcut for the second integral, after 20 years the probability of exiting state 0 directly to state 3 is the proportion of its transition force over the total force of transition out of state 0, or $0.005/\bigl(0.005+2(0.03)\bigr)$, and the probability of surviving 20 years is $e^{-20[0.005+2(0.01)]}$, so the probability at time 0 of going directly from state 0 to state 3 after 20 years is $e^{-0.5}(0.005/0.065)$.

20.6. The question asks for the probability of moving to state 2, (x) dead and (y) alive, within 10 years. We need $\int_0^{10} {}_{10}p_{xy}^{00}\,\mu_{x+t:y+t}^{02}\,\mathrm{d}t$.

$$\int_0^{10} {}_{10}p_{xy}^{00}\,\mu_{x+t:y+t}^{02}\,\mathrm{d}t = \int_0^{10} e^{-(0.03+0.02+0.005)t}(0.02) = 0.02\left(\frac{1-e^{-0.55}}{0.055}\right) = \boxed{0.153836}$$

20.7.

$$\begin{aligned}
{}_{10}p_{xy}^{03\text{ direct}} &= \int_0^{10} 0.02e^{-0.07(t)}\left(\frac{50-t}{50}\right)\mathrm{d}t\\
&= \frac{1}{50}\int_0^{10} e^{-0.07t}\,\mathrm{d}t - \int_0^{10}\frac{1}{2500}te^{-0.07t}\,\mathrm{d}t
\end{aligned}$$

The first integral is

$$\int_0^{10} e^{-0.07t}\,\mathrm{d}t = \frac{1-e^{-0.7}}{0.07} = 7.1916$$

The second integral is

$$\int_0^{10} te^{-0.07t}\,\mathrm{d}t = \frac{1}{0.07^2}(1-1.7e^{-0.7}) = 31.7969$$

The probability they die simultaneously within 10 years is

$$\frac{7.1916}{50} - \frac{31.7969}{2500} = \boxed{0.1311}$$

20.8. The workbook solution can be downloaded here.

(a) We calculate all state probabilities, although only probabilities of states 0 and 1 are needed here. Then we multiply the sum of the probabilities of states 0 and 1 by 1,000,000.

(b) We calculate $\int_0^\infty {}_tp^{00}_{35:40}\,\mu^{03}_{35+t,40+t}$, with directly calculated values of ${}_tp^{00}_{35:40}$.

Lesson 30

Profit Tests: Universal Life

Reading: *Actuarial Mathematics for Life Contingent Risks* 3rd edition 14

In this lesson, we will first of all get familiar with universal life insurance, and understand how it works. Then we will discuss how to compute profit measures for it.

30.1 How universal life works

The traditional insurances we've dealt with throughout this manual have a fixed structure. The policyholder pays premiums that are predetermined, for a period that is fixed at issue. In return, the policyholder gets a fixed death benefit at death, and sometimes a maturity benefit. Behind the scenes, the insurance company holds assets to pay for the death benefits. The amount of these assets equals the reserve. If the policyholder stops paying premiums, often he is entitled to either receive the reserve or some portion of it, or to apply it to purchase paid up or extended term insurance. However, all this is happening behind the scenes; the policyholder is often unaware that the policy has a value.

Universal life (UL) is an unbundled, flexible form of insurance. The policy has an *account value*. The policyholder pays premiums, which are flexible, into the account. The account is charged for expenses and for the cost of providing a death benefit upon death, or the mortality charge, for the current period. The account earns interest.

Here is a schematic for how the account value is updated:

Starting Account Value	+	Premium	−	Expense Charge	−	Mortality Charge	+	Interest Credited	=	Ending Account Value

The company specifies the expense charge, the mortality charge, and the interest crediting rate.

Expense charges may be expressed as a percent of premium, a proportion of the face amount, or a fixed amount per policy. There may be a current schedule of expense charges, set by the insurer, subject to a guaranteed maximum.

> Expense charges are amounts the insurance company charges the policyholder. Do not confuse them with *expenses*, which is what the company pays in order to acquire or maintain a policy. An expense charge is part of the structure of the policy and is written into the insurance contract. An expense is not part of the policy. It is part of the company's cost of doing business.

The mortality charge is called the *cost of insurance (COI)*.

There are two types of death benefit patterns:

1. In a policy with a fixed death benefit equal to a specified amount, called ***Type A***, the death benefit is level, unless it must be increased (as discussed in the next paragraph). Thus, if the death benefit is 10,000 and the account value is 3000, the policyholder would receive 10,000 on death. The excess of the death benefit over the account value is called the "additional death benefit (ADB)". It is the net amount at risk.

   The net amount at risk cannot be negative, so if the account value is higher than the death benefit, the account value is paid. However, U.S. tax laws require that the death benefit be at least a certain multiple of the account value.[1] This requirement is meant to prevent a person from avoiding investment taxes by disguising an

[1] There are actually two options for satisfying the U.S. tax laws, but one of the options is more suitable for universal life. The corridor factors are one of the two requirements of that option.

investment product as an insurance product by including a token death benefit. This multiple of the account value, which is called the ***corridor factor***, ranges from 2.5 at ages 40 and below to 1.05 at age 90, and is 1 for ages 95 and above. In other words, the death benefit when the insured is age 40 or less must be at least 2.5 times the account value, and lower multiples apply at ages above 40. The death benefit is automatically increased when necessary to meet this rule.

2. In a policy with a death benefit equal to a specified amount plus the account value, called ***Type B***, the net amount at risk is level. The death benefit is the account value plus the fixed amount. The death benefit must be increased if the corridor factor requires it, but that is less likely to happen in this design. This design is similar to traditional policies with a payment of the reserve on death.

For either design, the term "face amount" is used for the specified amount. Thus the additional death benefit for Type A is face amount minus ending account value, and for Type B the additional death benefit is the face amount.

The cost of insurance under either design is the premium for a one-year term insurance providing the additional death benefit. In other words, it is the mortality rate times the additional death benefit discounted with interest. The mortality rate is set by the insurer and may be changed, subject to a guaranteed maximum. The cost of insurance is charged at the beginning of the year. In most of our examples and exercises, we will assume the death benefit is paid at the end of the year, so the COI is the mortality rate discounted with one year's interest. In reality, benefits are paid at the moment of death, and the discounting may be built into the mortality rate.

The *cost of insurance rate (COI rate)* is the mortality rate used for calculating the cost of insurance.

Interest is credited to the account. The insurer declares the credited interest rate, subject to a minimum specified in the contract. The interest is credited on all funds—beginning of year account value plus premium minus expense charges and the cost of insurance. The interest crediting rate does not have to equal the interest rate used for discounting the mortality rate used for computing the COI. However, when not stated otherwise, you should assume they are equal—whether in this manual or on the exam.[2]

> Mortality rates, interest rates, and expense rates used in updating the account value are *not* the assumptions that the company uses for pricing the product. While they may have some relationship to what the insurance company expects, they may not. They are simply rates selected by the insurance company to meet its profit objectives and to make its product competitive. *Do not confuse these rates with mortality, interest, and expense assumptions used for profit testing.*

Define the following notation:[3,4]

AV_t be the account value at time t.

P_t is the premium paid at the beginning of the t^{th} year.

e_t is the expense charge at the beginning of the t^{th} year.

q_{x+t-1} be the mortality rate used for calculating the cost of insurance.

i^q is the interest rate used for discounting the cost of insurance.

v_q is $1/(1+i^q)$

FA is the specified death benefit, also called the face amount. For a Type B policy this is paid in addition to the account value upon death, while for a Type A policy it is the amount paid upon death.

b_t is the death benefit paid if death occurs at time t. This is FA for a Type A policy and $\mathrm{FA}+\mathrm{AV}_t$ for a Type B policy.[5]

i^c is the interest crediting rate. We will usually call this i, without the superscript.

[2] To quote from the SOA notation and terminology note: "Unless otherwise stated in the question $i^q = i^c$". i^q and i^c are defined below.

[3] The subscripts on P and e in this lesson, while consistent with the textbook, are inconsistent with other lessons. P_t is the premium paid at the beginning of year t, not the premium paid at time t, the beginning of year $t+1$.

[4] Most of this notation is taken from the textbook. The SOA notation and terminology note mentions this notation, but says you're not responsible for it; it'll be defined as needed in any exam question that uses it.

[5] b_t is defined for completeness, but is not used in the following formulas.

We will assume that death benefits and surrender benefits are paid at the end of the period. This is the standard assumption for exam questions as well.[6]

For a Type B design, the additional death benefit is not a function of the account value. To compute account values recursively, add premium and subtract charges from the previous period's ending account value, then accumulate the result with interest. Since the COI rate for year t is q_{x+t-1}, the formula is

$$\mathrm{AV}_t = \left(\mathrm{AV}_{t-1} + P_t - e_t - \mathrm{FA}\ v_q\ q_{x+t-1}\right)(1+i^c) \tag{30.1}$$

If $i^q = i^c$, we can write this as

$$\mathrm{AV}_t = (\mathrm{AV}_{t-1} + P_t - e_t)(1+i) - \mathrm{FA}\ q_{x+t-1} \tag{30.2}$$

There is no need to memorize separate formulas for the case $i^q \neq i^c$. Simply replace the mortality rate q_{x+t} (for all t) with $q'_{x+t} = q_{x+t}(1+i^c)v_q$, and use the revised mortality rate in conjunction with i^c.

To understand these formulas, move the q_{x+t-1} term to the other side. For example, formula (30.2) becomes

$$(\mathrm{AV}_{t-1} + P_t - e_t)(1+i) = \mathrm{FA}\ q_{x+t-1} + \mathrm{AV}_t$$

The formula says the following:

The accumulation of initial account value plus premium minus expense charges pays the ending account value to those who survive and the death benefit to those who die.

The death benefit equals the specified amount FA plus the account value. If you keep this principle in mind, you will not need to memorize formulas!

The cost of insurance is the mortality rate times the face amount, discounted to the beginning of the year, or

$$\mathrm{COI}_t = v_q\, q_{x+t-1}\ \mathrm{FA} \tag{30.3}$$

For a Type B policy, if you wish to calculate account values recursively, it is easier to first compute the COI and then the account value.

A policyholder who surrenders the policy receives the account value minus a *surrender charge*. The surrender charge declines with time and may be 0 at later durations, and helps the insurer recover acquisition costs. The account value minus the surrender charge is the ***cash surrender value***. In this manual, it will sometimes be called the cash value.

If the account value goes down to 0, which can happen due to premium deposits inadequate to cover the mortality and expense charges, the policy lapses. Some policies have ***no-lapse guarantees***, also known as *secondary guarantees*. These guarantee that the policy won't lapse if the policyholder has paid a predetermined fixed amount of premium and continues paying it. For example, the policy may guarantee that it won't lapse even if the account value goes to 0 as long as the policyholder has paid 200 every year and continues to pay 200 each year in the future. The premium of 200 may be called the no-lapse premium.

A policyholder can borrow the account value through ***policy loans***.

The next example illustrates how a Type B universal life policy works.

EXAMPLE 30A For a Type B universal life policy of 100,000 on (55):

(i) The policyholder pays a premium of 4000 a year at the beginning of each year.
(ii) Expense charges are 40% of first year premium plus 500 in the first year, and 8% of renewal premiums plus 50 in renewal years.
(iii) The mortality assumption used for computing the COI is 120% of the Standard Ultimate Mortality Model. The COI interest rate is 5%.
(iv) Interest of 5% is credited annually.
(v) The account value is updated annually.

