Chapter 9 – Questions Sample

Thank you for downloading our Exam MFE/3F Online Sample Questions. Questions 50 through 59 from our practice questions for Chapter 9 are provided below, followed by the corresponding full solutions. At the beginning of each solution, gray boxes indicate the question's degree of difficulty, on a scale of 1 to 5.

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Question 50

An investor takes the following two-part position:

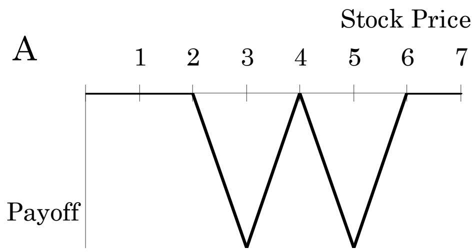
(i) Sells one \$2-strike put option, buys two \$3-strike put options, and sells one \$4-strike put option.

(ii) Sells one \$4-strike call option, buys two \$5-strike call options, and sells one \$6-strike call option.

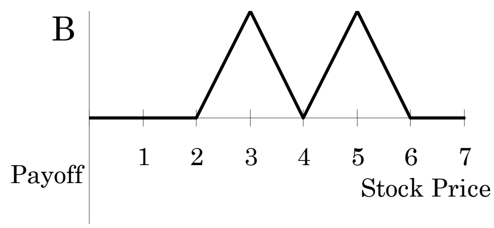
All of the options have the same underlying stock and they all expire in one year.

Which of the following payoff graphs corresponds with the investor's position at the expiration of the options?

A



B



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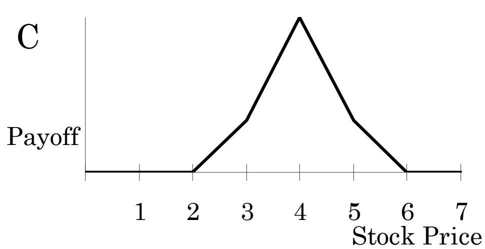
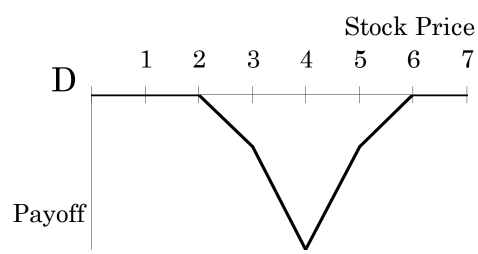
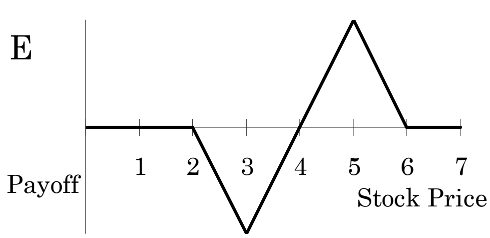


D



E



Question 51

The price, strike price, and time until expiration are given below for 3 European call options on the same nondividend paying stock.

	Option Price	Strike Price	Expiration
Option A	\$8.00	\$50.00	1 year
Option B	\$7.70	\$52.00	1.5 years
Option C	\$7.50	\$53.00	2.0 years

An arbitrageur sees an arbitrage opportunity and therefore buys or sells exactly one of Option B at time 0. Subsequently, the actual stock prices emerge as described in the table below:

Time	Stock Price
1 year	\$50.00
1.5 years	\$52.50
2.0 years	\$52.50

The continuously compounded risk-free rate of return is 6%.

Arbitrage profits are accumulated at the risk-free rate of return. Determine the value of the arbitrage profits at the end of 2 years.

A \$0.30 B \$0.83 C \$1.08 D \$1.31 E \$1.83

Question 52

Near market closing time on a given day, you lose access to stock prices, but some European call and put prices for a stock are available as follows:

Strike Price	Call Price	Put Price
$45	$12	$4
$55	$7	$9
$60	$4	$12

All 6 options have the same expiration date. The risk-free interest rate is zero.

After reviewing the information above, Jill tells Sabrina and Kelly that one could use the following zero-cost portfolio to obtain arbitrage profit: Short one put option with strike price 45; long 3 put options with strike price 55; lend $1; and short some number of put options with strike price 60.

Sabrina claims that the following zero-cost portfolio can produce arbitrage profit: Long one call option with strike price 45; short 3 call options with strike price 55; lend $1; and long some number of call options with strike price 60.

