Flag for Review

Record Notes

Communicate with the Professor

Monitor Difficulty

Only three assets A, B and F are being traded in the market.

Asset	A	B	F
Return	R_A	R_B	R_F
Expected return	5%	10%	3%
Variance of return	1%	9%	0%

The correlation between R_A and R_B is 0.5.

Determine the expected return of the market portfolio M, an efficient portfolio formed by only risky assets A and B.

A 6.7%

B 7.6%

C 8.5%

D 9.4%

E 10.3%

Helpful Strategies to Get You Started

Help Me Start

Derive the relationship between $E[R_P]$ and $SD(R_P)$, where R_P is the return of any portfolio P.

Comprehensive Solutions and Alternatives when Available

Solution

We know that F is a risk-free asset since $Var(R_F) = 0$.

Let $R_P = x_A R_A + x_B R_B$, where $x_A + x_B = 1$, be the return of any portfolio P formed by only A and B.

The expected return of P is $E[R_P] = 0.05x_A + 0.1(1 - x_A) = 0.1 - 0.05x_A$.

This gives $x_A = \frac{0.1 - E[R_P]}{0.05}$.

The standard deviation of return of P is

$SD(R_P)$

$= \sqrt{0.01x_A^2 + 0.09(1 - x_A)^2 + 2(0.5)(0.1)(0.3)x_A(1 - x_A)}$

$= \sqrt{0.01x_A^2 + 0.09(1 - x_A)^2 + 0.03x_A(1 - x_A)}$

$= \sqrt{0.07x_A^2 - 0.15x_A + 0.09}$

The relationship between $E[R_P]$ and $SD(R_P)$ is:

$SD(R_P) = \sqrt{0.07\left(\frac{0.1 - E[R_P]}{0.05}\right)^2 - 0.15\left(\frac{0.1 - E[R_P]}{0.05}\right) + 0.09}$

$= \sqrt{28E[R_P]^2 - 2.6E[R_P] + 0.07}$

Now consider any portfolio Q formed by F and P. The relationship between $E[R_Q]$ and $SD(R_Q)$ is:

$E[R_Q] = R_F + \frac{E[R_P] - R_F}{SD(R_P)} SD(R_Q)$

The slope is $k = \frac{E[R_P] - R_F}{SD(R_P)} = \frac{E[R_P] - 0.03}{\sqrt{28E[R_P]^2 - 2.6E[R_P] + 0.07}}$.

We need to find the value of $E[R_P]$ such that k is maximized. Take derivative with respect to $E[R_P]$, we have:

$k' = \left(28E[R_P]^2 - 2.6E[R_P] + 0.07\right)^{-\frac{3}{2}}(-0.46E[R_P] + 0.031) = 0$

$E[R_P] = 0.06739$ or 6.739%

The slope k is the highest when $E[R_P] = E[R_M]$, for which we have the capital market line, where E is any efficient portfolio, and M is the market portfolio formed by only A and B:

$E[R_E] = R_F + \frac{E[R_M] - R_F}{SD(R_M)} SD(R_E)$

Graphs and Other Solution Techniques Demonstrated when Applicable

Hence, $E[R_M] = 0.06739$ or 6.739%.