Exam C
Practice Exam #1

These practice exams should be used during the month prior to your exam.

This practice exam contains **20 questions**, of equal value,
corresponding to about a **2 hour** exam.

Each problem is similar to a problem in my study guides, sold separately.
Solutions to problems are at the end of each practice exam.

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Exam C, Practice Exam #1

1. One has observed the following distribution of insureds by number of claims:

Number of Claims	0	1	2	3	4	5	6 & +	All
Number of Insureds	511	912	727	342	81	8	0	2581

A Binomial Distribution with m = 5 is fit via the Method of Moments.
Which of the following is an approximate 90% confidence interval for q?
A. [0.290, 0.292]
B. [0.288, 0.294]
C. [0.286, 0.296]
D. [0.284, 0.298]
E. [0.282, 0.300]

2. You are given the following:
Prior to observing any data, you assume that the claim frequency rate per exposure has mean = 0.08 and variance = 0.12.
A full credibility standard is devised that requires the observed sample frequency rate per exposure to be within 10% of the expected population frequency rate per exposure 99% of the time.
You observe 112 claims on 1,000 exposures.
Estimate the number of claims you expect for these 1000 exposures next year.
A. 89 B. 91 C. 93 D. 95 E. 97

3. You are given the following:

- The losses in 2004 follow a LogNormal Distribution, with parameters $\mu = 5$ and $\sigma = 2.5$.
- Assume that losses increase by 5% from 2004 to 2005, 3% from 2005 to 2006, and 6% from 2006 to 2007.

What is the increase between 2004 and 2007 in the expected number of claims exceeding a 10,000 deductible?
A. Less than 4%
B. At least 4%, but less than 6%
C. At least 6%, but less than 8%
D. At least 8%, but less than 10%
E. At least 10%

4. The random variable N has a mixed distribution:
(i) With probability 0.3, N has a binomial distribution with $q = 0.2$ and $m = 3$.
(ii) With probability 0.5, N has a binomial distribution with $q = 0.4$ and $m = 4$.
(iii) With probability 0.2, N has a binomial distribution with $q = 0.6$ and $m = 5$.
What is Prob(N = 2)?
A. 21% B. 22% C. 23% D. 24% E. 25%

5. You observe the following 10 losses:

142	132	400	521	611
311	931	222	161	501

A distribution: $F(x) = 1 - (x/100)^{-\alpha}$, $x > 100$, is fit to this data via percentile matching at the 75th empirically smoothed percentile estimate. Determine the value of α.
A. less than 1.0
B. at least 1.0 but less than 1.3
C. at least 1.3 but less than 1.6
D. at least 1.6 but less than 1.9
E. at least 1.9

6. William M. Lowe, consulting actuary, works on each assignment in intervals.
The length in hours of these intervals has an Exponential Distribution with a mean of 2.
William bills each work interval at $500 per hour, excluding any fraction of an hour.
So if for example, a work interval lasts 2.7 hours, then the client is only billed $1000.
The number of work intervals per assignment is distributed as
a zero-truncated Geometric Distribution with $\beta = 4$.
Determine the average amount William bills per assignment.
A. Less than $3700
B. At least $3700, but less than $3800
C. At least $3800, but less than $3900
D. At least $3900, but less than $4000
E. At least $4000

7. A Tweedie Distribution can be represented as a Compound Poisson Distribution with a Gamma severity. If the Poisson frequency has a mean of λ, and the Gamma severity has parameters α and θ, determine the coefficient of variation of the Tweedie Distribution.

A. $\sqrt{\frac{\alpha}{\alpha+1}}$ B. $\sqrt{\frac{(\alpha+1)}{\lambda\,\alpha}}$ C. $\sqrt{\frac{\alpha(\alpha+1)}{\lambda}}$ D. $\sqrt{\frac{\lambda\,\alpha}{\alpha+1}}$ E. $\sqrt{\lambda\alpha(\alpha+1)\theta}$

8. An insurance company wishes to estimate its agent retention rate using data on all agents hired between 8 and 10 years ago. You are given:

- Using the Nelson-Aalen estimator, the company estimates the proportion of agents remaining after 4 years of service as S(4) = 0.60.
- Four agents resigned with between 4 and 6 years of service, each at a different length of service.
- Eleven of the studied agents have been employed by the insurance company for at least 6 years.