Develop annual account values for the first 5 years, assuming the policy remains in force for 5 years.

[6] To quote the SOA notation and terminology note, "Death benefits and surrender benefits are paid at the end of the period, after the account value at the end of the period has been calculated."

Solution: In the first year, 4000 is deposited, and expense charges are $0.4(4000) + 500 = 2100$. Since $q_{55} = 0.001993$, the cost of insurance is $1.2(0.001993)(100{,}000)/1.05 = 227.77$. After these charges, the account value is $4000 - 2100 - 227.77 = 1672.23$. This earns interest at 5%, so the ending account value is $1672.23(1.05) = 1755.84$.

In the second year, 4000 is deposited, and expense charges are $0.08(4000) + 50 = 370$. The COI is

$$\frac{1.2(0.002212)(100{,}000)}{1.05} = 252.80$$

□

After these charges, the account value is $1755.84 + 4000 - 370 - 252.80 = 5133.04$. This earns 5% interest, for a final account value of $5133.04(1.05) = 5389.69$.

You should be able to verify the remainder of the development of the account value, displayed in the following table:

Year	Premium	Expense	Cost of Insurance	Interest	Account Value
1	4000	2100	227.77	83.61	1755.84
2	4000	370	252.80	256.65	5389.69
3	4000	370	281.03	436.93	9175.60
4	4000	370	312.69	624.65	13117.56
5	4000	370	348.34	819.96	17219.17

Quiz 30-1 For a Type B universal life policy of 10,000 on (45):

(i) At time 5, the account value is 125.
(ii) The mortality rate used for computing the cost of insurance is 110% of the Standard Ultimate Mortality Model.
(iii) Expense charges are 50 plus 2% of premium.
(iv) $i = 0.05$ is used for accumulating the account value and discounting the cost of insurance.
(v) The surrender charge in the sixth year is 100.
(vi) The policy lapses if the cash surrender value is below 0.

Calculate the minimum premium needed to avoid lapse of the policy in the sixth year.

For Type A, computing the COI is more complicated, since the additional death benefit depends on the ending account value, but the ending account value depends on the COI. The following formula, based on the schematic on page 571, applies to any universal life policy:

$$(\mathrm{AV}_{t-1} + P_t - e_t - \mathrm{COI}_t)(1+i) = \mathrm{AV}_t \tag{30.4}$$

But

$$\mathrm{COI}_t = v_q q_{x+t-1}(\mathrm{FA} - \mathrm{AV}_t)$$

Thus we have to solve two simultaneous equations to obtain AV_t. The best way to do this is to remember the principle mentioned above:

> *The accumulation of initial account value plus premium minus expense charges pays the ending account value to those who survive and the death benefit to those who die.*

For a Type A policy, whose death benefit is FA, this means (assuming $i^q = i^c$)

$$(\mathrm{AV}_{t-1} + P_t - e_t)(1+i) = q_{x+t-1}\,\mathrm{FA} + p_{x+t-1}\,\mathrm{AV}_t$$
$$(\mathrm{AV}_{t-1} + P_t - e_t)(1+i) = q_{x+t-1}\,\mathrm{FA} + (1 - q_{x+t-1})\,\mathrm{AV}_t$$
$$\mathrm{AV}_t = \frac{(\mathrm{AV}_{t-1} + P_t - e_t)(1+i) - q_{x+t-1}\,\mathrm{FA}}{1 - q_{x+t-1}} \tag{30.5}$$

If $i^q \neq i^c$, we replace q_{x+t-1} with $v_q\, q_{x+t-1}(1+i^c)$ and get

$$\mathrm{AV}_t = \frac{(\mathrm{AV}_{t-1} + P_t - e_t - v_q\, q_{x+t-1}\,\mathrm{FA})(1+i^c)}{1 - v_q\, q_{x+t-1}(1+i^c)} \tag{30.6}$$

In formulas (30.5) and (30.6), $q_{x+t-1}\,\mathrm{FA}$ is *not* the COI. The formulas are simply a method for calculating AV_t, derived by solving the set of two simultaneous equations mentioned above. The COI is $v_q q_{x+t-1}(\mathrm{FA} - \mathrm{AV}_t)$. If you are given the COI, you do *not* divide by anything to roll forward the account value. *To go from one account value to the next one, always use formula* (30.4), regardless of whether the policy is Type A or Type B.

Formula (30.5) is similar to the reserve recursion formula for traditional products.

Comparing the two formulas here to (30.1) and (30.2), the difference is the denominators; by dividing by something less than 1, the result is a higher account value, since the COI is charged on a smaller additional death benefit. In the formulas for Type B insurances, the face amount is paid to those who die and the account value to everybody, so no division by p_x is done.

The COI is $v_q\, q_{x+t-1}(\mathrm{FA} - \mathrm{AV}_t)$, or

$$\mathrm{COI}_t = \frac{v_q\, q_{x+t-1}\big(\mathrm{FA} - (\mathrm{AV}_{t-1} + P_t - e_t)(1+i^c)\big)}{1 - v_q\, q_{x+t-1}(1+i^c)} \tag{30.7}$$

and when $i^q = i^c$,

$$\mathrm{COI}_t = \frac{v q_{x+t-1}\big(\mathrm{FA} - (\mathrm{AV}_{t-1} + P_t - e_t)(1+i)\big)}{1 - q_{x+t-1}} \tag{30.8}$$

For a Type A policy, if you need the COI, it is easier to first calculate the ending account value.

Example 30B For a Type A universal life policy of 100,000 on (60):

(i) The cost of insurance is based on $q_x = 0.001x$.
(ii) Interest credited and assumed in the cost of insurance is 5%.
(iii) The account value at the end of 4 years is 15000.
(iv) The account value is updated annually.
(v) The premium paid at the beginning of the fifth year is 1000.
(vi) Expense charges at the beginning of the fifth year are 50.

Calculate the cost of insurance for the fifth year.

Solution: Using formula (30.8),

$$\mathrm{COI}_5 = \frac{(0.064/1.05)\big(100{,}000 - (15{,}000 + 1{,}000 - 50)(1.05)\big)}{1 - 0.064} = \boxed{5421.41}$$

Although not required for the question, let's verify that to go from one account value to the next, we merely subtract the COI and do no division.

$$(15{,}000 + 1{,}000 - 50 - 5{,}421.41)(1.05) = 11{,}055.02$$

and the COI is

$$\frac{0.064(100{,}000 - 11{,}055.02)}{1.05} = 5{,}421.41$$

matching our calculated value. □

Quiz 30-2 For a Type B universal life policy of 50,000 on (55):

(i) The account value at the beginning of year 10 is 13,000.
(ii) The cost of insurance in the tenth year is 800.
(iii) The policyholder does not contribute anything in the tenth year.
(iv) Expense charges in the tenth year are 50.
(v) Interest credited is 5%.

Calculate the cost of insurance rate.

Quiz 30-3 For a Type A universal life policy of 100,000 on (50):

(i) The cost of insurance is based on mortality implied by $l_x = 100 - x$, $x \le 100$.
(ii) Interest on the cost of insurance is 4%.
(iii) Interest credited is 6%.
(iv) The initial premium is 40,000.
(v) Initial expense charges are 5,000.

Calculate the account value at the end of the first year.

EXAMPLE 30C [Same as Example 30A except for type] For a Type A universal life policy of 100,000 on (55):

(i) The policyholder pays a premium of 4000 a year at the beginning of each year.
(ii) Expense charges are 40% of first year premium plus 500 and 8% of renewal premiums plus 50.
(iii) The mortality assumption used for computing the COI is 120% of the Standard Ultimate Mortality Model. COI's are discounted at 5%.
(iv) Interest of 5% is credited annually. The account value is updated annually.

Develop annual account values for the first 5 years, assuming the policy remains in force for 5 years.

SOLUTION: Use equation (30.5). In the first year, the account value is

$$\mathrm{AV}_1 = \frac{(4000 - 2100)(1.05) - 100{,}000(1.2)(0.001191)}{1 - 1.2(0.001191)} = 1760.05$$

□

and the COI is $1.2(0.001993)(100{,}000 - 1760.05)/1.05 = 223.76$. The following table shows the calculations for five years:

Year	Premium	Expense	Cost of Insurance	Interest	Account Value
1	4000	2100	223.76	83.81	1760.05
2	4000	370	239.12	257.55	5408.47
3	4000	370	255.11	489.17	9222.53
4	4000	370	271.38	629.06	13210.20
5	4000	370	287.80	827.62	17380.02

In real life, the account value is updated monthly. However, for simplicity, most of our examples and exercises will update the account value annually. In this manual, assume annual updating unless told otherwise. On the exam, the question will state the updating period.

Recall that the death benefit must be at least as high as the account value times the corridor factor. If a question in this manual or on the exam does not state a corridor factor, assume that the corridor does not come into play. Let's discuss how to calculate account values and costs of insurance when there is a corridor factor.

First of all, calculate the account value ignoring the corridor factor. If the resulting account value times the corridor factor is less than or equal to the death benefit, you're done. Otherwise, let γ be the corridor factor and set the death benefit equal to $\gamma\,\mathrm{AV}_t$. Use the boxed principle, which we'll repeat for a third time:

The accumulation of initial account value plus premium minus expense charges pays the ending account value to those who survive and the death benefit to those who die.

This means (assuming $i^q = i^c$)

$$(\mathrm{AV}_{t-1} + P_t - e_t)(1+i) = q_{x+t-1}\gamma\,\mathrm{AV}_t + p_{x+t-1}\,\mathrm{AV}_t$$
$$(\mathrm{AV}_{t-1} + P_t - e_t)(1+i) = q_{x+t-1}\gamma\,\mathrm{AV}_t + (1-q_{x+t-1})\,\mathrm{AV}_t = \mathrm{AV}_t\big(1 + q_{x+t-1}(\gamma-1)\big)$$
$$\mathrm{AV}_t = \frac{(\mathrm{AV}_{t-1} + P_t - e_t)(1+i)}{1 + q_{x+t-1}(\gamma-1)} \tag{30.9}$$

If $i^q \neq i$, we replace q_{x+t-1} with $v_q\,q_{x+t-1}(1+i^c)$ to obtain

$$\mathrm{AV}_t = \frac{(\mathrm{AV}_{t-1} + P_t - e_t)(1+i^c)}{1 + v_q\,q_{x+t-1}(1+i^c)(\gamma-1)} \tag{30.10}$$

The cost of insurance is then $v_q\,q_{x+t-1}$ times $(\gamma-1)\,\mathrm{AV}_t$, or

$$\mathrm{COI}_t = \frac{v_q\,q_{x+t-1}(\gamma-1)(\mathrm{AV}_{t-1} + P_t - e_t)(1+i^c)}{1 + v_q\,q_{x+t-1}(1+i^c)(\gamma_t-1)} \tag{30.11}$$

Example 30D For a Type A universal life policy of 13,000 on (25):

(i) The account value at the beginning of the sixth year is 5000.
(ii) The account value is updated monthly.
(iii) The corridor factor is 2.5.
(iv) The policyholder pays a premium of 1000 at the beginning of the sixth year.
(v) Monthly expense charges during the sixth year are 0.5% of premium plus 3.
(vi) The monthly cost of insurance rate is 0.00201.
(vii) Interest is credited and discounted at $i^{(12)} = 0.06$.

