Chapter 2 Questions Sample – Comparing Options

Questions 2.16 through 2.21 from Chapter 2 are provided below as a Sample of our Questions, followed by the corresponding full Solutions. At the beginning of each solution, gray boxes indicate the question's degree of difficulty, on a scale of 1 to 5.

*Our MFE/3F Questions contain approximately **650** exam-style questions, and the full solutions to these questions can be either purchased as a hardcopy or downloaded free from our website, www.ActuarialBrew.com.*

Question 2.16

An investor takes the following two-part position:

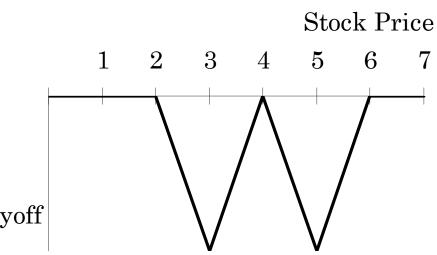
(i) Sells one \$2-strike put option, buys two \$3-strike put options, and sells one \$4-strike put option.

(ii) Sells one \$4-strike call option, buys two \$5-strike call options, and sells one \$6-strike call option.

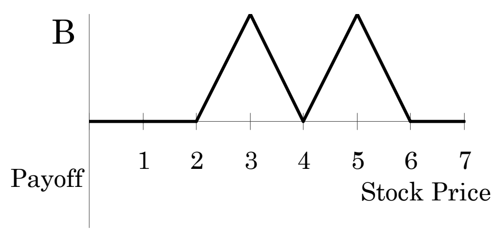
All of the options have the same underlying stock and they all expire in one year.

Which of the following payoff graphs corresponds with the investor's position at the expiration of the options?

A



B



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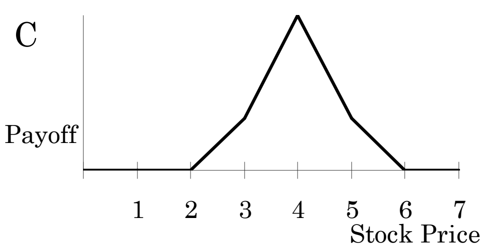
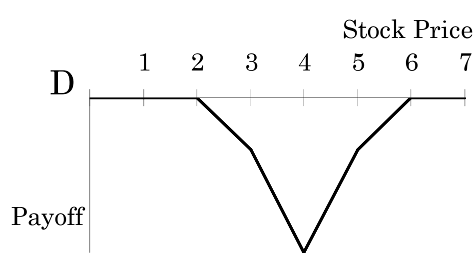
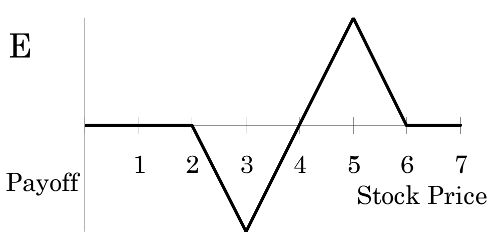


D



E



Question 2.17

The price, strike price, and time until expiration are given below for 3 European call options on the same nondividend paying stock.

	Option Price	Strike Price	Expiration
Option A	\$8.00	\$50.00	1 year
Option B	\$7.70	\$52.00	1.5 years
Option C	\$7.50	\$53.00	2.0 years

An arbitrageur sees an arbitrage opportunity and therefore buys or sells exactly one of Option B at time 0. Subsequently, the actual stock prices emerge as described in the table below:

Time	Stock Price
1 year	\$50.00
1.5 years	\$52.50
2.0 years	\$52.50

The continuously compounded risk-free rate of return is 6%.

Arbitrage profits are accumulated at the risk-free rate of return. Determine the value of the arbitrage profits at the end of 2 years.