Kelly claims that the following zero-cost portfolio can produce arbitrage profit: Long 2 calls and short 2 puts with strike price 60; long 1 call and short 1 put with strike price 45; lend $2; and short some calls and long the same number of puts with strike price 55.

Which of the following statements is correct?

A Only Jill is correct.

B Only Sabrina is correct.

C Only Kelly is correct.

D Only Sabrina and Kelly are correct.

E None of them is correct.

Question 53

Consider a claim that matures at time 1. The payoff of the claim is the minimum of the following: the price of Stock 1, the price of Stock 2, and $17. $S_j(t)$ denotes the price of one share of stock *j* at time *t*, so the payoff of the claim is:

$$\text{Minimum}\left[S_1(1),\ S_2(1),\ 17\right]$$

The current value of this claim is $15.50.

A one-year European option gives its owner the right to sell either Stock 1 or Stock 2 at a price of $17 one year from now. The price of this option is $0.67.

Calculate the continuously compounded risk-free rate of return.

A 1% B 2% C 3% D 4% E 5%

Question 54

Consider a claim that matures at time 1. The payoff of the claim is the maximum of the following: the price of Stock 1, the price of Stock 2, and \$22. $S_j(t)$ denotes the price of one share of stock *j* at time *t*, so the payoff of the claim is:

$$\text{Maximum}\left[S_1(1),\ S_2(1),\ 22\right]$$

The current value of this claim is \$23.34.

A one-year European option gives its owner the right to buy either Stock 1 or Stock 2 at a price of \$17 one year from now. The price of this option is \$2.83.

Calculate the continuously compounded risk-free rate of return.

A 7% B 8% C 9% D 10% E 11%

Question 55

Consider a claim that matures at time 3. The payoff of the claim is the maximum of the following: the price of Stock 1, the price of Stock 2, and \$25. $S_j(t)$ denotes the price of one share of stock *j* at time *t*, so the payoff of the claim is:

$$\text{Maximum}\left[S_1(3),\ S_2(3),\ 25\right]$$

The current value of this claim is \$25.50.

The continuously compounded risk-free rate of return is 8% per year.

A three-year European option gives its owner the right to buy either Stock 1 or Stock 2 at a price of \$25 three years from now. Calculate the current price of this option.

A \$0.50 B \$1.58 C \$2.42 D \$5.83 E \$6.28

Question 56

A rainbow put option has a payoff in 5 years that is based on the prices of Stock X and Stock Y in 5 years. The rainbow put option's payoff is the minimum of the following: 2 times the price of Stock X, 3 times the price of Stock Y, and \$25. The current price of this rainbow put option is \$12.67.

The continuously compounded risk-free interest rate is 5%.

Consider a 5-year European option that gives its holder the right to sell either 2 shares of Stock X or 3 shares of Stock Y for \$25. Calculate the current price of this option.

A 5.30 B 6.80 C 7.50 D 9.60 E 12.30

Question 57

You are given:

(i) $C(K,T)$ denotes the current price of a K-strike T-year European call option on a nondividend-paying stock.

(ii) $P(K,T)$ denotes the current price of a K-strike T-year European put option on the same stock.

(iii) S denotes the current price of the stock.

(iv) The continuously compounded risk-free interest rate is r, and $r > 0$.

Which of the following is (are) correct?

I $0 \le P(85,T) - P(80,T) \le 5e^{-rT}$

II $75e^{-rT} \le P(75,T) - C(80,T) + S \le 80e^{-rT}$

III $80e^{-rT} \le P(75,T) - C(80,T) + S \le 85$

A I only B II only C III only D I and II only E I and III only

Question 58

The following information was known at time 0:

(i) The continuously compounded risk-free interest rate is 9%.

(ii) The price of a stock is \$80.

(iii) The stock pays 2 discrete dividends.

(iv) The stock pays a discrete dividend of \$2 in 3 months.

(v) The stock pays a second discrete dividend of \$1.90 in 9 months.

At time 0, a 1-year American call option was written on the stock. The strike price of the American call option was \$86.

At time 2, you are informed that the call option did not expire worthless. Determine when the call was exercised.

A Time 0

B Time 3 months

C Time 9 months

D Time 12 months

E There is not enough information provided to answer this question.

Question 59

Let $S(t)$ be the stock price at time t. The current time is time 0.