Calculate the cost of insurance for the first month of the sixth year.

Solution: Expense charges are $0.005(1000) + 3 = 8$. Ignoring the corridor,

$$\mathrm{AV}_{5\,1/12} = \frac{(5000 + 1000 - 8)(1.005) - 0.00201(13000)}{1 - 0.00201} = 6007.91$$

and $13{,}000 < 2.5(6007.91)$.

With the corridor factor and using formula (30.9),

$$\mathrm{AV}_{5\,1/12} = \frac{(5000 + 1000 - 8)(1.005)}{1 + 0.00201(1.5)} = 6003.86$$

The cost of insurance is $0.00201(1.5)(6003.86)/1.005 =$ **18.01**. □

Table 30.1 summarizes account value and COI formulas.

30.2 Profit tests

Profit tests for universal life are similar to ones for traditional products; the only difference is that the account value must be projected. When projecting cash flows for a profit test, we make assumptions for interest, mortality, and expenses, the same way we did for traditional life insurance. *These assumptions are not the same as the expense charges and crediting rates used to calculate account values.* The insurer builds in margins. The insurer charges higher mortality

Table 30.1: Summary of Universal Life Formulas

Type A

$$\mathrm{AV}_t = \frac{(\mathrm{AV}_{t-1} + P_t - e_t - v_q\, q_{x+t-1}\,\mathrm{FA})(1+i^c)}{1 - v_q\, q_{x+t-1}(1+i^c)} \tag{30.6}$$

$$\mathrm{AV}_t = \frac{(\mathrm{AV}_{t-1} + P_t - e_t)(1+i) - q_{x+t-1}\,\mathrm{FA}}{1 - q_{x+t-1}} \qquad \text{if } i^q = i^c \tag{30.5}$$

$$\mathrm{COI}_t = \frac{v_q\, q_{x+t-1}\big(\mathrm{FA} - (\mathrm{AV}_{t-1} + P_t - e_t)(1+i^c)\big)}{1 - v_q\, q_{x+t-1}(1+i^c)} \tag{30.7}$$

$$\mathrm{COI}_t = \frac{v q_{x+t-1}\big(\mathrm{FA} - (\mathrm{AV}_{t-1} + P_t - e_t)(1+i)\big)}{1 - q_{x+t-1}} \qquad \text{if } i^q = i^c \tag{30.8}$$

Type B

$$\mathrm{AV}_t = \big(\mathrm{AV}_{t-1^c} + P_t - e_t - v_q\, q_{x+t-1}\ \mathrm{FA}\big)\,(1+i) \tag{30.1}$$

$$\mathrm{AV}_t = (\mathrm{AV}_{t-1} + P_t - e_t)(1+i) - q_{x+t-1}\ \mathrm{FA} \qquad \text{if } i^q = i^c \tag{30.2}$$

$$\mathrm{COI}_t = v_q\, q_{x+t-1}\,\mathrm{FA} \tag{30.3}$$

If corridor applies

$$\mathrm{AV}_t = \frac{(\mathrm{AV}_{t-1} + P_t - e_t)(1+i^c)}{1 + v_q\, q_{x+t-1}(1+i^c)(\gamma - 1)} \tag{30.10}$$

$$\mathrm{AV}_t = \frac{(\mathrm{AV}_{t-1} + P_t - e_t)(1+i)}{1 + q_{x+t-1}(\gamma - 1)} \qquad \text{if } i^q = i^c \tag{30.9}$$

$$\mathrm{COI}_t = \frac{v_q\, q_{x+t-1}(\gamma - 1)(\mathrm{AV}_{t-1} + P_t - e_t)(1+i)}{1 + v_q\, q_{x+t-1}(1+i^c)(\gamma - 1)} \tag{30.11}$$

Simplified version of account value formulas
Let the accumulated fund be $(\mathrm{AV}_{t-1} + P_t - e_t)(1+i)$. Then:

- For Type A, this amount pays AV_t to those who survive and the face amount to those who die.
- For Type B, this amount pays AV_t to everybody plus the face amount to those who die.
- If the corridor applies, this amount pays the AV_t to everybody plus $(\gamma - 1)\,\mathrm{AV}_t$ to those who die, where γ is the corridor factor.

rates than it expects to have, and charges higher expense rates than it expects to incur (except that expected initial expenses may be higher than charged), and credits lower interest than it expects to earn. That's where profits come from.

To perform a profit test, we need to know the (benefit) reserve. The net premium reserve requires a prospective calculation, which means making assumptions on future premiums and death benefits. This is complicated, and if you follow the SOA's life insurance track, you will learn in later courses how the reserve is calculated. We do not want to discuss net premium reserve calculations for universal life in this course, so we will make a simple approximation. *We will assume that the reserve equals the account value.* In the next section, we'll discuss certain reserves that are held by the company in addition to the account value.

We assume that all surrenders occur at the end of the year, after all the deaths. If associated single-decrement rates of surrender are given, the probability of surrender is computed by multiplying the surrender rate by the probability of survival from death in the current year. For example, if the mortality rate is 0.01 and the surrender rate is $q'^{(\text{withdrawal})} = 0.10$, then the probability of surrender is $(0.10)(1 - 0.01) = 0.099$.

For a Type B policy, there are three classes of policyholders:

- Those who die get the death benefit plus the account value. In addition, there may be costs in processing the death claim, known as "settlement expenses". The "expected death benefit", or EDB, is the mortality rate times the sum of the face amount, the account value, and settlement expenses for death claims.

- Those who surrender get the account value, possibly minus a surrender charge. In addition, there may be costs in processing the surrender claim, even if nothing is paid. The "expected surrender benefit", or ESB, is the surrender probability (surrender rate times the complement of the mortality probability) times the account value minus the surrender charge, but not less than 0, plus settlement expenses for surrender claims.

- Those who survive get the account value. The expected (ending) account value, or EAV, is the ending account value times 1 minus the death and surrender probabilities.

This categorization is logical. However, it may be more convenient to do many exercises involving computation of profit differently. As an example of a different way of summing up all the payment components, consider the following:

- Everyone gets the account value.

- Those who die get the face amount plus settlement expenses for death.

- Those who surrender get settlement expenses for surrender minus the surrender charge, assuming the surrender charge is no higher than the account value.

A breakdown like this may make the computation easier when you're trying to back out the surrender charge (for example).

For a Type A policy, those who die do not get the account value. Otherwise, everything is the same as for a Type B policy. In the second way of summing up the payment components, the first component would give the account value to those who do not die, and the other two components would be the same.

> ⚠ Settlement expenses, are part of an insurance company's expenses and are taken into account when computing profit. On the other hand, they must be ignored when computing account values. Account values are computed based on the insurance contract only. The insurance contract never specifies an additional charge for settlement expenses, as these would look illogical to a policyholder. (Imagine a policyholder complaining: Why am I being charged for mortality on 100,100 of face amount if my face amount is 100,000?)

Let's continue Example 30A.

Example 30E In Example 30A, you are also given:

(i) Surrender charges are 5000, 4000, 3000, 2000, and 1000 in years 1–5 respectively.
(ii) The mortality assumption is 100% of the Standard Ultimate Mortality Model.
(iii) The independent surrender rate assumption is that surrenders occur at the rate of 10% in the first year, 5% in years 2–5.
(iv) Pre-contract expenses are 90% of first year premium.
(v) Expenses are 4% of premium in all years except the first.
(vi) The interest assumption is $i = 0.06$.

Calculate the profit vector and profit signature for the first 5 years.

Solution: Keep in mind that mortality, expense, and interest assumptions are not used for the same purpose as COI, expense charges, and interest crediting rate in the contract. Let's repeat the warning.

> Mortality rates, interest rates, and expense rates used in updating the account value are *not* the assumptions that the company uses for pricing the product. While they may have some relationship to what the insurance company expects, they may not. They are simply rates selected by the insurance company to meet its profit objectives and to make its product competitive. *Do not confuse these rates with mortality, interest, and expense assumptions used for profit testing.*

The change in account value is part of the profit. Rather than calculating change in account value, we'll create a column with the beginning account value, which will be income, and a column with the ending account value for the survivors, which will be an expense. The former will earn interest.

Initial expenses are 0.9(4000) = 3600.

In the first year, the initial account value is 0 and the premium is 4000. This earns interest at 6%, or 240.

We denote expected death benefit costs with EDB. They are computed as

$$\text{EDB}_t = q'^{(d)}_{x+t-1}(\text{AV}_t + \text{FA}_t + \text{Expense for death})$$

Note that the death benefit is the total face amount including the account value. The specified death benefit amount is added to the end-of-year account value. This procedure is different from the procedure for updating account values (formula (30.1)). When we updated account values, the COI rate was multiplied by the additional death benefit only (and also discounted). The account value was not multiplied by the probability of survival. Here, when performing a profit test, the expected death benefit is the assumed mortality rate times the full death benefit, and the expected account value (which we will soon discuss) is multiplied by the probability of survival. The two procedures are equivalent, but when we were updating account values, we wanted to derive the updated account value (and therefore didn't want the account value times the probability of survival), whereas when doing a profit test we would like death benefits and reserves per policy in force at the beginning of the year.

In our example, we are assuming no expenses to process a death benefit, and the first year account value is 1755.84, so we multiply 0.001993 by 101,755.84: 0.001993(101,755.84) = 202.80.

We denote the expected surrender benefit costs with ESB. They are computed as

$$\text{ESB}_t = (1 - q'^{(d)}_{x+t-1})q'^{(w)}_{x+t-1}(\text{CV}_t + \text{Expense for surrender})$$

where $q'^{(d)}_{x+t-1}$ is the assumed mortality rate,[7] $q'^{(w)}_{x+t-1}$ is the assumed surrender rate, CV_t is the cash value at the *end* of year t, and expenses incurred in processing a surrender (which may occur even if the cash value is 0) are added. We are assuming no expenses, and in the first year, since the surrender charge is higher than the ending account value of 1755.84, ESB is 0.

[7] All surrenders are assumed to occur at the end of the year, whereas deaths occur during the year. Therefore, $q^{(d)}_{x+t-1} = q'^{(d)}_{x+t-1}$; during the year, there is only one decrement. However, $q^{(w)}_{x+t-1} = (1 - q^{(d)})q'^{(w)}$ since all surrenders occur after all deaths have occurred.

We denote expected account values at the end of the year by EAV. This is computed as:

$$\text{EAV} = \text{AV}_t(1 - q'^{(d)}_{x+t-1})(1 - q'^{(w)}_{x+t-1})$$

In other words, the withdrawals are assumed to occur at the end of the year, after all the deaths occur. We get $(1 - 0.001993)(1 - 0.1)(1755.84) = 1577.11$.

Profit is the sum of the previous account value, premiums, and interest, minus expenses, EDB, ESB, and EAV.

You can easily verify the other four years of the calculation. We'll go through the third year calculation to illustrate a non-zero ESB.