A \$0.30 B \$0.83 C \$1.08 D \$1.31 E \$1.83

Question 2.18

Near market closing time on a given day, you lose access to stock prices, but some European call and put prices for a stock are available as follows:

Strike Price	Call Price	Put Price
$45	$12	$4
$55	$7	$9
$60	$4	$12

All 6 options have the same expiration date. The risk-free interest rate is zero.

After reviewing the information above, Jill tells Sabrina and Kelly that one could use the following zero-cost portfolio to obtain arbitrage profit: Short one put option with strike price 45; long 3 put options with strike price 55; lend $1; and short some number of put options with strike price 60.

Sabrina claims that the following zero-cost portfolio can produce arbitrage profit: Long one call option with strike price 45; short 3 call options with strike price 55; lend $1; and long some number of call options with strike price 60.

Kelly claims that the following zero-cost portfolio can produce arbitrage profit: Long 2 calls and short 2 puts with strike price 60; long 1 call and short 1 put with strike price 45; lend $2; and short some calls and long the same number of puts with strike price 55.

Which of the following statements is correct?

A Only Jill is correct.

B Only Sabrina is correct.

C Only Kelly is correct.

D Only Sabrina and Kelly are correct.

E None of them is correct.

Question 2.19

You are given:

(i) $C(K,T)$ denotes the current price of a K-strike T-year European call option on a nondividend-paying stock.

(ii) $P(K,T)$ denotes the current price of a K-strike T-year European put option on the same stock.

(iii) S denotes the current price of the stock.

(iv) The continuously compounded risk-free interest rate is r, and $r > 0$.

Which of the following is (are) correct?

I $0 \le P(85,T) - P(80,T) \le 5e^{-rT}$

II $75e^{-rT} \le P(75,T) - C(80,T) + S \le 80e^{-rT}$

III $80e^{-rT} \le P(75,T) - C(80,T) + S \le 85$

A I only B II only C III only D I and II only E I and III only

Question 2.20

The following information was known at time 0:

(i) The continuously compounded risk-free interest rate is 9%.

(ii) The price of a stock is $80.

(iii) The stock pays 2 discrete dividends.

(iv) The stock pays a discrete dividend of $2 in 3 months.

(v) The stock pays a second discrete dividend of $1.90 in 9 months.

At time 0, a 1-year American call option was written on the stock. The strike price of the American call option was $86.

At time 2, you are informed that the call option did not expire worthless. Determine when the call was exercised.

A Time 0

B Time 3 months

C Time 9 months

D Time 12 months

E There is not enough information provided to answer this question.

Question 2.21

An insurance company sells single premium deferred annuity contracts with returns linked to a stock index, the time t value of one unit of which is denoted by $S(t)$. The contracts offer a minimum guaranteed return rate of 4%. At time 0, a single premium of P is paid by the policyholder and $P \times y\%$ is deducted by the insurance company. Thus, at the contract maturity date, T, the insurance company will pay the policyholder:

$$P(1-y\%) \times Max\left[\frac{S(T)}{S(0)},\ 1.04^T\right]$$

You are given the following information:

(i) The contract will mature in 2 years.

(ii) Dividends are not incorporated in the stock index. That is, the stock index is constructed such that the stock dividends are not reinvested.

(iii) The continuously compounded dividend yield of the index is 7% per year.

(iv) $S(0) = 50$

(v) The price of a 2-year European put option on the index with strike price of \$54.08 is \$8.96.

(vi) The continuously compounded risk-free interest rate is 6%.

Determine $y\%$ so that the insurance company does not make or lose money on this contract.