The current price of a special option is $\hat{C}(K)$. Its only payoff occurs in 1 year and is defined to be:

$$Max\left[0, S(1) - K + 1\right] - Max\left[0, S(1) - K\right]$$

The current price of a different special option is $\hat{P}(K)$. Its only payoff occurs in 1 year and is defined to be:

$$Max\left[0, K - S(1)\right] - Max\left[0, K - 1 - S(1)\right]$$

You are given:

(i) $\hat{C}(109) = 0.3$

(ii) The continuously compounded risk-free interest rate is 22%.

Determine $\hat{P}(109)$.

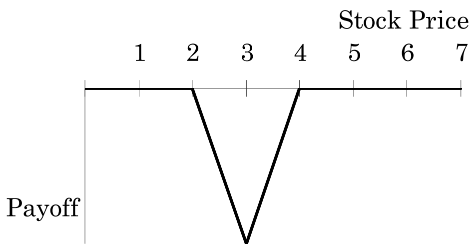
A 0.4 B 0.5 C 0.6 D 0.7 E 0.8

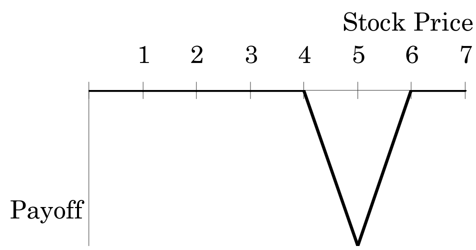
Chapter 9 – Solutions

Solution 50

A Chapter 9, Option Payoffs

A butterfly spread involves options with 3 different strike prices. Part one of the investor's position is a symmetric butterfly spread with strike prices \$2, \$3 and \$4. Part two of the investor's position is a symmetric butterfly spread with strike prices \$4, \$5 and \$6. The payoff when buying a butterfly spread is never less than zero. In this case, each butterfly spread was sold, which means the payoffs are less than zero. The payoff of the butterfly spread looks the same whether the position is composed of calls or puts. Part one of the position's payoff is shown as the bold line below at the left, and part two of the position's payoff is shown as the bold line below at the right.





When combined, the investor's position looks like Choice A.

Solution 51

D Chapter 9, Strike Price Grows Over Time

The prices of the options decrease as time to maturity increases. Therefore, if the strike price increases at a rate that is less than the risk-free rate, then arbitrage is available. Option B expires 0.5 years after Option A, so let's accumulate Option A's strike price for 0.5 years at the risk-free rate:

$$50e^{0.06\times 0.5} = 51.5227$$

Since the strike price of Option B is \$52, which is greater than \$51.5227, arbitrage is not indicated by the prices of Option A and Option B.

Option C expires 0.5 years after Option B, so let's accumulate Option B's strike price for 0.5 years at the risk-free rate:

$$52e^{0.06\times 0.5} = 53.5836$$

Since the strike price of Option C is \$53, the strike price grows from time 1 to time 1.5 at a rate that is less than the risk-free rate of return. Consequently, arbitrage can be earned by purchasing Option C and selling Option B (i.e., buy low and sell high).

The arbitrageur buys the 2-year option for \$7.50 and sells the 1.5-year option for \$7.70 The difference of \$0.20 is lent at the risk-free rate of return.

The 1.5-year option

After 1.5 years, the stock price is \$52.50. Therefore, the 1.5-year option is exercised against the arbitrageur. The arbitrageur borrows a share of stock and sells it for the strike price of \$52. As a result, at the end of 2 years the arbitrageur owes the share of stock and has the accumulated value of the \$52. This position results in the following cash flow at the end of 2 years:

$$-52.50+52e^{0.06\times 0.5}=1.0836$$

The 2-year option

The stock price of \$52.50 at the end of 2 years is less than the strike price of the 2-year option, which is \$53. Therefore, the 2-year call option expires worthless, and the resulting cash flow is zero.

