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EA-1 Exam

110 Original Questions and Solutions

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ABOUT THIS MANUAL

This manual contains original questions. They are not old examination questions. Note that the Joint Board announced that beginning in 2007, commutation functions will be tested on the EA-1 examination. There have been approximately two questions on each of the EA-1 examinations that have been given since the Joint Board announcement that required the use of commutation functions. Some of the questions in this manual use commutation functions in their solution.

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EA-1 Review Questions

Question 1

An initial deposit is made to a fund on 1/1/1998 of $1,000, with subsequent deposits at the end of every calendar quarter of $1,000. For the first five years, interest is earned at an annual effective rate of 10%. For the remaining eight years, interest is earned at an annual rate of discount convertible semiannually of 12%.

In what range is the accumulated value of the fund as of 12/31/2010?

(A) Less than $120,000
(B) $120,000 but less than $123,000
(C) $123,000 but less than $126,000
(D) $126,000 but less than $129,000
(E) $129,000 or more

Question 2

Value of a pension fund on 1/1/2000: $30,000
Contribution to the pension fund on 6/30/2000: $10,000
Value of the pension fund on 12/31/2000: $50,000

In what range is the rate of return for 2000?

(A) Less than 26%
(B) 26% but less than 27%
(C) 27% but less than 28%
(D) 28% but less than 29%
(E) 29% or more

Question 3

Beginning January 1, 2000, 15 annual deposits of $1,500 are made to a savings account. Beginning January 1, 2015, annual withdrawals of $4,500 are made until the savings account is completely exhausted immediately after the withdrawal made on January 1, 2029.

In what range is the effective annual interest rate earned by the account?

(A) Less than 6.5%
(B) 6.5% but less than 7.0%
(C) 7.0% but less than 7.5%
(D) 7.5% but less than 8.0%
(E) 8.0% or more

Question 4

Deposits to a fund:	$1,000
Number of deposits:	40
Date of first deposit:	January 1, 2000
Frequency of deposit:	Quarterly
Rate of interest:	8% per year, compounded quarterly

Twenty withdrawals of $3,000 each are made annually, beginning on January 1, 2010.

In what range is the fund balance on January 1, 2030?

(A) Less than $110,000
(B) $110,000 but less than $120,000
(C) $120,000 but less than $130,000
(D) $130,000 but less than $140,000
(E) $140,000 or more

Question 5

A pension fund earns a gross effective annual interest rate of $j\%$.

At the end of each quarter, a charge equal to .25% of the fund balance is withdrawn. After this deduction, the fund earns a net effective annual interest rate of 10%. There are no deposits to the fund and no withdrawals other than this charge.

In what range is $j\%$?

(A) Less than 10.8%
(B) 10.8% but less than 10.9%
(C) 10.9% but less than 11.0%
(D) 11.0% but less than 11.1%
(E) 11.1% or more

Question 6

Selected values:

$\ddot{a}_{\overline{n}|} = 7.2077$ $\ddot{a}_{\overline{n+1}|} = 7.3069$

In what range is the effective annual rate of interest?

(A) Less than 13.5%
(B) 13.5% but less than 14.0%
(C) 14.0% but less than 14.5%
(D) 14.5% but less than 15.0%
(E) 15.0% or more

Question 7

$\ddot{a}_{\overline{n}|} = 6.76$ $(1 + i)^n = 2.59$

In what range is $100s_{\overline{2n}|}$?

(A) Less than 5,700
(B) 5,700 but less than 5,725
(C) 5,725 but less than 5,750
(D) 5,750 but less than 5,775
(E) 5,775 or more

Question 8

Smith enrolls in a four year college on October 1, 2000. Quarterly tuition payments of $3,000 per quarter are due on the first of day of each quarter (October 1, January 1, and April 1). No payment is due on July 1.

The effective annual interest rate is 8%.

In what range is the present value as of October 1, 2000 of the tuition payments for the four years?