Determine the Nelson-Aalen estimate of S(6).
(A) Less than 0.46
(B) At least 0.46, but less than 0.48
(C) At least 0.48, but less than 0.50
(D) At least 0.50, but less than 0.52
(E) At least 0.52

9. You are given the following:

- Claims follow a Poisson Distribution, with a mean of 19 per year.
- The size of claims are given by an Exponential Distribution with mean 540.
- Frequency and severity are independent.

Given that during a year there are 12 claims of size less than 1000, what is the expected number of claims during that year?
(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

10. You are given:

(i) The number of claims for each policyholder has a binomial distribution with parameters m = 10 and q.

(ii) The prior distribution of q is beta with parameters a = 3, b (unknown), and θ = 1.

(iii) A randomly selected policyholder had the following claims experience:

Year	Number of Claims
1	4
2	x

(iv) The Buhlmann credibility estimate for the expected number of claims in Year 2 based on the Year 1 experience is 3.5.

(v) The Buhlmann credibility estimate for the expected number of claims in Year 3 based on the Year 1 and Year 2 experience is 4.0.

Determine x.

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

11. A random number 0.702 is generated from a uniform distribution on the interval (0, 1). Using the Inverse Transform Algorithm, determine the simulated value of a random draw from a LogNormal Distribution with μ = 8.21 and σ = 2.40.

A. Less than 13,000
B. At least 13,000, but less than 14,000
C. At least 14,000, but less than 15,000
D. At least 15,000, but less than 16,000
E. At least 16,000

12. You are given:

- Albino Insurance provides insurance to Unlucky Urkel for losses due to business interruption.
- The number of business interruptions suffered in a year by Unlucky Urkel is a Negative Binomial distribution with $r = 4$ and $\beta = 0.1$.
- The distribution of losses Unlucky Urkel suffers due to a single business interruption is:

x	Probability of x
10,000	0.3
20,000	0.2
30,000	0.2
40,000	0.1
50,000	0.1
100,000	0.1

- The number of business interruptions and the amounts of losses are independent.
- There is an annual deductible of 25,000.

What is the expected annual amount paid by Albino Insurance?

A. less than 5500
B. at least 5500 but less than 6000
C. at least 6000 but less than 6500
D. at least 6500 but less than 7000
E. at least 7000

13. You are given the following information:

- There are three types of drivers with the following characteristics:

Type	Portion of Drivers of This Type	Poisson Annual Claim Frequency
Good	50%	3%
Bad	30%	5%
Ugly	20%	10%

- A driver is observed to have two claims over an 8 year period.

Use Buhlmann Credibility to predict this driver's future annual claim frequency.

(A) 6% (B) 7% (C) 8% (D) 9% (E) 10%

14. The size of claim distribution for any particular policyholder is Normal with mean m and standard deviation 70. The m values of the portfolio of policyholders have probability density function:

$$f(m) = \frac{\exp[-\frac{(m-300)^2}{3200}]}{40\sqrt{2\pi}}, -\infty < m < \infty.$$

An insured has 3 claims of sizes 220, 250, and 310. What is the probability that this insured will have an expected future mean severity m between 240 and 260?

A. 12% B. 13% C. 14% D. 15% E. 16%

15. You are given:

(i) All members of a mortality study are observed from birth.
Some leave the study by means other than death.

(ii) $s_8 = 13$.

(iii) The following Kaplan-Meier product-limit estimates were obtained:
$S_n(y_7) = 0.800$, $S_n(y_8) = 0.720$, $S_n(y_9) = 0.612$

(iv) Between times y_8 and y_9, 17 observations were censored.

(v) Assume no observations were censored at the times of deaths.

Determine s_9.

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

16. You are given:

(i) The following are nine observed claim amounts:
200 500 500 1000 1000 1500 2000 3000 5000

(ii) An exponential distribution with $\theta = 1500$ is hypothesized for the data.