The net cash flow

The net cash flow at the end of 2 years is the sum of the accumulated value of the \$0.20 that was obtained by establishing the position, the \$1.0836 resulting from the 1.5-year option, and the \$0.00 resulting from the 2-year option:

$$0.20e^{0.06\times 2}+1.0836+0.00=1.3091$$

Solution 52

D Chapter 9, Arbitrage

Let X be the number of puts with a strike price of \$60 that are sold for Jill's portfolio. The fact that the net cost of establishing the portfolio is zero allows us to solve for X:

$$-P(45)+3P(55)+1-P(60)\times X=0$$
$$-4+3\times 9+1-12X=0$$
$$X=2$$

An arbitrage strategy does not allow the cash flow at expiration to be negative. But suppose that only the \$60-strike put option is in-the-money at expiration. Since Jill is short the \$60-strike put option, this results in a negative cash flow. In particular, if the stock price at expiration is between \$55 and \$59, then the payoff from Jill's strategy will be negative. For example, if the stock price at expiration is \$57, the payoff from Jill's strategy will be:

$$-[\text{45-strike payoff}]+3[\text{55-strike payoff}]-2[\text{60-strike payoff}]+[\text{Proceeds from loan}]$$
$$0+0-2(60-57)+1=-5$$

Since Jill's strategy can result in a negative payoff at expiration, Jill's strategy is not arbitrage.

Let Y be the number of calls with a strike price of \$60 that are purchased for Sabrina's portfolio. The fact that the net cost of establishing the portfolio is zero allows us to solve for Y:

$$C(45) - 3C(55) + 1 + C(60) \times Y = 0$$
$$12 - 3 \times 7 + 1 + 4Y = 0$$
$$Y = 2$$

The table below shows that regardless of the stock price at time T, Sabrina's payoff is positive. Therefore, Sabrina is correct.

Sabrina's Portfolio		**Time T**			
Transaction	**Time 0**	$S_T < 45$	$45 \le S_T \le 55$	$55 \le S_T \le 60$	$60 < S_T$
Buy 1 of $C(45)$	–12.00	0.00	$S_T - 45$	$S_T - 45$	$S_T - 45$
Sell 3 of $C(55)$	3(7.00)	0.00	0.00	$-3(S_T - 55)$	$-3(S_T - 55)$
Buy 2 of $C(60)$	–2(4.00)	0.00	0.00	0.00	$2(S_T - 60)$
Lend \$1	–1.00	1	1	1	1
Total	0.00	1	$1 + S_T - 45$	$1 + 120 - 2S_T$	1

Let Z be the number of calls purchased and puts sold with a strike price of \$55 for Kelly's portfolio. Since the net cost of establishing the portfolio is zero, we can solve for Z:

$$2\left[C(60) - P(60)\right] + 1\left[C(45) - P(45)\right] + 2 - Z\left[C(55) - P(55)\right] = 0$$
$$2\left[4 - 12\right] + \left[12 - 4\right] + 2 - Z\left[7 - 9\right] = 0$$
$$-16 + 8 + 2 + 2Z = 0$$
$$Z = 3$$

In evaluating Kelly's portfolio, we can make use of the fact that purchasing a call option and selling a put option is equivalent to purchasing a prepaid forward on the stock and borrowing the present value of the strike price. We can see this by writing put-call parity as:

$$C_{Eur}(K,T) - P_{Eur}(K,T) = F_{0,T}^{P}(S) - Ke^{-rT}$$

Therefore, purchasing a call option and selling a put option results in a payoff of:

$$S_T - K$$

Since Kelly purchases offsetting amounts of puts and calls for any given strike price, we can use this result to evaluate her payoffs.

Kelly's Portfolio

Transaction	Time 0	Time T
Buy 2 of $C(60)$ & sell 2 of $P(60)$	$2(12.00-4.00)$	$2(S_T-60)$
Buy 1 of $C(45)$ & sell 1 of $P(45)$	$4.00-12.00$	S_T-45
Sell 3 of $C(55)$ & buy 3 of $P(55)$	$3(7.00-9.00)$	$-3(S_T-55)$
Lend \$2	–2.00	2
Total	0.00	2

Kelly's portfolio is certain to have a positive payoff at time T, so Kelly is correct.

Solution 53

E Chapter 9, Minimum of 2 Assets

The European option has a payoff of:

$$Max\left[17-Min\left(S_1(1),S_2(1)\right),0\right]$$

We recognize the European option as a put option on the minimum of the two stocks. Let's call the underlying asset X:

$$X(1)=Min\left[S_1(1),S_2(1)\right]$$

The price of the option is \$0.67:

$$P_{Eur}(X,17,1)=0.67$$

We are told that a claim that pays the minimum of $X(1)$ and 17 has a current value of \$15.50:

$$F_{0,1}^{P}\left(Min\left[S_1(1),\ S_2(1),\ 17\right]\right)=F_{0,1}^{P}\left(Min\left[X(1),\ 17\right]\right)=15.50$$