(A) Less than $30,000
(B) $30,000 but less than $31,000
(C) $31,000 but less than $32,000
(D) $32,000 but less than $33,000
(E) $33,000 or more

Question 9

A loan is to be repaid by 16 quarterly payments of \$50, \$100, \$150, ..., \$800, the first payment due three months after the loan is made.

The force of interest is 8%.

In what range is the total amount of interest paid over the life of the loan?

(A) Less than \$1,220
(B) \$1,220 but less than \$1,320
(C) \$1,320 but less than \$1,420
(D) \$1,420 but less than \$1,520
(E) \$1,520 or more

Question 10

Selected values:

$$s_{\overline{n}|} = 20 \qquad s_{\overline{2n}|} = 59$$

In what range is $s_{\overline{4n}|}$?

(A) Less than 180
(B) 180 but less than 210
(C) 210 but less than 240
(D) 240 but less than 270
(E) 270 or more

Question 11

The present value of a continuous annuity of 1 per year for *n* years is 4.

The force of interest is equal to 12.5%.

In what range is the accumulated value of a continuous annuity of 1 per year for 2*n* years?

(A) Less than 10
(B) 10 but less than 15
(C) 15 but less than 20
(D) 20 but less than 25
(E) 25 or more

Question 12

A perpetuity of $1,000 is paid each January 1, beginning on 1/1/2000, with increases of 4% each subsequent January 1. The annual effective rate of interest is 6%.

In what range is the present value of the annuity as of 1/1/2000?

(A) Less than $52,800
(B) $52,800 but less than $53,800
(C) $53,800 but less than $54,800
(D) $54,800 but less than $55,800
(E) $55,800 or more

Question 13

A perpetuity of $1,000 is paid each January 1, beginning on 1/1/2000, with increases of $100 each subsequent January 1. The annual effective rate of interest is 6%.

In what range is the present value of the annuity as of 1/1/2000?

(A) Less than $47,800
(B) $47,800 but less than $48,800
(C) $48,800 but less than $49,800
(D) $49,800 but less than $50,800
(E) $50,800 or more

Question 14

An annuity of $1,000 is paid each January 1, beginning on 1/1/2000, with increases of 5% each subsequent January 1. There are a total of 30 payments. The annual effective rate of interest is 7%.

In what range is the present value of the annuity as of 1/1/2000?

(A) Less than $22,000
(B) $22,000 but less than $23,000
(C) $23,000 but less than $24,000
(D) $24,000 but less than $25,000
(E) $25,000 or more

Solutions to
EA-1 Review Questions

Solution to Question 1

The quarterly interest rate for the first five years is $1.1^{1/4} - 1 = .024114$
The semiannual rate of discount for the last eight years is $.12/2 = .06$
The semiannual interest rate for the last eight years is $d/(1 - d) = .06/(1 - .06) = .063830$
The quarterly interest rate for the last eight years is $1.06383^{1/2} - 1 = .031421$
The accumulated balance is:

$$\$1{,}000\ddot{s}_{\overline{20}|\,.024114}(1.031421)^{32} + \$1{,}000 s_{\overline{33}|\,.031421} = \$126{,}296$$

Solution to Question 2

The annual rate of return is equal to i, where i is the effective annual rate of interest such that the value of the fund is equal to the accumulated value of the contributions:

$$50{,}000 = (30{,}000)(1 + i) + (10{,}000)(1 + i)^{1/2}$$
$$0 = (30{,}000)(1 + i) + (10{,}000)(1 + i)^{1/2} - 50{,}000$$

This can be written as:

$$0 = 30{,}000x^2 + 10{,}000x - 50{,}000 \qquad \text{(where } x = (1 + i)^{1/2}\text{)}$$
$$= 3x^2 + x - 5$$

Recall the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In the equation above, $a = 3$, $b = 1$, and $c = -5$

Substituting into the quadratic formula:

$$x = \frac{-1 \pm \sqrt{1^2 - (4)(3)(-5)}}{(2)(3)} = 1.135$$

Since $x = 1.135$ and $x = (1 + i)^{½}$,

$$(1 + i)^{½} = 1.135$$
$$1 + i = 1.288$$
$$i = 28.8\%$$

Note that on an exam, the endpoints of the ranges can be substituted into the equation of value if you do not remember the quadratic formula (or if a more complex equation of value is given).