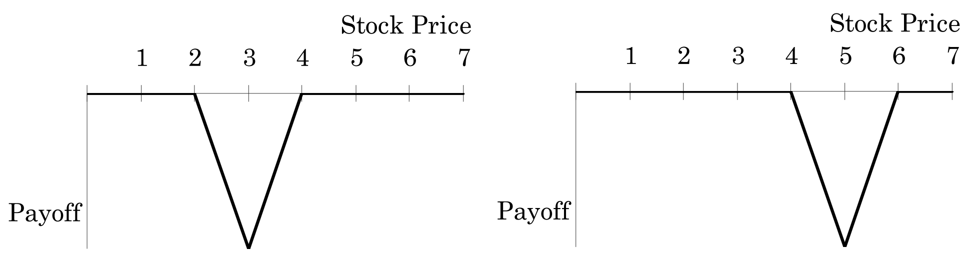
A 4.6% B 7.2% C 10.0% D 12.2% E 15.2%

Chapter 2 – Solutions

Solution 2.16

A Option Payoffs

A butterfly spread involves options with 3 different strike prices. Part one of the investor's position is a symmetric butterfly spread with strike prices \$2, \$3 and \$4. Part two of the investor's position is a symmetric butterfly spread with strike prices \$4, \$5 and \$6. The payoff when buying a butterfly spread is never less than zero. In this case, each butterfly spread was sold, which means the payoffs are less than zero. The payoff of the butterfly spread looks the same whether the position is composed of calls or puts. Part one of the position's payoff is shown as the bold line below at the left, and part two of the position's payoff is shown as the bold line below at the right.



When combined, the investor's position looks like Choice A.

Solution 2.17

D Comparing Options With Different Strikes and Maturities

The prices of the options decrease as time to maturity increases. Therefore, if the strike price increases at a rate that is less than the risk-free rate, then arbitrage is available. Option B expires 0.5 years after Option A, so let's accumulate Option A's strike price for 0.5 years at the risk-free rate:

$$50e^{0.06\times 0.5} = 51.5227$$

Since the strike price of Option B is \$52, which is greater than \$51.5227, arbitrage is not indicated by the prices of Option A and Option B.

Option C expires 0.5 years after Option B, so let's accumulate Option B's strike price for 0.5 years at the risk-free rate:

$$52e^{0.06\times 0.5} = 53.5836$$

Since the strike price of Option C is \$53, the strike price grows from time 1 to time 1.5 at a rate that is less than the risk-free rate of return. Consequently, arbitrage can be earned by purchasing Option C and selling Option B (i.e., buy low and sell high).

The arbitrageur buys the 2-year option for \$7.50 and sells the 1.5-year option for \$7.70 The difference of \$0.20 is lent at the risk-free rate of return.

The 1.5-year option

After 1.5 years, the stock price is \$52.50. Therefore, the 1.5-year option is exercised against the arbitrageur. The arbitrageur borrows a share of stock and sells it for the strike price of \$52. As a result, at the end of 2 years the arbitrageur owes the share of stock and has the accumulated value of the \$52. This position results in the following cash flow at the end of 2 years:

$$-52.50 + 52e^{0.06\times 0.5} = 1.0836$$

The 2-year option

The stock price of \$52.50 at the end of 2 years is less than the strike price of the 2-year option, which is \$53. Therefore, the 2-year call option expires worthless, and the resulting cash flow is zero.

The net cash flow

The net cash flow at the end of 2 years is the sum of the accumulated value of the \$0.20 that was obtained by establishing the position, the \$1.0836 resulting from the 1.5-year option, and the \$0.00 resulting from the 2-year option:

$$0.20e^{0.06\times 2} + 1.0836 + 0.00 = 1.3091$$

Solution 2.18

D Arbitrage

Let X be the number of puts with a strike price of \$60 that are sold for Jill's portfolio. The fact that the net cost of establishing the portfolio is zero allows us to solve for X:

$$-P(45) + 3P(55) + 1 - P(60)\times X = 0$$
$$-4 + 3\times 9 + 1 - 12X = 0$$
$$X = 2$$

An arbitrage strategy does not allow the cash flow at expiration to be negative. But suppose that only the \$60-strike put option is in-the-money at expiration. Since Jill is short the \$60-strike put option, this results in a negative cash flow. In particular, if the stock price at expiration is between \$55 and \$59, then the payoff from Jill's strategy will be negative. For example, if the stock price at expiration is \$57, the payoff from Jill's strategy will be:

$$-[\text{45-strike payoff}] + 3[\text{55-strike payoff}] - 2[\text{60-strike payoff}] + [\text{Proceeds from loan}]$$
$$0 + 0 - 2(60-57) + 1 = -5$$

Since Jill's strategy can result in a negative payoff at expiration, Jill's strategy is not arbitrage.