AV_{t-1}: 5389.69
Premium: 4000
Expense: $0.04(4000) = 160$
Interest: $0.06(5389.69 + 4000 - 160) = 268.46$
EDB: $0.002459(109{,}175.60) = 268.46$
ESB: $(1 - 0.002459)(0.05)(9175.60 - 3000) = 308.02$
EAV: $(1 - 0.002459)(1 - 0.05)(9175.60) = 8695.38$
Pr_t: $5389.69 + 4000 - 160 + 553.78 - 268.46 - 308.02 - 8695.38 = 511.61$

t	AV_{t-1}	Premium	Expense	Interest	EDB	ESB	EAV	Pr_t
0			3600					−3600.00
1	0	4000	0	240.00	202.80	0	1577.10	2460.09
2	1755.84	4000	160	335.75	233.12	69.33	5108.88	520.26
3	5089.69	4000	160	553.78	268.46	308.02	8695.38	511.61
4	9175.60	4000	160	780.94	309.49	554.36	12427.58	505.10
5	13117.56	4000	160	1017.45	357.28	808.49	16308.36	500.88

To calculate the profit signature, we multiply Pr_t by the persistency factors. The persistency rate to the second year is $(1-0.001993)(1-0.1) = 0.898206$. The persistency rate to the third year is $0.898206(1-0.002212)(1-0.05) = 0.851408$. And so on.

t	Persistency to $t-1$	Pr_t	Π_t	NPV_t
0		−3600.00	−3600.00	−3600.00
1	1.000000	2460.09	2460.09	−1363.55
2	0.898206	520.26	520.26	−977.36
3	0.851408	511.61	511.61	−650.09
4	0.806849	505.10	505.10	−371.74
5	0.764410	500.88	500.88	−134.00

The NPV's were calculated at a hurdle rate of 10%. The policy does not break even by year 5. □

To handle expenses for payment of death and surrender benefits, add them to the death and surrender benefit when computing EDB and ESB. For example, if there had been an expense of 100 in handling a surrender benefit, even if the surrender benefit is 0, then ESB in the first year in the above example would be $100(0.1)(1-0.001993) = 9.98007$. Surrender expenses are incurred even when the surrender benefit is 0.

Now let's continue Example 30C to see the computations for a policy with fixed death benefit.

Example 30F In Example 30C, you are also given:

(i) Surrender charges are 5000, 4000, 3000, 2000, and 1000 in years 1–5 respectively.
(ii) The mortality assumption is 100% of the Standard Ultimate Mortality Model.
(iii) The surrender assumption is that surrenders occur at the rate of 10% in the first year, 5% in years 2–5.
(iv) Pre-contract expenses are 90% of first year premium.
(v) Expenses are 4% of premium in all years except the first.
(vi) The interest assumption is $i = 0.06$.

Calculate the profit vector and profit signature for the first 5 years.

Solution: All the calculations are the same as in the previous example, except that the expected death benefit is the mortality rate times 100,000. Once again, the account value is not subtracted from 100,000, unlike when calculating the COI. So in year 3:

AV_{t-1}: 5408.47
Premium: 4000
Expense: $0.04(4000) = 160$
Interest: $0.06(5408.47 + 4000 - 160) = 554.91$
EDB: $0.002459(100{,}000) = 245.90$
ESB: $(1 - 0.002459)(0.05)(9222.53 - 3000) = 310.56$
EAV: $(1 - 0.002459)(1 - 0.05)(9222.53) = 8739.86$
Pr_t: $5408.47 + 4000 - 160 + 554.91 - 245.9 - 310.36 - 8739.86 = 507.26$

The following table summarizes the calculation of NPV for 5 years, calculated at 10%.

t	AV_{t-1}	Premium	Expense	Interest	EDB	ESB	EAV	Pr_t	${}_{t-1}p_x$	Π_t	NPV_t
0			3600					−3600.00		−3600.00	−3600.00
1	0	4000	0	240.00	199.30	0	1580.89	2459.81	1.000000	2459.81	−1363.81
2	1760.05	4000	160	336.00	221.20	70.27	5126.68	517.91	0.898206	465.19	−979.36
3	5408.47	4000	160	554.91	245.90	310.36	8739.86	507.26	0.851408	431.89	−654.87
4	9222.53	4000	160	783.75	273.60	558.98	12515.36	498.34	0.806849	402.09	−380.24
5	13210.20	4000	160	1023.01	304.80	816.50	16460.70	491.21	0.764110	375.49	−147.09

Let's once again distinguish between the charges and credits used to compute the account value and the assumptions used to perform a profit test.

For computing account values, use the following:

1. Expense charges
2. Costs of insurance (which are based on a mortality table used specifically for this purpose)
3. Interest credits (and possibly a different interest rate used to discount the COI)

Ignore, among other things, surrender charges, surrender assumptions, surrender expenses, and expenses for settling death claims.

For performing profit tests, use the following:

1. Expense assumptions
2. Assumptions for settlement expenses for surrenders and deaths
3. Mortality assumptions
4. Surrender rate assumptions
5. Interest assumptions
6. Surrender charges

In addition, to calculate total gain or gain by source, use actual experience and compare the results to the assumed results. That means that you should use actual mortality experience, actual surrender experience, actual expenses, and actual investment results to compute actual profit, and then compare that to assumed profit.

Do not confuse account value calculation with profit testing!

Expense charges, COI, and interest credits are used only for calculating account values.

Expense assumptions (including assumptions for settlement expenses), mortality assumptions, and interest assumptions, as well as projected account values (which are computed using expense charges, COI's, and interest credits) are used to compute expected profit in profit tests.

Actual experience is used to calculate actual profit. To compute gain, calculate actual profit minus expected profit.

30.3 Comparison of traditional and universal life insurance

If a level premium is paid on a universal life contract, and level interest is credited, and $i^q = i^c$, then the account value at time n may be calculated using standard insurance and annuity functions, as follows. For the following calculations, the interest assumption is the credited interest rate and the mortality assumption is the COI rate.

1. Calculate the expected present value at issue of future premiums minus expenses.
2. Calculate the expected present value at issue of future COIs. This is the present value of an n-year term insurance.
3. Subtract the EPV of COIs from the EPV of premiums minus expenses.
4. Divide by an n-year pure endowment

Example 30G A universal life contract has the following provisions:

(i) COI rate is based on the Standard Ultimate Mortality Model.
(ii) Interest crediting rate is 5%.
(iii) Expense charges are 20% of premium in the first year and 3% of premium in all other years.
(iv) All charges and credits are performed annually.

A policyholder age 65 purchases a Type A contract with face amount 100,000 and pays 2000 in premium at the beginning of each year.

Determine the account value at the end of 10 years.

Solution: Based on the policy provisions, we want

$$\frac{2000\left(0.97\ddot{a}_{65:\overline{10}|} - 0.17\right) - 100{,}000A^{\;1}_{65:\overline{10}|}}{{}_{10}E_{65}}$$

We look up all the values we need in the Standard Ultimate Mortality Model.

$$\frac{2000\left(0.97(7.8435) - 0.17\right) - 100{,}000(0.62650 - 0.55305)}{0.55305} = \boxed{\mathbf{13{,}618}}$$

□

30.4 Reserves for no-lapse guarantees, and other comments on reserves

We've been assuming the reserve equals the account value. However, due to the surrender charge, perhaps the reserve should be lower. It's a little risky banking on the surrender charge, though.

Consider a policy with an effective no-lapse guarantee. A no-lapse guarantee is "effective" if the policyholder has paid the required premiums and thus is eligible for this guarantee if the account value goes to 0. Suppose the account value is 0. The policy doesn't lapse due to the guarantee, and the insurance company is obligated to pay the death benefit as long as the policyholder continues paying the premium. A reserve of the account value, or 0, would be inadequate. An appropriate reserve for the no-lapse guarantee would be the EPV of future benefits minus the EPV of the premiums needed to qualify for the no-lapse guarantee, minus the account value.

Example 30H A Type A universal life policy of 10,000 is sold to a person age 60. The policy has a no-lapse guarantee that guarantees that the policy will stay in force, even if the account value is 0, if the policyholder pays at least 250 in premium at the beginning of every year.

Policy charges are:

- Expense charge: 4% of premium every year.
- Cost of insurance: Based on 120% of the Standard Ultimate Mortality Model.
- Interest credit: 5% effective.

At time 10, the account value of the policy is 0, and the no-lapse guarantee is effective.

The gross premium reserve is calculated assuming expenses are 3% of premium every year, mortality follows the Standard Ultimate Mortality Model, and interest earned is 5% effective.

Calculate the gross premium reserve at time 10.

Solution: Using the Standard Ultimate Mortality Model, $A_{70} = 0.42818$ and $\ddot{a}_{70} = 12.0083$. The reserve is

$$ {}_{10}V = 10{,}000(0.42818) - 250(0.97)(12.0083) = \boxed{1369.79} $$ □

If the account value is not 0, the reserve for the no-lapse guarantee is the excess (if greater than 0) of the EPV of benefits minus premiums under the no-lapse guarantee over the account value.

Example 30I In the previous example, assume that the account value at time 10 is 1000.

Calculate the reserve for the no-lapse guarantee at time 10 before the premium and charges at time 10 are accounted for.

Solution: The account value is 1000.

In the previous example, we computed the total reserve as 3071.44, so the reserve for the no-lapse guarantee is $1369.79 - 1000.00 = \boxed{369.79}$. □

Exercises

How universal life works

30.1. For a universal life policy on (45) with death benefit of 100,000 plus the account value:

(i) Expense charges are 35% of premium in the first year and 10% of premium in the second year.
(ii) Cost of insurance is based on $q_{45} = 0.01$ and $q_{46} = 0.012$, with benefit payments assumed to occur at the end of the year. Cost of insurance is discounted at 5%.
(iii) Surrender charges are 5000 in year 1 and 3000 in year 2.
(iv) The insurer credits interest of 5% each year.
(v) The policyholder pays a premium of 4000 each year.

Calculate the cash value at the end of the second year for this policyholder.

Exercises continue on the next page . . .

30.2. For a Type A universal life policy of 50,000 on (50):

(i) Expense charges are 35% of premium plus 100 in the first year and 5% of premium plus 50 in renewal years.
(ii) Cost of insurance is based on $l_x = 100 - x$, with benefit payments assumed to occur at the end of the year.
(iii) The account value at the beginning of the sixth year, before any premium is paid, is 8,500.
(iv) The policyholder pays a premium of 1000 at the beginning of the sixth year.
(v) The insurer credits 5% interest in the sixth year.

Calculate the account value at the end of the sixth year.

30.3. For a Type A universal life policy of 100,000 on (50), you are given:

(i) At issue, the policyholder pays a premium of 10,000.
(ii) Expense charges in the first year are 500.
(iii) Interest in the first year is credited at 5%.
(iv) The cost of insurance is based on Makeham mortality with $A = 0.001$, $B = 0.0001$, and $c = 1.05$.
(v) The cost of insurance is discounted at 3%.

Calculate the account value at the end of the first year.

30.4. For a Type B universal life policy of 200,000 on (55):

(i) The account value at the beginning of the ninth year is 45,000.
(ii) Expense charges are 35% of first year premiums and 10% of renewal premiums.
(iii) The cost of insurance in the ninth year is based on $q_{63} = 0.01$. Death benefits are assumed to be paid at the end of the year.
(iv) Premium of 5000 is paid at the beginning of the ninth year.
(v) The account value at the end of the ninth year is 49,480.