Using $i = 28\%$, $(30{,}000)(1.28) + (10{,}000)(1.28)^{½} = 49{,}354$
Using $i = 29\%$, $(30{,}000)(1.29) + (10{,}000)(1.29)^{½} = 50{,}058$

Since 50,000 is between 49,354 and 50,058, the effective rate of interest must be between 28% and 29%.

Solution to Question 3

The accumulated balance on 1/1/2015 must be equal to the present value of the withdrawals. The equation of value is:

$$1{,}500\ddot{s}_{\overline{15}|} = 4{,}500\ddot{a}_{\overline{15}|}$$
$$\ddot{s}_{\overline{15}|}/\ddot{a}_{\overline{15}|} = 4{,}500/1{,}500$$
$$(1 + i)^{15} = 3$$
$$\ln((1 + i)^{15}) = \ln(3)$$
$$15 \ln(1 + i) = \ln(3)$$
$$\ln(1 + i) = \ln(3)/15 = .073241$$
$$e^{\ln(1+i)} = e^{.073241}$$
$$1 + i = 1.076$$
$$i = .076, \text{ or } 7.6\%$$

Note that on an exam, the endpoints of the ranges can be substituted into the equation of value if you do not remember or are not comfortable with logarithms.

Using $i = 7.5\%$, $1{,}500\ddot{s}_{\overline{15}|} = 4{,}500\ddot{a}_{\overline{15}|} \Rightarrow 42{,}116 = 42{,}702$
Using $i = 8.0\%$, $1{,}500\ddot{s}_{\overline{15}|} = 4{,}500\ddot{a}_{\overline{15}|} \Rightarrow 43{,}986 = 41{,}599$

Since the accumulated value is less than the present value of the disbursements using an interest rate of 7.5% and the present value exceeds the accumulated value when the interest rate is increased to 8%, the effective rate of interest must be between 7.5% and 8.0%.

Solution to Question 4

The quarterly effective rate of interest is $i^{(4)}/4 = .08/4 = .02$.
The annual effective rate of interest is $(1.02)^4 - 1 = .082432$

The accumulated balance on 1/1/2030 is:

$(1{,}000\ddot{s}_{\overline{40}|\,.02} \times 1.082432^{20}) - 3{,}000\ddot{s}_{\overline{20}|\,.082432} = 147{,}708$

Solution to Question 5

F = Fund value at beginning of year
Balance after first quarter = $F(1 - .0025)(1 + j^{(4)}/4)$
Balance after second quarter = $F(1 - .0025)^2(1 + j^{(4)}/4)^2$
Balance after third quarter = $F(1 - .0025)^3(1 + j^{(4)}/4)^3$
Balance after fourth quarter = $F(1 - .0025)^4(1 + j^{(4)}/4)^4 = F(1.1)$
Since $(1 + j^{(4)}/4)^4 = 1 + j$, $(1 - .0025)^4(1 + j) = 1.1 \Rightarrow j = 11.11\%$

Solution to Question 6

$\ddot{a}_{\overline{n+1}|} = 1 + v + v^2 + ... + v^{n-1} + v^n = \ddot{a}_{\overline{n}|} + v^n \Rightarrow \quad v^n = 7.3069 - 7.2077 = .0992$
$\ddot{a}_{\overline{n}|} = (1 - v^n)/d = (1 - .0992)/d \quad \Rightarrow \quad d = .9008/7.2077 = .124977$
$i = d/(1 - d) = .124977/(1 - .124977) = .142827$, or 14.2827%