Let Y be the number of calls with a strike price of \$60 that are purchased for Sabrina's portfolio. The fact that the net cost of establishing the portfolio is zero allows us to solve for Y:

$$C(45) - 3C(55) + 1 + C(60) \times Y = 0$$
$$12 - 3 \times 7 + 1 + 4Y = 0$$
$$Y = 2$$

The table below shows that regardless of the stock price at time T, Sabrina's payoff is positive. Therefore, Sabrina is correct.

Sabrina's Portfolio		**Time *T***			
Transaction	**Time 0**	$S_T < 45$	$45 \le S_T \le 55$	$55 \le S_T \le 60$	$60 < S_T$
Buy 1 of $C(45)$	–12.00	0.00	$S_T - 45$	$S_T - 45$	$S_T - 45$
Sell 3 of $C(55)$	3(7.00)	0.00	0.00	$-3(S_T - 55)$	$-3(S_T - 55)$
Buy 2 of $C(60)$	–2(4.00)	0.00	0.00	0.00	$2(S_T - 60)$
Lend \$1	–1.00	1	1	1	1
Total	0.00	1	$1 + S_T - 45$	$1 + 120 - 2S_T$	1

Let Z be the number of calls purchased and puts sold with a strike price of \$55 for Kelly's portfolio.

Since the net cost of establishing the portfolio is zero, we can solve for Z:

$$2\left[C(60) - P(60)\right] + 1\left[C(45) - P(45)\right] + 2 - Z\left[C(55) - P(55)\right] = 0$$
$$2\left[4 - 12\right] + \left[12 - 4\right] + 2 - Z\left[7 - 9\right] = 0$$
$$-16 + 8 + 2 + 2Z = 0$$
$$Z = 3$$

In evaluating Kelly's portfolio, we can make use of the fact that purchasing a call option and selling a put option is equivalent to purchasing a prepaid forward on the stock and borrowing the present value of the strike price. We can see this by writing put-call parity as:

$$C_{Eur}(K,T) - P_{Eur}(K,T) = F_{0,T}^{P}(S) - Ke^{-rT}$$

Therefore, purchasing a call option and selling a put option results in a payoff of:

$$S_T - K$$

Since Kelly purchases offsetting amounts of puts and calls for any given strike price, we can use this result to evaluate her payoffs.

Kelly's Portfolio

Transaction	Time 0	Time T
Buy 2 of $C(60)$ & sell 2 of $P(60)$	$2(12.00-4.00)$	$2(S_T-60)$
Buy 1 of $C(45)$ & sell 1 of $P(45)$	$4.00-12.00$	S_T-45
Sell 3 of $C(55)$ & buy 3 of $P(55)$	$3(7.00-9.00)$	$-3(S_T-55)$
Lend $2	–2.00	2
Total	0.00	2

Kelly's portfolio is certain to have a positive payoff at time T, so Kelly is correct.

Solution 2.19

D Bounds on Option Prices

From Propositions 1 and 2, we see that Statement I is true:

Proposition 1: $P(K_2) \geq P(K_1)$ for $K_1 < K_2$

$\Rightarrow \quad 0 \leq P(85) - P(80)$

Proposition 2: $P_{Eur}(K_2) - P_{Eur}(K_1) \leq (K_2 - K_1)e^{-rT}$ for $K_1 < K_2$