(iii) The goodness of fit is to be assessed by a p-p plot and a D(x) plot.

Let (s, t) be the coordinates of the p-p plot for a claim amount of 2000.

Determine (s - t) + D(2000).

(A) -0.010 (B) -0.005 (C) 0.000 (D) 0.005 (E) 0.010

17. You are given the following:

- θ is a parameter in a probability distribution function.
- There are three available estimators for θ: X, Y, and Z.
- The standard deviations of the estimators are:

  StdDev(X) = 90
  StdDev(Y) = 50
  StdDev(Z) = 110
- The difference between the expected value of the estimator and the true parameter are:

  Bias(X, θ) = 65

  Bias(Y, θ) = -100

  Bias(Z, θ) = 10
- You wish to rank the estimators in order to minimize the mean square error (MSE).

Choose the ranking of MSE(X), MSE(Y), and MSE(Z) from lowest to highest.
A. MSE(X) < MSE(Y) < MSE(Z)
B. MSE(X) < MSE(Z) < MSE(Y)
C. MSE(Y) < MSE(Z) <MSE(X)
D. MSE(Z) < MSE(Y) < MSE(X)
E. MSE(Z) < MSE(X) < MSE(Y)

18. Use the special algorithm for the (a, b, 0) class to simulate an observation from a Binomial Distribution with m = 4 and q = 0.3.
(Simulate the time of the first claim and the times between occurrences of subsequent claims.)
Use the following pseudorandom numbers as necessary: 0.28, 0.33, 0.46, 0.60, 0.12, 0.89.
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

19. An actuary Bailey Simon observed a sample of policyholders during the interval from age 77 to age 78 and found that 172 of them died in this age interval.
Based on the assumption of a constant hazard rate in this age interval, Bailey obtained a maximum likelihood estimate of 0.937 for the conditional probability that a policyholder alive at age 77 survives to age 78.
Using the delta method, calculate the estimate of the standard deviation of this maximum likelihood estimator.
(A) 0.005 (B) 0.007 (C) 0.009 (D) 0.011 (E) 0.013

20. Losses follow a mixture of two independent distributions A and B. You are given:
(i) Distribution A is Exponential.
(ii) Distribution B is Exponential, with a mean greater than that of Distribution A.
(iii) Weight 0.6 is assigned to distribution A.
(iv) The mean of the mixture is 180.
(iv) The variance of the mixture is 43,200.
Estimate the probability of a loss of size greater than 500, using the method of moments.
(A) 6.6% (B) 6.8% (C) 7.0% (D) 7.2% (E) 7.4%

END OF PRACTICE EXAM

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Solutions:

1. D. The estimated mean is:
{(0)(511) + (1)(912) +(2)(727) + (3)(342) + (4)(81) + (5)(8)} / 2581 = 1.4552.
Thus the point estimate of q is: $\overline{X}$/m = 1.4552/5 = 0.2910.

Var[$\hat{q}$] = Var[$\overline{X}$ / 5] = Var[$\overline{X}$]/5^2 = (Var[X]/n) / 25 = (Var[X]/2581)/25 = Var[X]/64,525
= 5(q)(1-q) / 64,525 = (0.2910)(1 - 0.2910)/12,905 = 0.00001599.

Standard Deviation is: $\sqrt{0.00001599}$ = 0.0040.

Thus an approximate 90% confidence interval for q is: 0.291 ± (1.645)(0.0040) = **0.291 ± 0.007.**
Alternately, here method of moments is equivalent to the method of maximum likelihood.

f(x) = (5!/{(5-x)!x!}) $q^x(1-q)^{5-x}$. ln f(x) = xlnq + (5-x)ln(1-q) + ln[5!] - ln[(5-x)!] - ln[x!].

$\frac{\partial \ln f(x)}{\partial q}$ = x/q - (5-x)/(1-q). $\frac{\partial^2 \ln f(x)}{\partial q^2}$ = -x/q^2 - (5-x)/$(1-q)^2$.

Information = -n E[$\frac{\partial^2 \ln f(x)}{\partial q^2}$] = -n{-E[x]/$q^2$ - (5-E[x])/$(1-q)^2$} =

(2581){1.4552/0.2910^2 + (5 - 1.4522)/$(1 - 0.2910)^2$} = 62,569.