Solution to Question 7

$\ddot{a}_{\overline{n}|} = (1 - v^n)/d = (1 - 1/2.59)/d = 6.76 \quad \Rightarrow \quad d = .0900814$
$i = d/(1 - d) = .09988$
$100s_{\overline{2n}|} = (100)[(1.09988^{2n} - 1)/.09988] = 5{,}715$

Solution to Question 8

The quarterly effective rate of interest is $(1 + i)^{1/4} - 1 = 1.08^{1/4} - 1 = .019427$.

The present value of the payments due each October 1, January 1, and April 1, as of October 1, is:

$3{,}000\ddot{a}_{\overline{3}|\,.019427} = 8{,}829.58$

The three payments made each school year can be thought of as annual payments of \$8,829.58 made each October 1. The present value of these payments as of October 1, 2000 is:

$8{,}829.58\ddot{a}_{\overline{4|}\,.08} = 31{,}584$

Solution to Question 9

Convert the force of interest to a quarterly effective interest rate.
$i^{(4)}/4 = (1 + \delta/100{,}000)^{25{,}000} - 1 = (1 + .08/100{,}000)^{25{,}000} - 1 = .020201$
Note that the force of interest can just be considered a nominal annual rate of interest convertible many, many times per year (in this case 100,000, although any large number would yield the same result).

Amount of loan = $(50)(Ia)_{\overline{16|}\,.020201} = 5{,}474$
Total payments = 50 + 100 + 150 + ... + 800 = 50(1 + 2 + ... + 16) = 6,800
Total interest = Total payments - Amount of loan = 6,800 - 5,474 = 1,326

Solution to Question 10

$s_{\overline{2n|}} = s_{\overline{n|}} + (s_{\overline{n|}})(1 + i)^n \quad \Rightarrow \quad (1 + i)^n = (59 - 20)/20 = 1.95$
$s_{\overline{4n|}} = s_{\overline{2n|}} + (s_{\overline{2n|}})(1 + i)^{2n} = 59 + (59)(1.95^2) = 283.35$

Solution to Question 11

$4 = \bar{a}_{\overline{n|}} = (1 - v^n)/\delta = (1 - v^n)/.125 \quad \Rightarrow \quad v^n = .5$
$\bar{s}_{\overline{2n|}} = ((1 + i)^{2n} - 1)/\delta = ((1/.5)^2 - 1)/.125 = 24$

Solution to Question 12

Set up a series representing the payment flow. This is:

$1{,}000 + 1{,}000(1.04)(1/1.06) + (1{,}000)(1.04)^2(1/1.06)^2 + \ldots$
$= 1{,}000\,[1 + (1.04)(1/1.06) + (1.04)^2(1/1.06)^2 + \ldots]$

The common factor of (1.04)(1/1.06) can be rewritten as 1.04/1.06 = .981132. The series can then be written as $1{,}000\,[1 + .981132 + .981132^2 + \ldots]$. Using the summation for an infinite series (with .981132 as the common factor, the series is equal to 1,000 [1/(1 - .981132)] = 52,999.79.

Note that the formula for the summation of an infinite series is:

$x + x^2 + x^3 + \ldots = x/(1 - y)$, where y is the common factor needed to multiply each term of the series to get to the next term

An alternative method to solving this is to rewrite the common factor of (1.04)(1/1.06) as 1/(1.06/1.04) = 1/1.019231. This is equivalent to a discount factor (v) with an interest rate of 1.9231%. The series then becomes a perpetuity due equal to $1{,}000\ddot{a}_{\overline{\infty}|}$ = (1,000)(1/.019231)(1.019231) = 52,999.38. The pennies difference in the answers are due to rounding.