We can express the value of the claim as the value of the strike asset minus the value of the put option:

$$F_{t,T}^{P}\left(Min[X_T,Q_T]\right)=F_{t,T}^{P}(Q)-P_{Eur}(X_t,Q_t,T-t)$$

$$F_{0,1}^{P}\left(Min[X(1),17]\right)=17e^{-r}-0.67$$

$$15.50=17e^{-r}-0.67$$

$$16.17=17e^{-r}$$
$$r=0.05$$

Solution 54

A Chapter 9, Maximum of 2 Assets

The European option has a payoff of:

$$Max\left[Max\left(S_1(1),S_2(1)\right)-22,0\right]$$

We recognize the European option as a call option on the maximum of the two stocks. Let's call the underlying asset *X*:

$$X(1)=Max\left[S_1(1),S_2(1)\right]$$

The price of the option is \$2.83:

$$C_{Eur}(X,22,1)=2.83$$

We are told that a claim that pays the maximum of $X(1)$ and 22 has a current value of \$23.34:

$$F_{0,1}^{P}\left(Max\left[S_1(1),\ S_2(1),\ 22\right]\right)=F_{0,1}^{P}\left(Max\left[X(1),\ 22\right]\right)=23.34$$

We can express the value of the claim as the value of the call option plus the value of the strike asset:

$$F_{t,T}^{P}\left(Max[X_T,Q_T]\right)=C_{Eur}(X_t,Q_t,T-t)+F_{t,T}^{P}(Q)$$

$$F_{0,1}^{P}\left(Max[X_1,22]\right)=2.83+22e^{-r}$$

$$23.34=2.83+22e^{-r}$$

$$20.51=22e^{-r}$$

$$r=0.0701$$

Solution 55

D Chapter 9, Maximum of 2 Assets

The European option has a payoff of:

$$Max\left[Max\left(S_1(3),S_2(3)\right)-25,0\right]$$

We recognize the European option as a call option on the maximum of the two stocks. Let's call the underlying asset *X*:

$$X(3)=Max\left[S_1(3),S_2(3)\right]$$

We are told that a claim that pays the maximum of $X(3)$ and 25 has a current value of \$25.50:

$$F_{0,3}^{P}\left(Max\left[S_1(3),\ S_2(3),\ 25\right]\right)=F_{0,3}^{P}\left(Max\left[X(3),\ 25\right]\right)=25.50$$

We can express the value of the claim as the value of the call option on X plus the value of the strike asset:

$$F_{t,T}^{P}\left(Max[X_T, Q_T]\right) = C_{Eur}(X_t, Q_t, T-t) + F_{t,T}^{P}(Q)$$
$$F_{0,3}^{P}\left(Max[X_3, 25]\right) = C_{Eur}(X_3, 25e^{-3\times 0.08}, 3) + 25e^{-3\times 0.08}$$
$$25.50 = C_{Eur}(X_3, 25e^{-3\times 0.08}, 3) + 19.6657$$
$$C_{Eur}(X_3, 25e^{-3\times 0.08}, 3) = 5.8343$$

Solution 56

B Chapter 9, Maximum of 2 Assets

The European option has a payoff of:

$$Max\left[25 - Min\left(2X(5), 3Y(5)\right), 0\right]$$

We recognize the European option as a put option on the minimum of the two stocks. Let's call the underlying asset Z:

$$Z(5) = Min\left(2X(5), 3Y(5)\right)$$

The rainbow option pays the minimum of $Z(5)$ and 25, and it has a current value of \$12.67:

$$F_{0,5}^{P}\left(Min\left[2X(5),\ 3\mathrm{Y}(5),\ 25\right]\right) = F_{0,5}^{P}\left(Min\left[Z(5),\ 25\right]\right) = 12.67$$

We can express the value of the claim as the value of the prepaid forward price of the strike asset minus the value of the European put option:

$$F_{t,T}^{P}\left(Min[Z_T, Q_T]\right) = F_{t,T}^{P}(Q) - P_{Eur}(Z_t, Q_t, T-t)$$
$$F_{0,5}^{P}\left(Min[Z(5), 25]\right) = 25e^{-5\times 0.05} - P_{Eur}(Z_t, 25e^{-5\times 0.05}, 5)$$
$$12.67 = 19.4700 - P_{Eur}(Z_t, 25e^{-5\times 0.05}, 5)$$
$$P_{Eur}(Z_t, 25e^{-5\times 0.05}, 5) = 6.80$$