Var[$\hat{q}$] = 1/62,569 = 0.00001598. Proceed as before.

Comment: Similar to Q. 2.4 in “Mahler’s Guide to Fitting Frequency Distributions.”

2. A. P = 99%. Therefore, y = 2.576, since Φ(2.576) = 0.995 = (1+P)/2. k = 0.10.

Standard For Full Credibility is: $(y / k)^2$ (σ_f^2/μ_f) = $(2.576/0.1)^2$(0.12/0.08) = 995 claims,

or 995/0.08 = 12,438 exposures.

Z = $\sqrt{1000 / 12{,}438}$ = 28.4%.

Estimated future frequency is: (28.4%)(112/1000) + (71.6%)(.08) = 8.91%.
Expected number of future claims is: (1000)(8.91%) = **89**.
Alternately, using the expected number of claims of (0.08)(1000) = 80, and the standard for full credibility in terms of expected claims: Z = $\sqrt{80 / 995}$ = 28.4%. Proceed as before.

Comment: Similar to Q. 6.7 in “Mahler’s Guide to Classical Credibility.”
As stated at page 29 of “Credibility” by Mahler and Dean, when available one generally uses the number of exposures (1000) or the expected number of claims (80) in the square root rule, rather than the observed number of claims (112), since the observed number of claims is subject to random fluctuation.

3. E. The inflation factor is: (1.05)(1.03)(1.06) = 1.1464.
In 2007, the losses follow a LogNormal Distribution, with parameters
μ = 5 + ln(1.1464) = 5.137, and σ = 2.5.

In 2004, S(10000) = 1 - Φ[ln(10,000) - 5)/2.5] = 1 - Φ(1.68) = 1 - 0.9535 = 0.0465.

In 2007, S(10000) = 1 - Φ(ln(10,000) - 5.137)/2.5) = 1 - Φ(1.63) = 1 - 0.9484 = 0.0516.

The increase in the expected number of claims exceeding a 10,000 deductible is:
0.0516/0.0465 - 1 = **11%**.
Alternately, a deductible of 10,000 in 2007 corresponds to a deductible of:
10000/1.1464 = 8723 in 2004.

In 2004, S(8723) = 1 - Φ[ln(8723) - 5)/2.5] = 1 - Φ(1.63) = 1 - 0.9484 = 0.0516.

The increase is: 0.0516/0.0465 - 1 = **11%**.
Comment: Similar to Q. 36.87 (4B, 5/99, Q.21) in "Mahler's Guide to Loss Distributions."

4. E. For q = 0.2 and m = 3, f(2) = (3)(0.2^2)(0.8) = 0.096.

For q = 0.4 and m = 4, f(2) = (6)(0.6^2)(0.4^2) = 0.3456.

For q = 0.6 and m = 5, f(2) = (10)(0.4^3)(0.6^2) = 0.2304.
Probability that the mixed distribution is 2 is:
(0.3)(0.096) + (0.5)(0.3456) + (0.2)(0.2304) = **0.2477**.
Comment: Similar to Q. 17.61 (SOA M, 11/06, Q.39 & 2009 Sample Q.288)
in "Mahler's Guide to Frequency Distributions."

5. A. The estimated 75th percentile is the (10 +1)(0.75) = 8.25 claim from smallest to largest.
The eighth claim is 521 and the ninth claim is 611.
Linearly interpolating, the estimated 75th percentile is: (0.75)(521) + (0.25)(611) = 543.5.

Set 1 - $(543.5/100)^{-\alpha}$ = 0.75.

Then $5.435^{-\alpha}$ = 0.25. Taking logarithms: -α ln(5.435) = ln(0.25).

Solve for α = -ln(0.25) / ln(5.435) = **0.82**.

Alternately, this is a Single Parameter Pareto Distribution with θ = 100.

As shown in Appendix A: $\text{VaR}_p(X) = \theta\,(1-p)^{-1/\alpha}$.