Solution to Question 13

This can be thought of as a level perpetuity due of $1,000 per year, with an increasing perpetuity immediate (since the increases do not start until the end of the first year or the beginning of the second year) beginning at $100 per year and increasing by $100 each year. The present value of these payments is:

$$1{,}000\ddot{a}_{\overline{\infty}|} + 100(Ia)_{\overline{\infty}|} = 1{,}000(1/.06)(1.06) + 100[1/.06 + (1/.06)^2] = 47{,}111.11$$

Note that $(Ia)_{\overline{\infty}|} = 1/i + 1/i^2$

Solution to Question 14

This is a geometric progression, rather than an arithmetic progression. The standard formula for an increasing annuity will not help you here. Write out what you are looking for.

$$\begin{aligned} PV &= 1{,}000 + (1{,}000)(1.05)v + \dots + (1{,}000)(1.05^{29})v^{29} \\ &= 1{,}000 \times [1 + 1.05v + \dots + (1.05v)^{29}] \end{aligned}$$

At this point, you have two options. The first option is to treat the sum in brackets as a geometric series, and use the summation for such a series. Recall that this summation is:

$$1 + x + x^2 + x^3 + \dots + x^{n-1} = [(1 - x^n)]/(1 - x)$$

In this case, x = $1.05v$. So,

$$PV = 1{,}000 \times [(1 - (1.05v)^{30}]/(1 - 1.05v) = 23{,}125$$

The second option is to convert the common factor of $1.05v$ to an equivalent interest rate. Recall that v can also be written as $1/(1 + i)$. So, $1.05v$ can be written as $1.05 \times 1/1.07$. Alternatively, this can be written as 1/(1.07/1.05), or 1/1.019048. This is equivalent to a v where the interest rate is 1.9048%. Therefore, the present value becomes:

$$\begin{aligned} PV &= 1{,}000 \times (1 + v + \dots + v^{29}) \qquad \text{(where } i = .019048\text{)} \\ &= 1{,}000 \times \ddot{a}_{\overline{30}|} \\ &= 1{,}000 \times 23.125 \\ &= 23{,}125 \end{aligned}$$

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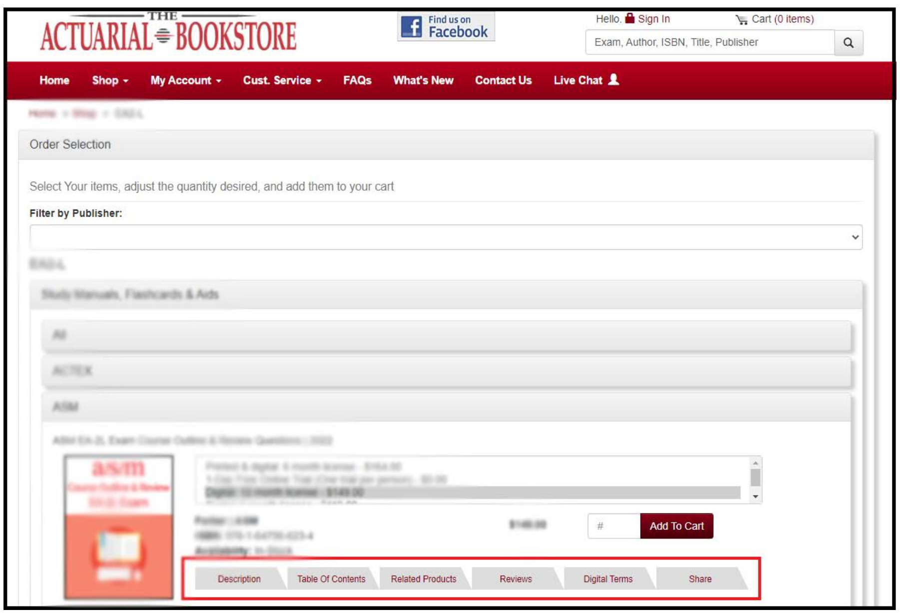

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