Determine the interest crediting rate.

30.5. For a Type B universal life policy of 100,000 on (40):

(i) Expense charges are 6% of premium plus 100.
(ii) Cost of insurance is based on 120% of the mortality rate computed using $\mu_x = 0.005(1.01^x)$, with benefits paid at the middle of the year.
(iii) Interest credited is 4.5% every year.
(iv) The policyholder paid premiums of 1000 at time 8 and 500 at time 9.
(v) The account value at time 10 is 12,000.

Determine the account value at time 8.

Exercises continue on the next page . . .

30.6. [**Based on old sample question**] For two universal life insurance policies issued on (60), you are given:

(i) Policy 1 is a Type A Universal Life with face amount 100,000.
(ii) Policy 1 is a Type B Universal Life with face amount 100,000.

For each policy:

(i) Death benefits are paid at the end of the month of death.
(ii) Account values are calculated monthly.
(iii) Level monthly premiums of G are payable at the beginning of each month. Past premiums may have been different from G, and may not have been the same for both policies.
(iv) Mortality rates for calculating the cost of insurance:
   a. Follow the Standard Ultimate Mortality Model.
   b. Assume UDD for fractional ages.
(v) Interest is credited at a monthly effective rate of 0.004.
(vi) The interest rate used for accumulating and discounting in the cost of insurance calculation is a monthly effective rate of 0.004.
(vii) Level expense charges of E are deducted at the beginning of each month.

At the end of the 36$^{\text{th}}$ month the account value for Policy 1 equals the account value for Policy 2.

Calculate the ratio of the account value for Policy 1 at the end of the 37$^{\text{th}}$ month to the account value of Policy 2 at the end of the 37$^{\text{th}}$ month.

30.7. [**Based on old sample question**] For a Type A universal life insurance of 100,000 on (50) you are given:

(i) Death benefits are paid at the end of the year of death if (50) dies prior to age 70.
(ii) The account value is calculated annually.
(iii) Level annual premiums are payable at the beginning of each year.
(iv) Mortality rates for calculating the cost of insurance follow the Standard Ultimate Mortality Model.
(v) Interest is credited at an annual effective rate of 0.05.
(vi) The interest rate used for accumulating and discounting in the cost of insurance calculation is an annual effective rate of 0.05.
(vii) Expense deductions are:
   - 50 at the beginning of each year; and
   - 5% of each annual contribution.

Calculate the level annual premium that results in an account value of 0 at the end of the 20$^{\text{th}}$ year.

30.8. For a Type A universal life policy of 100,000 on (35):

(i) The account value is at time 10 is 40,000.
(ii) A premium of 10,000 is paid.
(iii) Expense charges are 500.

The policy credits interest of 5% in the eleventh year.

The cost of insurance in the eleventh year is based on the mortality rate of the Standard Ultimate Mortality Model.

The corridor factor at age 46 is 2.12.

Calculate the cost of insurance for the eleventh year.

Exercises continue on the next page . . .

30.9. For a Type A universal life policy of 50,000 on (40), you are given:

(i) The account value at time 19 is 25,000.
(ii) There are no expense charges for the 20th year.
(iii) Interest in the 20th year is credited at 6%.
(iv) The cost of insurance in the 20th year is based on $q_{59} = 0.01$.
(v) The corridor factor at age 60 is 1.3.

Calculate the highest premium that can be paid without increasing the death benefit due to the corridor factor.

30.10. A universal life policy is sold to a person age 45. The death benefit is 50,000. You are given the following information concerning charges and credits:

(i) 5% of premium is charged at the beginning of each year.
(ii) The cost of insurance rate for age 49 is $q_{49} = 0.00546$.
(iii) Interest is credited at 5% effective.
(iv) The account value is updated annually.

The policyholder contributes 1000 at the beginning of the fifth year. At the end of the fifth year, the account value is 6134.

Determine the account value at the end of the fourth year.

(A) 5120 (B) 5150 (C) 5180 (D) 5210 (E) 5240

30.11. For a Type A universal life policy with death benefit 100,000 on (60):

(i) $\mathrm{AV}_{10} = 12{,}000$
(ii) Premium of 2000 is paid at the beginning of the eleventh year.
(iii) The expense charge at the beginning of the eleventh year is 100.
(iv) Interest is credited at 0.05 effective.
(v) The cost of insurance is 913.47.
(vi) The cost of insurance is computed with discounting at $i_q = 0.04$.

Determine the cost of insurance rate q_{70}.

(A) 0.0100 (B) 0.0105 (C) 0.0110 (D) 0.0115 (E) 0.0125

30.12. For a Type A universal life policy on (50) with death benefit 10,000:

(i) $\mathrm{AV}_{19} = 8000$
(ii) Expense charges are 3% of premium.
(iii) The cost of insurance rate in the 21st year is 0.015.
(iv) Interest is credited at 0.05 effective, and the COI is discounted at the same rate.
(v) The account value is updated annually.
(vi) The corridor factor at age 70 is 115%

Determine the maximum premium that can be paid at the beginning of the 20th year without forcing an increase in the death benefit due to the corridor.

(A) 300 (B) 309 (C) 318 (D) 327 (E) 336

Exercises continue on the next page . . .

30.13. For a Type A universal life policy on (30) with death benefit 50,000:

(i) $\text{AV}_{10} = 20{,}000$
(ii) Expense charges are 100 plus 3% of premium.
(iii) The cost of insurance rate in the 10th year is 0.005.
(iv) Interest is credited at 0.05 effective.
(v) The cost of insurance is discounted at $i_q = 0.03$.
(vi) The account value is updated annually.
(vii) The corridor factor at age 40 is 250%.
(viii) A premium of 5000 is paid at the beginning of the 10th year.

Determine AV_{10}.

(A) 25,600 (B) 25,700 (C) 25,800 (D) 25,900 (E) 26,000

30.14. For a Type A universal life policy on (x) with death benefit 100,000, you are given the following charges and credits for the tenth year:

	Current	Guaranteed
Expense	3% of premium plus 50	5% of premium plus 100
Interest	0.05	0.03
Mortality q_{x+9}	0.01	0.02

The account value at the beginning of the tenth year is 8250. A premium of 1000 is paid at the beginning of the tenth year.

Determine the absolute difference at the end of the tenth year between the guaranteed account value and the account value based on current charges and credits.

(A) 1088 (B) 1152 (C) 1192 (D) 1256 (E) 1298

30.15. For a Type A universal life policy of 100,000 on (35):

(i) The cost of insurance rate is based on $\mu_x = 0.0002(1.05^x)$.
(ii) Interest is credited at 0.05 effective.
(iii) $\text{AV}_{10} = 30,000$.
(iv) No premium is paid, and no expense is charged, in the 11th year.

Determine the cost of insurance for the 11th year.

(A) 120.25 (B) 125.25 (C) 130.25 (D) 135.25 (E) 140.25

Exercises continue on the next page . . .

30.16. For a Type A universal life policy with death benefit 100,000 on (x):

(i) The cost of insurance rate is $q_x = 0.01$ for all x.
(ii) Interest is credited at 0.06 effective.
(iii) The cost of insurance is discounted at $i = 0.04$.
(iv) Expense charges in the first year are 20% of premium plus 300.
(v) Expense charges in all years other than the first are 5% of premium plus 100.
(vi) Expense charges are paid at the beginning of the year.
(vii) The account value is updated annually.
(viii) The policyholder pays P at the beginning of each of the first five years.
(ix) $\mathrm{AV}_5 = 10{,}000$.

Determine P.

(A) 2900 (B) 2920 (C) 2940 (D) 2960 (E) 2980

30.17. For a Type A universal life policy of 10,000 on (50):

(i) The account value at the start of the fifth year is 5000.
(ii) The policyholder pays a premium of 2000 at the start of the year.
(iii) Annual expense charges are 200.
(iv) The cost of insurance rate is $q_{54} = 0.01$.
(v) The corridor factor at age 55 is 150%.
(vi) $i^c = i^q = 0.05$
(vii) The account value is updated annually.

(a) Calculate the cost of insurance in year 5.

(b) If there were no corridor, would the cost of insurance be higher or lower? Why?

30.18. [**Worksheet question**] The workbook can be downloaded here.

For a Type A universal life insurance on a person aged 50 with face amount 50,000, the contract provisions are:

(i) Account value is updated monthly.
(ii) Interest crediting rate is $i^{(12)} = 0.048$ each year.
(iii) Monthly cost of insurance rates are shown in the spreadsheet.
(iv) Expense charge is 25% of premium up to 2000 and 5% of the excess over 2000 in the first year, 5% of premium in renewal years.

Corridor percentages for each month are shown in the spreadsheet.

The policyholder pays 5000 at the beginning of each year.

Calculate the account values and the sum insured for each month of the first 10 years.

Exercises continue on the next page . . .

Profit tests

30.19. For a Type B universal life policy of 200,000 on (50), you are given the following table of charges and profit test assumptions for the 11th year:

Expense charge	10% of premium plus 100	Expense assumption	5% of premium plus 100
Cost of insurance	Based on $q_{60} = 0.008$	Mortality assumption	$q_{60} = 0.006$
Interest credited	0.05	Interest earned	0.055

The cost of insurance and the profit test assume death benefits are paid at the end of the year. There is no surrender charge.

Let $\Pr_{11}$ be the profit per policy in force if a policyholder pays a premium of x, and $\Pr^*_{11}$ be the profit per policy in force if a policyholder pays a premium of $x + 1000$.

Determine $\Pr^*_{11} - \Pr_{11}$.

30.20. A profit test is conducted for a Type A universal life policy of 100,000 on (55).

(i) You are given the following charges for the sixth year:
  (a) Expense charge: 8% of premium plus 100.
  (b) Cost of insurance is based on a mortality rate of 0.008, with benefits paid at the end of the year.
  (c) Surrender charge is 1000.
  (d) Interest is credited at 5%.

(ii) You are given the following assumptions for the sixth year:
  (a) Expenses paid at beginning of year: 5% of premium plus 50.
  (b) Mortality rate is 0.005.
  (c) Surrender rate is 0.07.
  (d) Cost of processing a death claim is 500.
  (e) Cost of processing a surrender (even if cash value is zero) is 100.
  (f) Interest is earned at 5%.

(iii) The account value at the beginning of the sixth year is 9,500.

(iv) The policyholder pays a premium of 1500 at the beginning of the sixth year.

Calculate $\Pr_6$, the profit in year 6.

Exercises continue on the next page . . .

30.21. A profit test is conducted for a Type B universal life policy of 100,000 on (60).

(i) You are given the following charges for the sixth year:
 (a) Expense charge: 5% of premium plus 50.
 (b) Cost of insurance is based on 105% of the Standard Ultimate Mortality Model.
 (c) Surrender charge is 1000.
 (d) Interest is credited at 5%.
(ii) You are given the following assumptions for the sixth year:
 (a) Expenses paid at beginning of year: 5% of premium plus 50.
 (b) Mortality rate is 100% of the Standard Ultimate Mortality Model.
 (c) Surrender rate is 10% in the first year, 6% in renewal years.
 (d) Cost of processing a death claim is 200.
 (e) Cost of processing a surrender (even if cash value is zero) is 50.
 (f) Interest is earned at 5.5%.
(iii) The account value at the beginning of the sixth year is 7,500.
(iv) The policyholder pays a premium of 2000 at the beginning of the sixth year.