Set 543.5 = $\text{VaR}_{0.75}(X)$ = (100) $(0.25)^{-1/\alpha}$. $\Rightarrow$ α = -ln(0.25) / ln(5.435) = **0.82**.

Comment: Similar to Q. 23.2 in "Mahler's Guide to Fitting Loss Distributions."

6. C. If William works 0.6 hours he charges nothing.
So he starts charging when $x \geq 1$. If $x < 1$ he charges nothing.
Thus, the expected hours billed per work interval is:
$S(1) + S(2) + S(3) + ... = e^{-1/2} + e^{-2/2} + e^{-3/2} + ... = e^{-1/2} / (1 - e^{-1/2}) = 1 / (e^{0.5} - 1) = 1.541$.
Mean number of work intervals per assignment is the mean of
a zero-truncated Geometric distribution: $\dfrac{\beta}{1 - 1/(1+\beta)} = \dfrac{4}{1 - 1/5} = 5$.

The average amount William bills per assignment is: (\$500)(5)(1.541) = **\$3853**.
Alternately, the number of work intervals per assignment is 1 + a Geometric Distribution with $\beta = 4$.
Mean number of work intervals per assignment is: 1 + 4 = 5. Proceed as before.
Comment: Similar to Q. 33.33 (CAS3, 5/06, Q.38) in “Mahler’s Guide to Loss Distributions.”
The expected hours billed per work interval is what is called the curtate expectation of life at zero:
$e_0 = \sum_{t=1}^{\infty} S(t)$. The curtate expectation of life for a person alive at age x is the expected number of complete years remaining to live, the expected number of birthdays that the person will celebrate.

7. B. Mean = λ(1st moment of severity) = $\lambda\alpha\theta$.

Variance = λ(2nd moment of severity) = $\lambda\alpha(\alpha+1)\theta^2$.

$$CV = \sqrt{\frac{\lambda\alpha(\alpha+1)\theta^2}{(\lambda\alpha\theta)^2}} = \sqrt{\frac{(\alpha+1)}{\lambda\,\alpha}}.$$

Comment: Similar to Q. 5.12 in “Mahler’s Guide to Aggregate Distributions”.
The Tweedie Distribution is not on the syllabus of this exam.
Nevertheless, you are given enough information to solve this question.

8. A. $H(4) = -\ln S(4) = -\ln(0.6) = 0.5108$.
$H(6) = H(4) + 1/15 + 1/14 + 1/13 + 1/12 = 0.5108 + 0.2984 = 0.8092$.
$S(6) = \exp[-H(6)] = e^{-0.8092} =$ **0.445**.
Alternately, $S(6) = S(4) \exp[1/15 + 1/14 + 1/13 + 1/12] = 0.6\ e^{-0.2984} =$ **0.445**.
Comment: Similar to Q. 4.31 in “Mahler’s Guide to Survival Analysis.”
H(4) refers to the cumulative hazard rate at age 4.
H(6) refers to the cumulative hazard rate at age 6.
Between age 4 and age 6 we have four events, each at separate ages.
Here we are not assuming that events can only occur at integer ages.
In this case, the event is an agent resigning; this is analogous to a death.
"age" is really length of service of the agent with the company.
Let us assume the "deaths" were at ages: 4.4, 4.5, 4.9, and 5.3.
Then we get added into H(4) a term of $1/r_i$ at each of these four ages of deaths.

9. B. The large and small claims are independent Poisson Distributions. Therefore, the observed number of small claims has no effect on the expected number of large claims.

For the Exponential, $S(x) = \exp[-x/\theta]$. $S(1000) = e^{-1000/540} = 0.157$.

Thus the expected number of large claims is: (19)(0.157) = 3.0.

Given that we observe 12 small claims, the expected number of claims in total is: 12 + 3.0 = **15.0**.

Comment: Similar to Q. 4.11 in "Mahler's Guide to Frequency Distributions."

10. B. A Beta-Binomial; Bayes Analysis equals Buhlmann Credibility in this case.

Based on the first year of data: a' = 3 + 4 = 7, and b' = b + (10 - 4) = b + 6.

Therefore, the estimate for q is:

a'/(a' + b') = 7 / (7 + b + 6).