$\Rightarrow \quad P_{Eur}(85) - P_{Eur}(80) \leq (85-80)e^{-rT}$

We make use of put-call parity for Statements II and III:

$$C(K) + K^{-rT} = S + P(K)$$

When the strike price for a call is increased, its price goes down, so the first inequality in Statement II is true:

$$P(75) - C(75) + S = 75e^{-rT} \quad \Rightarrow \quad P(75) - C(80) + S \geq 75e^{-rT}$$

When the strike price for a put is decreased, its price goes down, so the second inequality in Statement II is true:

$$P(80) - C(80) + S = 80e^{-rT} \quad \Rightarrow \quad P(75) - C(80) + S \leq 80e^{-rT}$$

As we saw in Statement II (directly above), the first inequality in Statement III is false:

$$P(80) - C(80) + S = 80e^{-rT} \quad \Rightarrow \quad P(75) - C(80) + S \leq 80e^{-rT}$$

The second inequalty in Statement III is true (but Statement III is still false):

$$P(80) - C(80) + S = 80e^{-rT} \quad \Rightarrow \quad P(75) - C(80) + S \leq 80e^{-rT} \quad \Rightarrow \quad P(75) - C(80) + S \leq 85$$

Solution 2.20

D Early Exercise of an American Call

If it is optimal to exercise an American call prior to maturity, then the early exercise takes place just before a dividend payment. Therefore the call is not exercised at time 0, and Choice A is not the correct answer.

The call is not exercised early if the present value of the interest on the strike exceeds the present value of the dividends:

$$K - Ke^{-r(T-t)} > PV_{t,T}(div) \qquad \Rightarrow \qquad \text{Don't exercise early}$$

Let's consider time 0.25:

$$86 - 86e^{-0.09(1-0.25)} > 2 + 1.9e^{-0.09(0.75-0.25)}$$
$$5.6134 > 3.8164$$

Since the present value of the interest on the strike exceeds the present value of the dividends, the call option is not exercised at time 0.25, so Choice B is not the correct answer.

Let's consider time 0.75:

$$86 - 86e^{-0.09(1-0.75)} > 1.9$$
$$1.9134 > 1.9$$

Since the present value of the interest on the strike exceeds the present value of the dividends, the call option is not exercised at time 0.75, so Choice C is not the correct answer.

Since the call was exercised, it must have been exercised at maturity, at the end of 12 months. Therefore, the correct answer is Choice D.

Solution 2.21

A Application of Option Pricing Concepts

We are told that the price of a European put option with a strike price of $54.08 has a value of $8.96. The payoff of the put option at time 2 is:

$$Max[0, 54.08 - S(2)]$$

Once we express the payoff of the single premium deferred annuity in terms of the expression above, we will be able to obtain the price of the annuity.

The payoff at time 2 is:

$$\begin{aligned}\text{Time 2 Payoff} &= P(1-y\%)\times Max\left[\frac{S(2)}{50},\ 1.04^2\right]\\ &= P(1-y\%)\times\frac{1}{50}Max\left[S(2),\ 50\times 1.04^2\right]\\ &= P(1-y\%)\times\frac{1}{50}Max\left[S(2),\ 54.08\right]\\ &= P(1-y\%)\times\frac{1}{50}\left\{Max\left[0,\ 54.08-S(2)\right]+S(2)\right\}\end{aligned}$$

The current value of a payoff of $Max\left[0,54.08-S(2)\right]$ at time 2 is \$8.96. The current value of $S(2)$ is its prepaid forward price:

$$F_{0,2}^{P}(S)=e^{-2\delta}S(0)=e^{-2\times 0.07}\times 50=43.4679$$

Therefore, the current value of the payoff is:

$$\text{Current value of payoff } = P(1-y\%)\times\frac{1}{50}\left\{8.96+43.4679\right\}$$

For the company to break even on the contract, the current value of the payoff must be equal to the single premium of P:

$$\begin{aligned}&P(1-y\%)\times\frac{1}{50}\left[8.96+43.4679\right]=P\\ &(1-y\%)\times 1.04856=1\\ &y\%=4.631\%\end{aligned}$$