Calculate Π_6, the profit signature in year 6.

30.22. A profit test is conducted for a Type B universal life policy of 50,000 on (35).

(i) Expense charges are 5% of premium plus 250.
(ii) Cost of insurance is based on $q_{35+t} = 0.005 + 0.001t$.
(iii) There are no surrender charges.
(iv) Interest is credited at 5% in the 11$^{\text{th}}$ year and 4.5% in the 12$^{\text{th}}$ year.
(v) Renewal expenses are 4% of premium plus 100.
(vi) Mortality rate is $q_{35+t} = 0.002 + 0.001t$.
(vii) Surrender rate is 4% each year.
(viii) Cost of processing a death claim is 200.
(ix) Cost of processing a surrender is 100.
(x) Interest of 5% is earned in the 11$^{\text{th}}$ and 12$^{\text{th}}$ years.
(xi) Death benefits are assumed to be paid at the end of the year.
(xii) The account value at the beginning of the 11$^{\text{th}}$ year is 2000.
(xiii) The policyholder pays no premium in the 11$^{\text{th}}$ and 12$^{\text{th}}$ years.

Calculate the profit in year 12 per policy in force at the beginning of the year.

30.23. For a Type B universal life policy of 50,000 on (40):

(i) $\text{AV}_4 = 10{,}000$.
(ii) $\text{AV}_5 = 12{,}000$.
(iii) Premium of 1800 is paid at the beginning of the fifth year.
(iv) Expenses of 100 are paid at the beginning of the fifth year.
(v) Assumed mortality rate for the fifth year is 0.005.
(vi) Assumed surrender rate for the fifth year is 0.05.
(vii) There are no settlement expenses.
(viii) Profit in the fifth year per policy in force at the beginning of the year is 93.5.

Determine the interest assumption for the fifth year.

(A) 0.05 (B) 0.0525 (C) 0.055 (D) 0.0575 (E) 0.06

Exercises continue on the next page . . .

Reserves for no-lapse guarantees

30.24. For a Type A universal life policy with death benefit of 100,000 on (50) with a no lapse guarantee, the guarantee assures that the policy will not lapse if the policyholder pays an annual premium of at least 1000 every year. You are given

(i) At time 5, the account value is 3000.
(ii) Reserve basis assumptions are:
- Mortality: Standard Ultimate Mortality Model
- Interest: 5%

(iii) The cost of insurance rate at age 55 is 0.01.
(iv) The expense charge in year 6 is 50.
(v) The interest crediting rate in year 6 is 4%.
(vi) Account values are updated annually.
(vii) The policyholder has paid premiums of at least 700 in the first 5 years.
(viii) The policyholder pays 700 at the beginning of the sixth year.

The reserve for the no-lapse guarantee is the excess of the reserve for the guaranteed death benefits, computed using reserve basis assumptions and taking into account the premiums required to maintain the no-lapse guarantee, over the account value.

Calculate the reserve for the no-lapse guarantee at the beginning of the 6th year after premiums have been paid and charges for expenses and COI have been made.

Additional old SOA Exam ALTAM questions: S23:3, F23:5, F24:5

Written answer sample questions: 49, 50, 51, 52, 53, 54

Solutions

30.1. Expenses are 0.35(4000) = 1400 in year 1 and 0.1(4000) = 400 in year 2.
Cost of insurance is 0.01(100,000)/1.05 = 952.381 in year 1 and 0.012(100,000)/1.05 = 1142.857 in year 2.
Interest is 0.05(4000 − 1400 − 952.381) = 82.381 in year 1 and 0.05(1730 + 4000 − 400 − 1142.857) = 209.357 in year 2.

Year t	AV_{t-1}	Premium	Expense	COI	Interest	AV_t
1	0.00	4000	1400	952.381	82.381	1730.00
2	1730.00	4000	400	1142.857	209.357	4396.50

The cash value at the end of the second year is 4396.50 − 3000 = **1396.50**.

30.2. The account value is

$$\mathrm{AV}_6 = \frac{(8500 + 1000 - 100)(1.05) - 50{,}000/45}{44/45} = \boxed{8957.95}$$

As a check, the COI is $(50{,}000 - 8957.95)/\bigl(45(1.05)\bigr) = 868.61$, and the account value is $(8500 + 1000 - 100 - 868.61)(1.05) = 8957.95$.

30.3. The mortality rate for the first year is

$$q_{50} = 1 - \exp\left(-0.001 - 0.0001(1.05^{50})\left(\frac{0.05}{\ln 1.05}\right)\right) = 0.00217281$$

By formula (30.6), the end-of-year account value is

$$\mathrm{AV}_1 = \frac{(10{,}000 - 500 - 0.00217281(100{,}000)/1.03)(1.05)}{1 - 0.00217281(1.05/1.03)} = \boxed{\mathbf{9775.15}}$$

30.4. Premium minus expense charges is $5000 - 500 = 4500$. The cost of insurance is $0.01(200{,}000)/(1+i) = 2000/(1+i)$. Interest is credited to the account balance plus premiums minus expense charges and cost of insurance. So we solve the following for i:

$$\left(45{,}000 + 4500 - \frac{2000}{1+i}\right)(1+i) = 49{,}480$$
$$49{,}500(1+i) - 2000 = 49{,}480$$
$$1 + i = \frac{51{,}480}{49{,}500} = 1.04$$
$$i = \boxed{\mathbf{0.04}}$$

30.5. Let's work out the cost of insurance. Before the 1.2 factor,

$$p_{48} = \exp\left(-\frac{0.005(1.01^{48})(1.01-1)}{\ln 1.01}\right) = 0.9919314$$
$$p_{49} = \exp\left(-\frac{0.005(1.01^{49})(1.01-1)}{\ln 1.01}\right) = 0.9918510$$

Cost of insurance is calculated by dividing mortality by half a year's interest, since death benefits are assumed to be paid in the middle of the year.

$$\mathrm{COI}_{48} = 100{,}000\left(\frac{1.2(1-0.9919314)}{1.045^{0.5}}\right) = 947.16$$
$$\mathrm{COI}_{49} = 100{,}000\left(\frac{1.2(1-0.9918510)}{1.045^{0.5}}\right) = 956.59$$

Expense charges are $100 + 0.06(1000) = 160$ in the ninth year and $100 + 0.06(500) = 130$ in the tenth year. We're ready to work backwards.

$$\mathrm{AV}_9 = \frac{12{,}000}{1.045} + 956.59 + 130 - 500 = 12{,}069.847$$
$$\mathrm{AV}_8 = \frac{12{,}069.85}{1.045} + 947.16 + 160 - 1000 = \boxed{\mathbf{11{,}657.253}}$$

30.6. Compare the recursive formulas for account value for Type A (fixed death benefit) and Type B (specified amount plus account value) policies. The formula for Type A divides by $1-q$, where q is the mortality rate for the period, while the formula for Type B doesn't; otherwise, the formulas are the same. Policy 1 is Type A and Policy 2 is Type B, so the ratio for a one-month recursion is $1/(1 - {}_{1/12}q_{63})$, which under UDD is $1/(1-\frac{1}{12}(q_{63})) = 1/(1-0.00473/12) = \boxed{\mathbf{1.000394}}$.

30.7. Since premiums and death benefits are level, interest is fixed at 6%, and the ending account value is 0, this policy is equivalent to a 20-year term policy at 5%. We evaluate the premium after expenses with the Standard Ultimate Mortality Model.

$$A^{1}_{50:\overline{20|}} = 0.38844 - 0.34824 = 0.04020$$
$$\ddot{a}_{50:\overline{20|}} = 14.8428$$
$$P = \frac{100{,}000(0.04020)}{14.8428} = 313.02$$

This net premium is after the expense charges, so we must gross it up. $P = 0.95G - 50$, so $G = (P+50)/0.95 = \boxed{\mathbf{382.12}}$.

30.8. If AV_{11} is calculated ignoring the corridor, we get, with $q_{45} = 0.000771$,

$$\mathrm{AV}_{11} = \frac{(40{,}000 + 10{,}000 - 500)(1.05) - 0.004(100{,}000)}{1 - 0.000771} = 51{,}937.94$$

and the death benefit is less than 2.12 times the account value. With the corridor,

$$\begin{aligned}\mathrm{AV}_{11} &= 49{,}500(1.05) - 0.000771(1.12\,\mathrm{AV}_{11})\\ \mathrm{AV}_{11} &= \frac{51{,}975}{1 + 0.000771(1.12)} = 51{,}930.16\end{aligned}$$

and the COI is $0.000771(51{,}930.16)(1.12)/1.05 = \boxed{42.71}$. The death benefit for the eleventh year, which is not needed for this exercise, is $2.12(51{,}930.16) = 110{,}092$.

30.9. The highest account value before triggering the corridor is $50{,}000/1.3 = 38{,}461.54$. The account value at time 20 is

$$\begin{aligned}\mathrm{AV}_{20} &= \frac{(25000 + P)(1.06) - 0.01(50{,}000)}{1 - 0.01}\\ 38{,}461.54(0.99) &= (25{,}000 + P)(1.06) - 500\\ 38{,}576.92 &= (25{,}000 + P)(1.06)\\ P &= \frac{38{,}576.92}{1.06} - 25{,}000 = \boxed{11{,}393.33}\end{aligned}$$

30.10. Using the formula relating account values,

$$\begin{aligned}\mathrm{AV}_5 &= \frac{(\mathrm{AV}_4 + P - E)(1 + i) - q_{49}FA}{1 - q_{49}}\\ 6134 &= \frac{\big(\mathrm{AV}_4 + 1000(0.95)\big)(1.05) - 50{,}000(0.00546)}{1 - 0.00546}\\ \mathrm{AV}_4 &= \frac{6134(1 - 0.00546) + 50{,}000(0.00546)}{1.05} - 950 = \boxed{5120} \qquad \textbf{(A)}\end{aligned}$$

30.11. Use formula (30.7).

$$\begin{aligned}\mathrm{COI}_t &= \frac{v_q\, q_{x+t-1}\big(\mathrm{FA} - (\mathrm{AV}_{t-1} + P_t - E_t)(1 + i)\big)}{1 - v_q\, q_{x+t-1}(1 + i)}\\ 913.47 &= \frac{q_{70}\big(100{,}000 - 13{,}900(1.05)\big)/1.04}{1 - 1.05q_{70}/1.04}\\ 913.47 - 913.47\left(\frac{1.05}{1.04}\right)q_{70} &= 82{,}120.19q_{70}\\ q_{70} &= \frac{913.47}{83{,}042.45} = \boxed{0.011} \qquad \textbf{(C)}\end{aligned}$$

30.12. The maximum account value is $10{,}000/1.15 = 8695.65$. The premium generating this account value is solved from

$$\begin{aligned}\frac{(\mathrm{AV}_{19} + 0.97P)(1 + i) - q_{69}FA}{1 - q_{69}} &= 8695.65\\ \frac{(8000 + 0.97P)(1.05) - 150}{0.985} &= 8695.65\\ 8000 + 0.97P = \frac{8695.65(0.985) + 150}{1.05} &= 8300.207\\ P = \frac{300.207}{0.97} &= \boxed{309.49} \qquad \textbf{(B)}\end{aligned}$$