The estimate of the number of claims is 10 times that:

3.5 = (10)(7) / (7 + b + 6). $\Rightarrow$ b = 7.

Based on the first two years of data: a' = 7 + x, and b' = 7 + (10 - 4) + (10 - x) = 23 - x.

The estimate of the number of claims is:

4 = (10)(7 + x) / (7 + x + 23 - x) = (10)(7 + x)/30. $\Rightarrow$ x = **5**.

Alternately, for the Beta-Binomial, K = (a+b)/m = (3 + b)/10.

Thus for one year of data, Z = 1/(1 + K) = 10/(13 + b). 1 - Z = (3 + b) / (13 + b).

The prior mean frequency is: 10 a / (a+ b) = 30 / (3 + b).

Thus the estimate using one year of data is:

$$3.5 = \frac{10}{13 + b} 4 + \frac{3 + b}{13 + b} \frac{30}{3 + b} = \frac{70}{13 + b}. \Rightarrow b = 7.$$

$\Rightarrow$ K = (a+b)/m = 1. prior mean = 30 / (3 + b) = 3.

For two years of data, Z = 2/(2 + K) = 2/3.

Thus the estimate using two years of data is:

4 = (2/3)(4+x)/2 + (1/3)(3). $\Rightarrow$ x = **5**.

Comment: Similar to Q. 7.44 (4, 5/07, Q.15) in "Mahler's Guide to Conjugate Priors."

11. B. Set $0.702 = F(x) = \Phi[(\ln(x) - 8.21)/2.4]$.

Using the Standard Normal Table, $\Phi(0.53) = 0.702$.

(To more decimal places 0.7019 rather than 0.702.)

Therefore 0.53 = (ln(x) - 8.21)/2.4.

Therefore x = exp[(0.53)(2.4) + 8.21] = **13,121.**

Comment: Similar to Q. 8.2 in "Mahler's Guide to Simulation."

12. C. The mean severity is:
(0.3)(10,000) + (0.2)(20,000) + (0.2)(30,000) + (0.1)(40,000) + (0.1)(50,000)
+ (0.1)(100,000) = 32,000.
The mean frequency is: (4)(0.1) = 0.4.
Therefore, prior to a deductible, the mean aggregate losses are: (0.4)(3,2000) = 12,800.
The probability of no claims is 1.1^{-4} = 0.6830.
The probability of one claim is: $(4)(0.1)(1.1^{-5})$ = 0.2484.
The probability of two claims is: $(4)(5)(0.1^2)(1.1^{-6})/2$ = 0.0564.
Therefore, the probability of no aggregate losses is 0.6830.
Aggregate losses of 10,000 correspond to one interruption costing 10,000,
with probability: (0.3)(.2484) = 0.0745.
Aggregate losses of 20,000 correspond to either one interruption costing 20,000, or two interruptions each costing 10,000, with probability: $(0.2)(0.2484) + (0.3^2)(0.0564)$ = 0.0547.
Prob[aggregate losses ≥ 25,000] = Prob[aggregate losses ≥ 30,000] =
1 - (0.6830 + 0.0745 + 0.0547) = 0.1878.
Therefore, the limited expected value of aggregate losses at 25,000 is:
(0)(0.6830) + (10,000)(0.0745) + (20,000)(0.0547) + (25,000)(0.1878) = 6534.
Thus the expected losses excess of 25,000 are: 12,800 - 6534 = **6266**.
Comment: Similar to Q. 11.42 (Course 151 Sample Exam #2, Q.22)
in “Mahler’s Guide to Aggregate Distributions.”
The annual deductible applies on an aggregate basis rather than to each event.
What we need to do here is figure out the expected aggregate dollars paid per year.
"What is the expected annual amount paid by Albino Insurance?"
A different question could have asked instead: "What is the expected amount paid by Albino Insurance in those years in which it makes a positive payment?"
The probability of Albino Insurance making a positive payment in a year is 0.1878.
Therefore, the expected amount paid by Albino Insurance in those years in which it makes a positive payment is: 6266/0.1878 = 33,365.