Solution 57

D Chapter 9, Bounds on Option Prices

From Propositions 1 and 2, we see that Statement I is true:

Proposition 1: $P(K_2) \geq P(K_1)$ for $K_1 < K_2$

$\Rightarrow \quad 0 \leq P(85) - P(80)$

Proposition 2: $P_{Eur}(K_2) - P_{Eur}(K_1) \leq (K_2 - K_1)e^{-rT}$ for $K_1 < K_2$

$\Rightarrow \quad P_{Eur}(85) - P_{Eur}(80) \leq (85 - 80)e^{-rT}$

We make use of put-call parity for Statements II and III:

$$C(K) + K^{-rT} = S + P(K)$$

When the strike price for a call is increased, its price goes down, so the first inequality in Statement II is true:

$$P(75) - C(75) + S = 75e^{-rT} \quad \Rightarrow \quad P(75) - C(80) + S \geq 75e^{-rT}$$

When the strike price for a put is decreased, its price goes down, so the second inequality in Statement II is true:

$$P(80) - C(80) + S = 80e^{-rT} \quad \Rightarrow \quad P(75) - C(80) + S \leq 80e^{-rT}$$

As we saw in Statement II (directly above), the first inequality in Statement III is false:

$$P(80) - C(80) + S = 80e^{-rT} \quad \Rightarrow \quad P(75) - C(80) + S \leq 80e^{-rT}$$

The second inequalty in Statement III is true (but Statement III is still false):

$$P(80) - C(80) + S = 80e^{-rT} \;\Rightarrow\; P(75) - C(80) + S \leq 80e^{-rT} \;\Rightarrow\; P(75) - C(80) + S \leq 85$$

Solution 58

D Chapter 9, Early Exercise of an American Call

If it is optimal to exercise an American call prior to maturity, then the early exercise takes place just before a dividend payment. Therefore the call is not exercised at time 0, and Choice A is not the correct answer.

The call is not exercised early if the present value of the interest on the strike exceeds the present value of the dividends:

$$K - Ke^{-r(T-t)} > PV_{t,T}(div) \qquad \Rightarrow \qquad \text{Don't exercise early}$$

Let's consider time 0.25:

$$86 - 86e^{-0.09(1-0.25)} > 2 + 1.9e^{-0.09(0.75-0.25)}$$
$$5.6134 > 3.8164$$

Since the present value of the interest on the strike exceeds the present value of the dividends, the call option is not exercised at time 0.25, so Choice B is not the correct answer.

Let's consider time 0.75:

$$86 - 86e^{-0.09(1-0.75)} > 1.9$$
$$1.9134 > 1.9$$

Since the present value of the interest on the strike exceeds the present value of the dividends, the call option is not exercised at time 0.75, so Choice C is not the correct answer.

Since the call was exercised, it must have been exercised at maturity, at the end of 12 months. Therefore, the correct answer is Choice D.

Solution 59

B Chapter 9, Put-Call Parity

We can rewrite the payoff of the first option with parentheses in the first *Max* funtion:

$$Max\left[0, S(1)-(K-1)\right]-Max\left[0, S(1)-K\right]$$

We can now see that the special option consists of a long position in a call with a strike price of $(K-1)$ and a short position in a call with a strike price of *K*:

$$\hat{C}(K)=C(K-1)-C(K)$$

Likewise, the second option consists of a long position in a put with a strike price of *K* and a short position in a put with a strike price of $(K-1)$:

$$\hat{P}(K)=P(K)-P(K-1)$$

We can add the value of the two special options together and use put-call parity to show that the value of the sum does not depend on *K*:

$$\begin{aligned}\hat{C}(K)+\hat{P}(K)&=C(K-1)-C(K)+P(K)-P(K-1)\\&=\left[C(K-1)-P(K-1)\right]+\left[P(K)-C(K)\right]\\&=\left[S(1)-(K-1)e^{-r}\right]+\left[Ke^{-r}-S(1)\right]\\&=1e^{-r}=e^{-0.22}=0.8025\end{aligned}$$

We can now find $\hat{P}(109)$:

$$\hat{C}(K)+\hat{P}(K)=0.8025$$
$$\hat{C}(109)+\hat{P}(109)=0.8025$$
$$0.3+\hat{P}(109)=0.8025$$
$$\hat{P}(109)=0.8025-0.3=0.5025$$