30.13. Obviously the corridor will be hit. By formula (30.10),

$$\begin{aligned}\mathrm{AV}_{10} &= \frac{(\mathrm{AV}_9 + P - E)(1+i)}{1 + v_q q_{39}(1+i)(1.5)} \\ &= \frac{24{,}750(1.05)}{1 + 0.005(1.05)(1.5)/1.03} = \boxed{25{,}790} \qquad \textbf{(C)}\end{aligned}$$

30.14. With current charges:

$$\mathrm{AV}_{10} = \frac{\big(8250 + 0.97(1000) - 50\big)(1.05) - 0.01(100{,}000)}{0.99} = 8715.66$$

With guaranteed charges:

$$\mathrm{AV}_{10} = \frac{\big(8250 + 0.95(1000) - 100\big)(1.03) - 0.02(100{,}000)}{0.98} = 7523.47$$

The difference is $\boxed{1192.19}$. **(C)**

30.15. First we derive q_{45}.

$$p_{45} = \exp\left(-Bc^{45}\left(\frac{c-1}{\ln c}\right)\right) = \exp\big(-0.0002(1.05^{45})(0.05)/\ln 1.05\big) = 0.998160$$

so $q_{45} = 0.001840$. The account value at time 11 is

$$\mathrm{AV}_{11} = \frac{30{,}000(1.05) - 0.001840(100{,}000)}{1 - 0.001840} = 31{,}373.74$$

and the COI is

$$\frac{0.001840(100{,}000 - 31{,}374.74)}{1.05} = \boxed{120.25} \qquad \textbf{(A)}$$

30.16. Since the cost of insurance is discounted at a different rate from the accumulation rate, effectively $q_x = 0.01(1.06/1.04) = 0.010192$. We use this rate for all calculations.

We will equate the present value of AV_5 to the sum of the present value of the premiums after percent-of-premium expense charges, minus the present value of insurance benefits and per policy expense charges. First, a 5-year pure endowment is

$${}_5E_x = \left(\frac{1 - 0.010192}{1.06}\right)^5 = 0.709945$$

A 5-year term insurance has EPV

$$\frac{q}{q+i}(1 - {}_5E_x) = \frac{0.010192}{0.070192}(1 - 0.709945) = 0.0421176$$

A 5-year temporary life annuity has EPV

$$\frac{1+i}{q+i}(1 - {}_5E_x) = \frac{1.06}{0.070192}(1 - 0.709945) = 4.380226$$

The present value of insurance benefits is 4211.76. The present value of per policy expense charges is $100(4.380226) + 200 = 638.0226$. The present value of 10,000 is 7099.45. The present value of 1 per year of premium after per-premium expenses is $0.95(4.380226) - 0.15 = 4.011214$. So the equation for P is

$$4.011214P - 4211.76 - 638.0226 = 7099.45$$

so $P = \boxed{2978.96}$. **(E)**

30.17.

(a) Ignoring the corridor,

$$\mathrm{AV}_5 = \frac{(5000+2000-200)(1.05)-100}{0.99} = 7111.11$$

but this violates the corridor. Using the corridor, so that the death benefit is 1.5 AV_5:

$$6800(1.05) - 0.01(1.5\,\mathrm{AV}_5) = 0.99AV_5$$

so $AV_5 = (6800)(1.05)/(0.99+0.015) = 7104.48$. Then the death benefit is 1.5(7104.48) = 10,656.72 and the COI is

$$\frac{(10{,}656.72-7104.48)(0.01)}{1.05} = \boxed{\mathbf{33.8308}}$$

(b) Ignoring the corridor would result in a lower death benefit, hence a lower cost of insurance.

30.18. The workbook solution can be downloaded here.

We use formula (30.5) for account values ignoring corridor and formula (30.9) to consider the corridor. The account value is the lower of the two calculations. The sum insured is the maximum of 50,000 and the account value multiplied by the corridor factor.

30.19. The account value is 1000 higher due to the extra premium, 100 lower due to the extra expense, and this is raised at 5% interest, so the account value is 900(1.05) = 945 higher.

Now for profits. There is an additional 1000 in profit from premium and an additional 50 in expense. There is additional interest of 0.055(1000 − 50) = 52.25. The account value is provided for everyone, whether they die, surrender, or persist, and that is an extra expense of 945. So the increase in profit is $1000-50+52.25-945 = \boxed{\mathbf{57.25}}$.

30.20. First we calculate AV_6. Expense charge is 0.08(1500) + 100 = 220. By equation (30.5),

$$\mathrm{AV}_6 = \frac{(9500+1500-220)(1.05)-0.008(100{,}000)}{1-0.008} = 10{,}603.83$$

Then the cost of insurance is 0.008(100,000−10,603.83)/1.05 = 681.11, but we don't need this for the profit calculation.

Now for profits. Expenses are 0.05(1500) + 50 = 125. Interest is 0.05(9500 + 1500 − 125) = 543.75. Expected death benefits, including processing costs, are 0.005(100,000 + 500) = 502.50. Expected surrender benefits, after the surrender charge and processing costs and based on AV_6, are (1 − 0.005)(0.07)(10,603.83 − 1000 + 100) = 675.87. Expected account value for survivors is (1 − 0.005)(1 − 0.07)(10,603.83) = 9812.25. Profit is

$$\mathrm{Pr}_6 = 9500+1500+543.75-125-502.50-675.87-9812.25 = \boxed{\mathbf{428.13}}$$

30.21. Let's project AV_6. Expense charge is 0.05(2000) + 50 = 150. Mortality rate $q_{65} = 0.005915$ in the Standard Ultimate Mortality Model, so cost of insurance is 1.05(0.005915)(100,000)/1.05 = 591.50. Interest credited is 0.05(7500 + 2000 − 150 − 591.50) = 437.93

$$\mathrm{AV}_6 = 7500+2000-150-591.50+437.93 = 9196.43$$

Now for the profit calculation.

- Expenses are 150.
- Interest earned is 0.055(7500 + 2000 − 150) = 514.25.
- Expected death benefits including processing costs are 0.005915(100,000 + 9196.43 + 200) = 647.08.
- Expected surrender benefits including processing costs are (1 − 0.005915)(0.06)(9196.43 − 1000 + 50) = 491.86.
- Expected account value for survivors is 9196.43(1 − 0.005915)(1 − 0.06) = 8593.51.

$$\mathrm{Pr}_6 = 7500+2000-150+514.25-647.08-491.86-8593.51 = 131.80$$

To convert this to a per issue basis, multiply it by ${}_5p_{60}$. The probability of survival from death is l_{65}/l_{60}, and the probability of not withdrawing is $0.9(0.94^4)$.

$$\Pi_6 = 131.80\left(\frac{94{,}479.7}{96{,}634.1}\right)(0.9)(0.94^4) = \boxed{\mathbf{90.65}}$$

30.22. We need to project the account value for 2 years, but to calculate profit only in the second year.

Expense charges each year are 250, due to lack of premium. Mortality rates for the charges are 0.015 and 0.016, so cost of insurance is 0.015(50,000)/1.05 = 714.29 and 0.016(50,000)/1.045 = 765.55. Then

$$\mathrm{AV}_{11} = 1.05(2000 - 250 - 714.29) = 1087.50$$
$$\mathrm{AV}_{12} = 1.045(1087.50 - 250 - 765.55) = 75.1875$$

For profit in year 12,

- Expenses are 100.
- Interest is 0.05(1087.50 − 100) = 49.375.
- Expected death benefits are 0.013(50,000 + 75.1875 + 200) = 653.58.
- Expected surrender benefits are (1 − 0.013)(0.04)(75.1875 + 100) = 6.92.
- Expected account value for survivors is 75.1875(1 − 0.013)(1 − 0.04) = 71.24.

Profit per policy in force at time 11 is

$$\mathrm{Pr}_{12} = 1087.50 + 49.375 - 100 - 653.58 - 6.92 - 71.24 = \boxed{305.14}$$

The large expense and mortality margins lead to a high profit.

30.23. Surrenders and survivors both get the account value, so no need to separate surrenders from survivors.

$$(\mathrm{AV}_4 + P - E)(1+i) - q_{44}(FA + \mathrm{AV}_5) - (1 - q_{44})\,\mathrm{AV}_5 = 93.5$$
$$(10{,}000 + 1800 - 100)(1+i) = 93.5 + 0.005(62{,}000) + 0.995(12{,}000) = 12{,}343.5$$
$$1 + i = \frac{12{,}343.5}{11{,}700} = 1.055$$
$$i = \boxed{0.055} \qquad \textbf{(C)}$$

30.24. The reserve for a 10-year term insurance on (55), taking into account future premiums but not the premium just paid, is

$$100{,}000A_55 - 1000(\ddot{a}_{55} - 1) = 100{,}000(0.23524) - 1000(16.0599 - 1) = 8464.10$$

The COI for the sixth year, using formula (30.8), is

$$\mathrm{COI}_6 = \frac{(0.01/1.04)\big(100{,}000 - (3000 + 700 - 50)(1.04)\big)}{(1 - 0.01)} = 934.38$$

The account value after premiums and charges is

$$3000 + 700 - 50 - 934.38 = 2715.62$$

The reserve for the no-lapse guarantee is 8464.10 − 2715.62 = $\boxed{5748.48}$.

Quiz Solutions

30-1. The Standard Ultimate Mortality Model has $q_{50} = 0.001209$. The COI is 10,000(0.0001209)(1.1)/1.05 = 12.67. Taking the surrender charge into account, we need the account value to be at least 100. Let P be the premium. Then

$$(125 + 0.98P - 50 - 12.67)(1.05) = 100$$
$$62.33 + 0.98P = \frac{100}{1.05} = 95.238$$
$$P = \boxed{33.58}$$

30-2. The cost of insurance rate times the net amount of risk discounted with interest is 800, so the cost of insurance rate is

$$\frac{800(1.05)}{50{,}000} = \boxed{\mathbf{0.0168}}$$

30-3. $q_{50} = 1/50 = 0.02$. Using formula (30.6),

$$\mathrm{AV}_1 = \frac{\big(40{,}000 - 5{,}000 - 0.02(100{,}000/1.04)\big)(1.06)}{1 - 0.02(1.06/1.04)} = \boxed{\mathbf{35{,}791.13}}$$

Part VII

Practice Exams

Here are 7 practice exams to help you test your knowledge, and to pinpoint areas you are weak in so you will know what to review.

The actual exam will have 60 points of questions and will allow 3 hours. There will be one worksheet questions and five other questions. The first question is a worksheet question. Thus the questions are not in order of topic nor in order of difficulty. The exams are not in order of difficulty either.

The worksheet questions on the practice exam attempt to imitate the exam style in length, worksheet formatting, style, and difficulty. You can download the worksheet file by clicking the link or accessing them through Actuarial University.

On the actual exam the solution to the worksheet question should be entered on the worksheet. This appears to imply that worksheet questions will not involve derivations or proofs of formulas, since it is rather tedious to write formulas in Excel; I wonder whether the worksheet program you get on an exam even has an equation editor. Nevertheless, a part of one of the worksheet questions provided here requires derivation of a formula, and of course you should do it on paper. While that part of the question would not be realistic for an exam, it will give you some practice working with the concepts.