13. B. EPV = 0.050.

VHM = $0.0032 - 0.05^2 = 0.0007$.

Type of Driver	A Priori Chance of This Type of Driver	Mean Annual Claim Freq.	Square of Mean Claim Freq.	Poisson Process Variance
Good	0.5	0.03	0.0009	0.03
Bad	0.3	0.05	0.0025	0.05
Ugly	0.2	0.10	0.0100	0.10
Average		0.050	0.00320	0.050

K = EPV / VHM = 0.05/0.0007 = 71.4.

Z = 8 / (8+71.4) = 10.1%.

Estimated frequency = (10.1%)(2/8) + (89.9%)(0.05) = **7.0%.**

Comment: Similar to Q. 9.5 in "Mahler's Guide to Buhlmann Credibility."

14. E. A Normal-Normal with prior Normal with mean 300 and standard deviation 40.
Where the greek letters refer to the prior Normal,
the posterior distribution is a Normal, with mean equal to:

$(L\sigma^2 + \mu s^2) / (C\sigma^2 + s^2) = \{(780)(40^2) + (300)(70^2)\} / \{(3)(40^2) + 70^2\} = 280.21$.

and variance equal to: $\sigma^2 s^2 / (C\sigma^2 + s^2) = (40^2)(70^2) / \{(3)(40^2) + 70^2\} = 808.25$.

Thus the probability that this insured will have an expected future mean severity between 240 and 260 is: $\Phi[(260 - 280.21)/\sqrt{808.25}] - \Phi[(240 - 280.21)/\sqrt{808.25}] =$

$\Phi(-0.71) - \Phi(-1.41) = 0.2389 - 0.0793 =$ **0.1596**.

Comment: Similar to Q. 10.12 in "Mahler's Guide to Conjugate Priors".

15. E. $0.72/0.8 = S_n(y_8)/S_n(y_7) = (r_8 - s_8)/r_8 = (r_8 - 13)/r_8. \Rightarrow 0.72r_8 = 0.8r_8 - 10.4. \Rightarrow r_8 = 130.$

$0.612/0.72 = S_n(y_9)/S_n(y_8) = (r_9 - s_9)/r_9. \Rightarrow 0.612r_9 = 0.72r_9 - 0.72s_9. \Rightarrow s_9 = 0.15r_9.$

However, $r_9 = r_8 - s_8 - 17 = 130 - 13 - 17 = 100. \Rightarrow s_9 = (0.15)(100) =$ **15**.

Comment: Similar to Q. 2.79 (4, 5/07, Q.38) in "Mahler's Guide to Survival Analysis."

16. D. The claim of size 2000 is the 7th out of 9, so the first coordinate of the p-p plot is:
$7/(9 + 1) = 0.7$. For the Exponential, $F(2000) = 1 - e^{-2000/1500} = 0.7364$.
Thus the point corresponding to the claim of size 2000 in the p-p plot is: (0.7, 0.7364).
The D(x) plot is the difference graph, the difference between the empirical and theoretical distribution functions. The empirical distribution function at 2000 is 7/9.
Therefore $D(2000) = 7/9 - 0.7364 = 0.0414$.
$(s - t) + D(2000) = (0.7 - 0.7364) + 0.0414 =$ **0.005**.
Comment: Similar to Q. 18.11 (4, 11/05, Q.31 & 2009 Sample Q.241)
in "Mahler's Guide to Fitting Loss Distributions."

17. E. In each case, MSE = Variance + Bias^2.
$MSE(X) = 90^2 + 65^2 = 12{,}325$.
$MSE(Y) = 50^2 + (-100)^2 = 12{,}500$
$MSE(Z) = 110^2 + 10^2 = 12{,}200$.
MSE(Z) < MSE(X) < MSE(Y).
Comment: Similar to Q. 26.71 (CAS3L, 11/11, Q.17)
in "Mahler's Guide to Fitting Loss Distributions."

18. C. $\lambda_k = c + d\,k$, where $d = \ln(1-q) = \ln(0.7) = -0.3567$, and $c = -md = -4\ln(0.7) = 1.4267$.

$\lambda_0 = 1.4267 + (-0.3567)(0) = 1.4267$.

Using the first random number, the time of the first claim is: $-\ln(1 - 0.28) / 1.4267 = 0.2303$.

$\lambda_1 = 1.4267 + (-0.3567)(1) = 1.0700$.

Using the next random number, the time between the 1st and 2nd claim is:
$-\ln(1 - 0.33) / 1.0700 = 0.3743$.
Thus the second claim occurs at: $0.2303 + 0.3743 = 0.6045$.

$\lambda_2 = 1.4267 + (-0.3567)(2) = 0.7133$.

Using the next random number, the time between the 2nd and 3rd claim is:
$-\ln(1 - 0.46) / 0.7133 = 0.8638$.
Thus the third claim occurs at: $0.6045 + 0.8638 = 1.4683 > 1$. Reject the 3rd claim.
We have simulated **two** claims from this Binomial Distribution.
In spreadsheet form, the entire calculation is:

i	Lambda	Uniform Random Number ui	Interevent Time si = -Ln(1-ui) / lambda	Cumulative Interevent Time Sum of si
1	1.4267	0.280	0.2303	0.2303
2	1.0700	0.330	0.3743	0.6045
3	0.7133	0.460	0.8638	1.4683

The third claim first exceeds time 1, so we have simulated **2** claims from this Binomial Distribution.
Comment: Similar to Q. 13.22 in "Mahler's Guide to Simulation."
Similar to Exercise 20.5 in Loss Models.

19. A. Under the assumption of a constant hazard rate on the age interval (j, j+1],
the exact exposure estimate, $\hat{q}_j = 1 - \exp[-d_j/e_j]$, corresponds to the maximum likelihood estimate.

Thus, $1 - 0.937 = 0.063 = 1 - \exp[-172 / e_{77}]$. $\Rightarrow e_{77} = 2643.2$.

For the exact exposures method over an interval of width one:
$\text{Var}[\hat{p}_x] = \text{Var}[\hat{q}_x] = \hat{p}_x^2\, d_x / e_x^2 = (0.937^2)\,(172) / 2643.2^2 = 0.000021615$.

$\sqrt{0.000021615} = \mathbf{0.00465}$.
Comment: Similar to Q. 8.23 (Exam C 2014 Sample Q.303)
in "Mahler's Guide to Survival Analysis."
More generally over an interval from a to b, for the exact exposure method:
$\hat{q} = 1 - \exp[-(b-a)\,d / e]$, and $\text{Var}[\hat{q}] = (1 - \hat{q})^2\,(b - a)^2\,d / e^2$. $\text{Var}[\hat{p}] = \text{Var}[\hat{q}]$.

This formula for $\text{Var}[\hat{q}]$ is derived using the delta method.

20. D. Mean of the mixed distribution is: $(\theta_A)(0.6) + (\theta_B)(0.4) = 180. \Rightarrow \theta_B = 450 - 1.5\theta_A$.

2nd Moment of the mixed distribution is: $(2\theta_A^2)(0.6) + (2\theta_B^2)(0.4) = 43{,}200 + 180^2 = 75{,}600$.

$\Rightarrow 0.6\theta_A^2 + 0.4(450 - 1.5\theta_A)^2 = 37{,}800. \Rightarrow 1.5\theta_A^2 - 540\theta_A + 43{,}200 = 0$.

$\Rightarrow \theta_A = \{540 \pm \sqrt{540^2 - (4)(1.5)(43{,}200)}\} / \{(2)(1.5)\} = 120 \text{ or } 240$.

If $\theta_A = 120$, then $\theta_B = 450 - 1.5\theta_A = 270 > \theta_A$. Okay.

If $\theta_A = 240$, then $\theta_B = 450 - 1.5\theta_A = 90 < \theta_A$. Not Okay.

$S(500) = 0.6e^{-500/120} + 0.4e^{-500/270} =$ **7.21%**.

Comment: Similar to Q. 9.19 in "Mahler's Guide to Fitting Loss Distributions."

While these solutions are believed to be correct, anyone can make a mistake.
If you believe you've found something that may be wrong, send any corrections or comments to:

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