Have fun working out these exams!

Practice Exam 1

1. [**Worksheet question**] (13 points) The workbook can be downloaded here.

For a fully discrete whole life insurance policy on (50) with face amount 100,000:

(i) Reserves are computed using the full preliminary term method, with mortality equal to the Standard Ultimate Mortality Model and $i = 0.04$.
(ii) Gross premium is 1200.
(iii) The assumed independent mortality rate is 80% of the Standard Ultimate Mortality Model.
(iv) The assumed independent surrender rate is 10% in the first year, 4% in other years. All surrenders occur at the end of the year.
(v) The cash surrender value equals the reserve.
(vi) The profit test interest rate is 0.06.
(vii) Expenses are 90% of gross premium in the first year, 6% of gross premium in years 2–10, and 3% of gross premium in year 11 and later.
(viii) 80% of first year premium is considered a precontract expense.

A profit test is performed. Only the first 30 years are included in the profit test.

(a) (2 points) The full preliminary term premium is 1388.22575. Calculate the reserves at times 1–30 using forward recursion, starting at time 0. You should get ${}_{30}V = 52{,}714$.

(b) (3 points) Calculate profits and the profit signature for years 0–30. You should get $\Pi_{30} = 256$ to the nearest integer.

(c) (2 points) The risk discount rate is 10%.

(i) Calculate the NPV.
(ii) Calculate the profit margin.

(d) (2 points) The reserve at time 29 is zeroized with respect to profit in year 30.

Calculate the revised NPV.

(e) (2 points) For a portfolio of 1000 policyholders who own this policy at time 4, 1 dies and 30 surrender the policy in the fifth year. Expenses are 80,000. The company earns 5.8% on its investment portfolio.

Calculate the gain in year 5 for this block of business.

(f) (2 points) Suppose that the cash surrender value at time t was $0.1t$ times the reserve for $t < 10$, and equal to the reserve for $t \geq 10$. Assume this change has no effect on surrender rates. Without doing calculations:

(i) State with reasons whether this change would increase the NPV, decrease the NPV, or have no effect on NPV.
(ii) State with reasons whether this change would increase the gain in year 5, decrease the gain in year 5, or have no effect on the gain in year 5.

2. (10 points) A disability insurance is modeled with a 4-state Markov chain having the following states:

0: Healthy
1: Disabled
2: Surrendered
3: Dead

Nonzero forces of transition for $x < 150$, where x is age, are

$$\mu_x^{01} = \frac{0.5}{150 - x}$$
$$\mu_x^{02} = \frac{0.4}{150 - x}$$
$$\mu_x^{03} = \frac{0.1}{150 - x}$$
$$\mu_x^{13} = 0.05$$

As a result of these forces of transition,

$$ {}_tp_x^{00} = \frac{150 - x - t}{150 - x} \qquad 0 \le x < 150$$

(a) (2 points) Calculate the probability that a person healthy at age 50 will become disabled within 10 years.

(b) (2 points) Calculate the probability that a person healthy at age 50 will be disabled at age 60.

(c) (3 points) A disability insurance pays 40,000 per year continuously while an individual is disabled until the individual's 70^{th} birthday.
Calculate the actuarial present value at $\delta = 0.05$ of a policy sold to a healthy individual age 50.

(d) (3 points) A disability insurance pays 40,000 per year continuously while an individual is disabled until the individual's 70^{th} birthday. Benefits are subject to a 6 month waiting period.
Calculate the actuarial present value at $\delta = 0.05$ of a policy sold to a healthy individual age 50.

Exam questions continue on the next page . . .

3. (8 points) An individual age x is subject to a serious disease. The individual's status is modeled using these three states;

0: Healthy
1: Diseased
2: Dead

Forces of transition between states are

$$\mu_t^{01} = 0.02 \qquad \mu_t^{02} = 0.01 \qquad \mu_t^{10} = 0.20 \qquad \mu_t^{12} = 0.35$$

The effective annual rate of interest is 0.05.

Use Euler's method with step 0.5 to solve Kolmogorov's forward equations for the state probabilities for the first two years.

(a) (2 points) Using approximate probabilities, calculate $\ddot{a}^{(2)00}_{x:\overline{2}|}$.

(b) (2 points) Using Woolhouse's formula to two terms, calculate $\ddot{a}^{(12)00}_{x:\overline{2}|}$.

Forces of transition for the first two years are estimated assuming constant forces of transition within each year, and using the following data:

- 92 lives in state 0 remained in state 0 for 2 years.
- 2 lives in state 0 died, one at time 0.8 and one at time 1.5.
- 3 lives in state 0 moved to state 1 at times 0.8, 1.2, 1.6. They all stayed there to time 2.
- 8 lives in state 1 remained in state 1 for 2 years.
- 2 lives in state 1 moved to state 0, one at time 0.4 and one at time 1.8. They all stayed there to time 2.
- 4 lives in state 1 died at times 0.3, 0.6, 1.2, 1.7

(c) (2 points)
  (i) Estimate μ_x^{01} and μ_{x+1}^{01}.
  (ii) Estimate the standard deviations of the estimates of μ_x^{01} and μ_{x+1}^{01}.

(d) (2 points) Using estimates of the forces of transition, calculate the probability of staying healthy for 2 years.

4. (8 points) For two independent lives:

(i) The first life is subject to a force of accidental death of 0.01 and a force of death due to other causes of 0.03.
(ii) The second life is subject to a force of accidental death of 0.005 and a force of death due to other causes of 0.02.
(iii) $\delta = 0.04$

(a) (2 points) Calculate the actuarial present value of an insurance paying 1000 upon the first death if the death is by accident.

(b) (2 points) Calculate the actuarial present value of an insurance paying 1000 upon the second death if the death is by accident.

(c) (2 points) Calculate the actuarial present value of an insurance paying 1000 upon the second death if both deaths were by accident.

(d) (2 points) Calculate the actuarial present value of an insurance paying 1000 upon the second death if the second life dies second.

Exam questions continue on the next page . . .

5. (9 points) An equity-linked insurance is issued to a life age 60. It matures at age 65. It has the following features:

(i) No front-end load.
(ii) A management charge of 2% of the fund value, deducted at the beginning of each year.
(iii) A surrender charge of 4%, 3%, 2%, 1% of the fund value in years 1, 2, 3, 4 respectively.

Upon death, the surrender charge is waived. The death benefit is paid at the end of the year.

You are given:

(i) Mortality rate is $q_{60+t} = 0.006 + 0.001t$, $t = 0, 1, 2, 3, 4$.
(ii) Independent surrender rate is 0.03 each year for the first 4 years.
(iii) Surrenders occur at the end of the year.

A single premium of 100,000 is paid.

(a) (1 point) The value of the waiver of the surrender charge may be evaluated as a decreasing insurance at a certain interest rate.
Determine the interest rate.

(b) (2 points) Calculate the value at issue of the waiver of the surrender charge.

For part (c) only, use the following information:

The company adds a GMMB to the policy. 110% of the fund is paid at maturity.

(c) (2 points) Calculate the value at issue of the GMMB.

For parts (d)–(e) only, use the following information:

The company enhances the GMDB. 120% of the fund is paid at the end of the year of death.

(d) (2 points) Calculate the value at issue of the enhanced GMDB, including the value of the waiver of the surrender charge.

(e) (2 points) Calculate the portion of the management charge, as a level percentage of the fund value, needed to fund the GMDB.

Exam questions continue on the next page . . .

6. (11 points) In a defined contribution plan, the employer and employee each contribute 3% of salary at the end of each year. You are given:

(i) Salaries increase 4% each year.
(ii) The pension fund earns 5% effective interest each year.

Vera enters the plan at age 30 at 70,000 salary. At retirement at age 65, Vera will convert the fund into a monthly whole life annuity-due priced with the following assumptions:

- Mortality follows the Standard Ultimate Mortality Model.
- $i = 0.05$
- Woolhouse's formula to two terms is used for fractional payments.

(a) (3 points) Calculate the replacement ratio.

(b) (3 points) For this part only, assume that Vera's salary increases 3% each year for the first 5 years, and 4% each year thereafter.
Calculate the replacement ratio if retirement occurs at age 65.

(c) (3 points) For this part only, assume that the pension fund earns 6% effective interest in all years.
Calculate the replacement ratio if retirement occurs at age 65.

(d) (2 points) Vera dies at age 55. Her husband, age 50, inherits the fund and purchases a monthly whole-life annuity-due. The assumptions used to calculate the single premium for this annuity are the same as the assumptions used by Vera.
Calculate the monthly benefit paid by the annuity.

Solutions to the above questions begin on page 755.

Appendices

Appendix A. Solutions to the Practice Exams

Practice Exam 1

1. The workbook solution can be downloaded here.
   (a) [**Section** 1.5] We use the recursive formula

$$ {}_tV = \frac{({}_{t-1}V + P)(1+i) - 100{,}000q_{50+(t-1)}}{1 - q_{50+(t-1)}} $$

   Under FPT, ${}_1V = 0$.

   (b) [**Section** 28.2] We calculate pricing mortality and then fill in the columns of the profit test.

   (c) [**Section** 28.3] We used the Excel NPV function. When using it, you must multiply the result by $1 + r$, because Excel discounts payment t for t years and we need to discount it $t - 1$ years. For premiums we multiplied premiums by survival probabilities and discount rates (so the NPV function, which doesn't consider survival probabilities, wouldn't work).

   (d) [**Section** 28.4] We subtract the profit for year 30 divided by $1 + i = 1.06$ from the initial reserve, which makes 30$^{\text{th}}$ year profit equal to 0 and increases 29$^{\text{th}}$ year profit. Then we calculate the NPV of the revised profit signature.

   (e) [**Lesson** 29] We calculate actual profit for the entire block (1000 policies); expenses for the entire block are 80,000, so policy-by-policy actual profit cannot be calculated. Then we subtract 1000 times expected profit per policy to obtain gain.

   (f) [**Lesson** 29]

      (i) Cash flows for years 2–10 would increase due to the surrender charge, increasing profits and therefore increasing NPV.

      (ii) The gain in year 5 would decrease. There are about 10 surrenders less than expected, so the company would not get the surrender charge for those policies.

2. (a) [**Section** 6.1]

$$ \int_0^{10} {}_tp_{50}^{00}\,\mu_{50+t}^{01}\,\mathrm{d}t = \int_0^{10}\left(\frac{100-t}{100}\right)\left(\frac{0.5}{100-t}\right)\mathrm{d}t $$
$$ = \int_0^{10}\frac{0.5\,\mathrm{d}t}{100} = \boxed{\mathbf{0.05}} $$

   (b) [**Section** 6.1]

$$ \int_0^{10} {}_tp_{50}^{00}\,\mu_{t0+t}^{01}\,{}_{10-t}p_{50+t}^{11}\,\mathrm{d}t = \int_0^{10}\left(\frac{100-t}{100}\right)\left(\frac{0.5}{100-t}\right)e^{-0.05(10-t)}\,\mathrm{d}t $$
$$ = 0.005e^{-0.5}\int_0^{10} e^{0.05t}\,\mathrm{d}t $$
$$ = \frac{0.005e^{-0.5}}{0.05}(e^{0.5}-1) = \boxed{\mathbf{0.039347}} $$